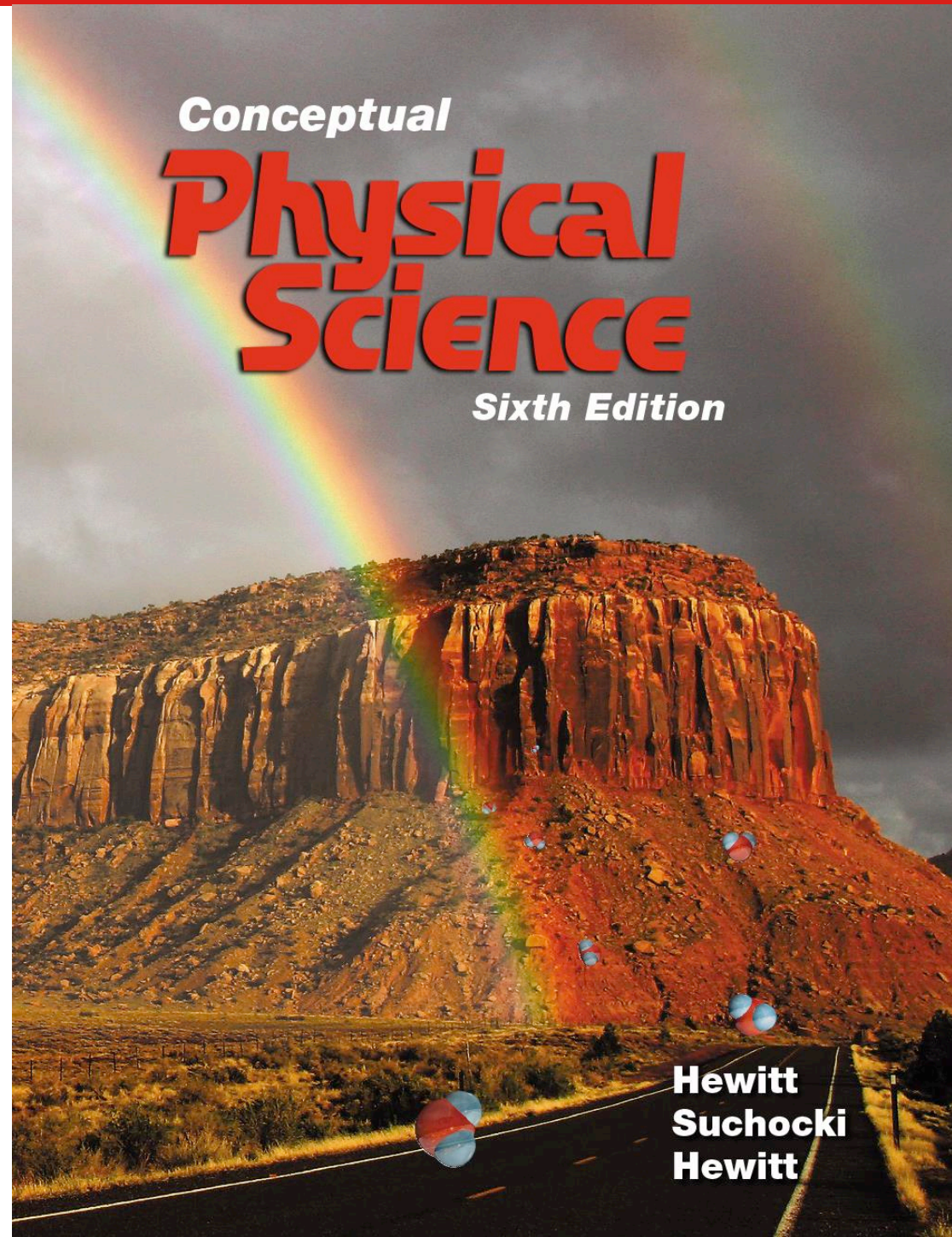


**Chapter 5:
Fluid
Mechanics**



Outline

- Density
- Pressure
- Buoyancy in a Liquid
- Archimedes' Principle
- Atmospheric Pressure
- Pascal's Principle
- Bernoulli's Principle

quiz 2 material
⊗ + part 2 of CH3

mass occupied per space
mass per vol

Density

Density → how much mass occupied
vol → how much space occupied

- **Density**: is the Measure of **compactness** which is calculated as mass per unit volume.
- In other words, **density is a measure of how much mass occupies a given space**. It is the amount of matter per unit volume. That is:

calculating mass

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

calculating by determining volume

$$\rho = \frac{m}{V}$$

- Units of density
 - grams per cubic centimeter (g/cm^3) → non SI-units but common
 - kilograms per cubic meter (kg/m^3) → the SI-units
- Note that $1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$

Density

- Densities of some materials

TABLE 5.1 DENSITIES

Material (kg/m³)

Solids

Iridium	22,650
Osmium	22,610
Platinum	21,090
Gold	19,300
Uranium	19,050
Lead	11,340
Silver	10,490
Copper	8,920
Iron	7,870
Aluminum	2,700
Ice	919

Liquids

Mercury	13,600
Glycerin	1,260
Seawater	1,025
Water at 4°C	1,000
Ethyl alcohol	785
Gasoline	680

Gases (g/cm³ at sea level)

Dry air:

at 0°C	1.29
at 10°C	1.25
at 20°C	1.21
Helium	0.178
Hydrogen	0.090
Oxygen	1.43

Density

$$\text{Density} = \frac{\text{Mass}}{\text{Vol}} = \frac{\text{kg/g}}{\text{m}^3/\text{cm}^3} \rightarrow \frac{\text{N}}{\text{m}^3}$$

- Weight density

$$\text{Weight Density} = \frac{\text{Weight}}{\text{Volume}}$$

- This quantity is commonly used when discussing liquid pressure.
- Units of weight density
 - kilograms per cubic meter (N/m^3) \rightarrow the SI-units

\hookrightarrow only measured through SI-units

Density

The density of 1 kilogram of iron is

- A. less on the Moon.
- B. the same on the Moon.
- C. greater on the Moon.
- D. All of the above.

Density

The density of 1 kilogram of iron is

- A. less on the Moon.
- B. the same on the Moon.**
- C. greater on the Moon.
- D. All of the above.

Explanation:

Both mass and volume of 1 kilogram of iron is the same everywhere, so density is the same everywhere.

Density

- The density of 2 kg of water = the density of 10 kg of water
 - because the ratio of mass to volume of a certain material is constant
- The density of an entire candy bar = the density of half candy bar *mass + volume are directly proportional*
- The density of 3 kg Lead > the density of 150 kg of Aluminum
 - Density depends on the material itself, not on how much of materials you compare

Density

- What happens to the volume of a loaf of bread when it is squeezed? what happens to the mass? What happens to the density?
 - when squeezing the bread, the volume will be reduced and the mass will stay the same, so the density will increase

volume reduced
density \uparrow

mass \rightarrow same
volume \rightarrow less



Notes on unit conversion

- $1 \text{ kg} = 1000 \text{ g}$
- $1 \text{ m} = 100 \text{ cm}$
- $1 \text{ m}^3 = 10^6 \text{ cm}^3 \rightarrow 1 \text{ m} = 100 \text{ cm}$, thus $1 \text{ m}^3 = (10^2)^3 \text{ cm}^3 = 10^6 \text{ cm}^3$
- The conversion from g/cm^3 to kg/m^3 (or the other way around) is done in steps:

$$1 \frac{\text{kg}}{\text{m}^3} = 1 \frac{\cancel{\text{kg}}}{\cancel{\text{m}^3}} \times \left[\frac{1000\text{g}}{1\cancel{\text{kg}}} \right] \times \left[\frac{1\cancel{\text{m}^3}}{10^6\text{cm}^3} \right]$$

Conversion factor
for kg

conversion factor
for m^3

- Therefore:

$$1 \frac{\text{kg}}{\text{m}^3} = \overset{\div 1000}{1} \frac{\text{g}}{1000 \text{ cm}^3}$$

$$1 \frac{\text{g}}{\text{cm}^3} = \overset{\times 1000}{1000} \frac{\text{kg}}{\text{m}^3}$$

Examples

- Calculate the density of an object that weighs 30 N and has a volume of 100 cm³

(Hint: check the units if they are SI-units or not, carry conversion if needed)

$$V = 100 \text{ cm}^3 = 100 \times 10^{-6} = 10^{-4} \text{ m}^3 \quad (1 \times 10^{-4} \text{ m}^3)$$

$$\text{weight density} = \frac{30 \text{ N}}{10^{-4} \text{ m}^3} = 3 \times 10^5 \frac{\text{N}}{\text{m}^3}$$

- Calculate the mass of an object that has a volume of 3000 cm³ and density of 1000 kg/m³

$$D = \frac{M}{V} = 1000 \frac{\text{kg}}{\text{m}^3}$$

kg
m³

$$V = 3000 \text{ cm}^3 = 3000 \times 10^{-6}$$

$$m = D \times V$$
$$m = 1000 \frac{\text{kg}}{\text{m}^3} \times$$
$$= 3 \text{ kg}$$

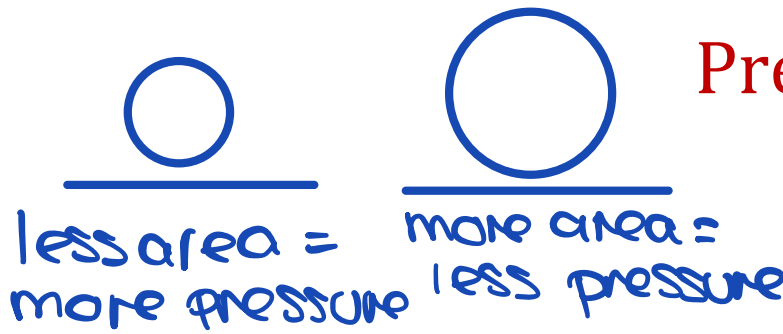
$$= 3 \times 10^{-3} \text{ m}^3$$

Pressure

* note: the pressure is a scalar quantity
(magnitude without a direction)

- **Pressure:** Force per unit area

$A \uparrow \quad P \downarrow$
 $A \downarrow \quad P \uparrow$
 $\left. \begin{array}{l} \rightarrow l \times w \\ \text{Force} \\ \perp \text{ to} \\ \text{area} \end{array} \right\}$



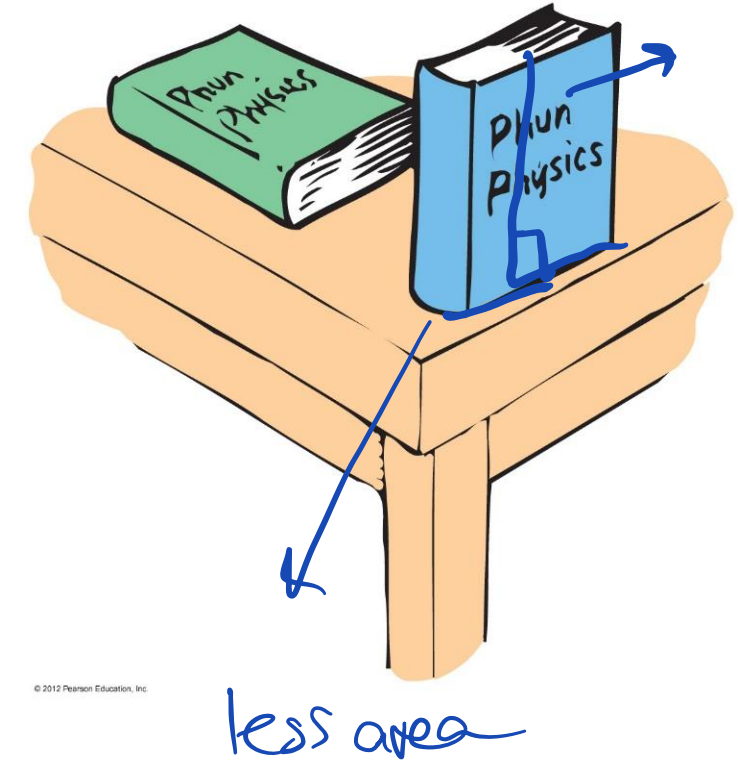
Pressure = $\frac{\text{Force}}{\text{Area}}$
 $P = \frac{F}{A}$

Units: $\text{N/m}^2 = \text{Pascal (Pa)}$

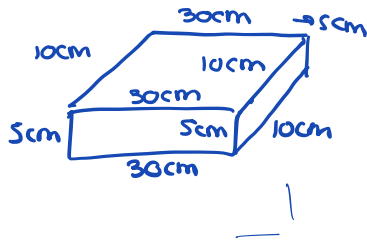
$P = \frac{F}{A} = \frac{N}{m^2} = \text{Pa}$

- Although the weight of both books is the same, the upright book exerts greater pressure against the table.

↓ less area



A brick of mass = 3 kg has its dimensions
30 cm x 10 cm x 5 cm



$$A \rightarrow m^2$$

$$1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$$

$$= 10^{-2} \times 10^{-2} \text{ m}$$

$$1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

a) Calculate the maximum pressure?

$$P = \frac{F}{A} \quad \frac{30}{\quad}$$

↳ indicates minimum area will be calculated

$$A = 5 \text{ cm} \times 10 \text{ cm} = 50 \text{ cm}^2$$

l x w

Pressure = $\frac{F}{\text{Area}}$
↙
Inverse relationship

$$50 \times 10^{-4} \text{ m}^2 = 5 \times 10^{-3} \text{ m}^2$$

max pressure =
min area
↓
L x W

OR

$$A = \frac{5}{100} \text{ m} \times \frac{10}{100} \text{ m} = 0.05 \times 0.1$$

$$= 5 \times 10^{-3} \text{ m}^2$$

$$P = \frac{F}{A} = \frac{3 \times 10}{5 \times 10}$$

m x a

$$\frac{30 \text{ N}}{5 \times 10^{-3} \text{ m}^2} \text{ m}^2$$

b) calculate minimum pressure

Area is max
inversely proportional
to pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$= 30 \times 10 = 300 \text{ cm}^2$$

$$300 \text{ cm}^2 \times 10^{-4} \text{ m}^2$$

$$\text{Pressure} = \frac{3 \times 10}{3 \times 10^{-2} \text{ m}^2} = \frac{30}{3 \times 10^{-2} \text{ m}^2} = 1000 \text{ Pa}$$

OR

$$\frac{30}{100} \times \frac{10}{100} = 0.3 \text{ m}^2 \quad \frac{30 \times 10}{100 \times 100}$$

$$= \frac{30}{0.3} = 1000 \text{ Pa}$$

$$0.3 \times 0.01 = 0.3 \text{ m}^2$$

$$\frac{30}{0.3} = 1000$$

An Elephant of mass = 5500 kg is standing on his 4 legs. Each leg has an area of 275 cm²

Find the pressure exerted by the elephant on the ground if:

a) standing on four legs

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \frac{\text{N}}{\text{m}^2}$$

$$1 \text{ cm}^2 = 10^{-4} \text{ m}^2 \\ = \frac{55000}{\quad}$$

$$4 \times 275 \text{ cm}^2$$

$$4 \times 275 \times 10^{-4} \text{ m}^2 = 0.11 \text{ m}^2 \rightarrow \text{Area}$$

5500 \rightarrow mass in kg $\rightarrow \times 10$ to form this value into Newton
5500 $\times 10 = 55000 \text{ N}$

Calculate pressure based on the results

$$\frac{55000}{0.11} = 500000 = 5 \times 10^5 \text{ Pa}$$

b) Standing on two legs

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \frac{\text{N}}{\text{m}^2}$$

$$2 \times 275 \text{ cm}^2$$

$$2 \times 275 \times 10^{-4} \text{ m}^2 = 0.055 \text{ m}^2 \rightarrow \text{Area}$$

5500 \rightarrow mass in kg $\rightarrow \times 10$ to form this value into Newton
5500 $\times 10 = 55000 \text{ N}$

$$P = \frac{m}{V}$$

Calculate pressure based on the results

$$\frac{55000}{0.055} = 1000000 = 1 \times 10^6 \text{ Pa}$$

Deriving formula of pressure due to liquid $\rho \times h \times g$

$$P = \frac{F}{A} = \frac{m \times g}{A} = \frac{(\rho \times \cancel{A} \times h) \times g}{\cancel{A}} = \frac{(\rho \times \cancel{A} \times h) \times g}{\cancel{A}}$$

$$m = \rho V \quad \frac{\rho \times A \times h \times g}{(\rho V) \times g \quad A}$$

$$m = (\rho \times V) \times g \quad P = \rho h g$$

Pa due to liquid

$$P_a = \rho h g$$

$$P = \frac{\rho \times \cancel{A} \times h \times g}{\cancel{A}}$$

$$P = \frac{m}{V}$$

$$\rho V = m$$

Pressure in a liquid

$$F/A$$

Liquid Pressure:

- Force per unit area that a liquid exerts on something
- Liquid pressure is caused by the weight of the liquid above

$$\begin{aligned} \text{Liquid pressure} &= \left(\overset{= \text{W density}}{\text{density} \times \text{acceleration of gravity}} \right) \times \text{depth} \\ &= \text{weight density} \times \text{depth} \end{aligned}$$

$$\text{Liquid pressure} = \rho g h$$

h : depth

g : acceleration of gravity = $10 \frac{\text{m}}{\text{s}^2}$

ρ : density

- Liquid pressure depends on the density of the liquid and the depth in the liquid. It does not depend on the surface area.

Liquid Pressure

- The force of gravity acting on the water in a tall tower produces pressure in pipes below that supply many homes with reliable water pressure.

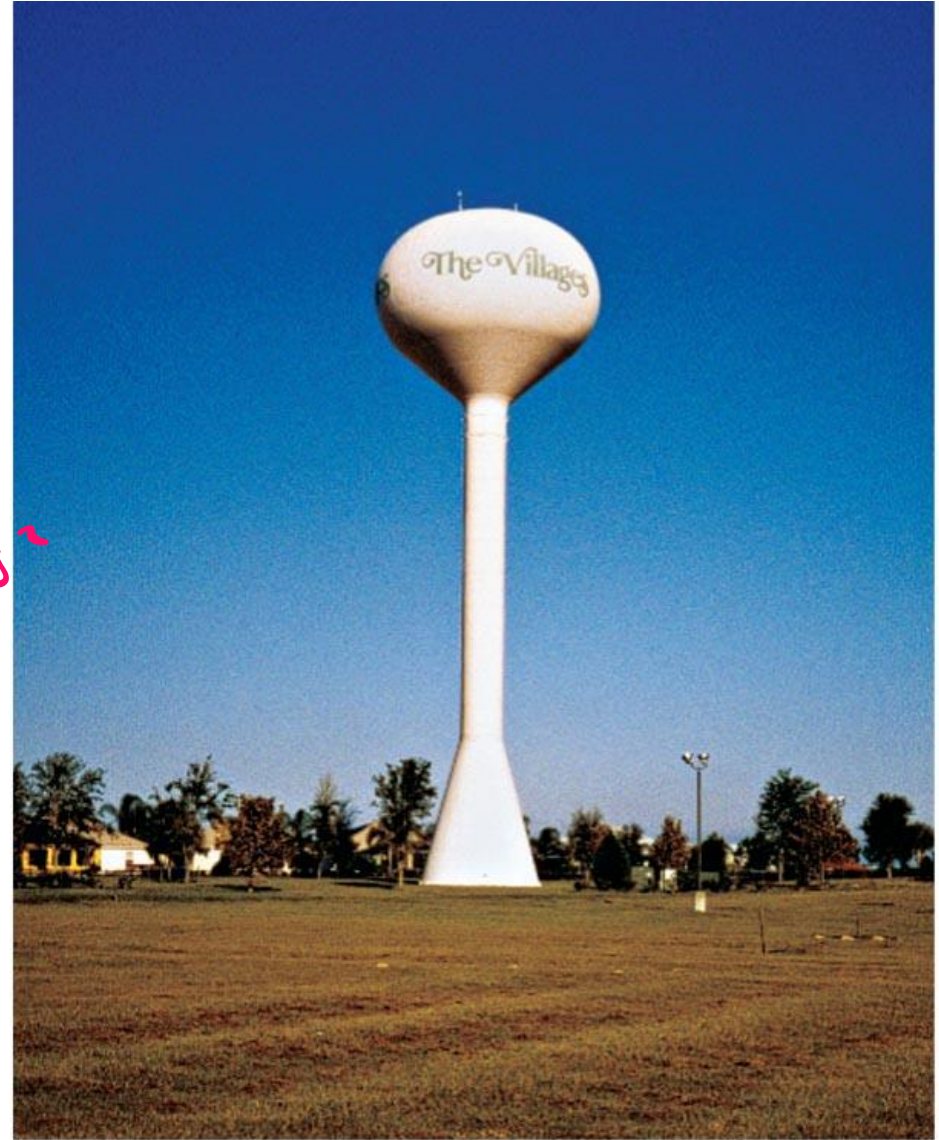
$$\text{weight} = 30\text{N} = 3\text{kg} \times 10\text{ m/s}^2$$

$$\text{volume} = 100\text{cm}^3$$

$$100 \times 10^{-6}\text{ m}^3$$

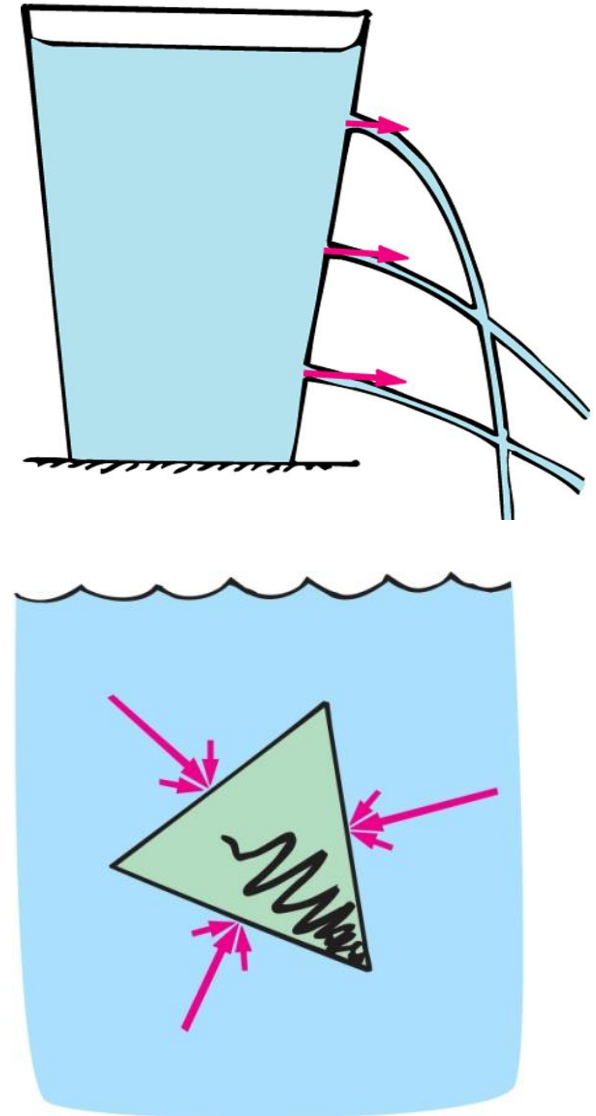
$$\rho = \frac{\text{mass}}{\text{volume}}$$

$$\rho_w = \frac{\text{weight}}{\text{vol}} = \frac{30\text{N}}{100 \times 10^{-6}\text{ m}^3}$$



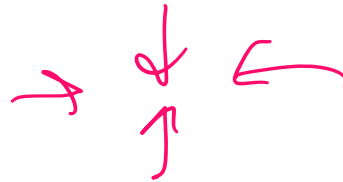
Liquid Pressure

- Pressure increases with depth in the liquid
- When liquid presses against any surface, its net force is directed perpendicular to the surface.



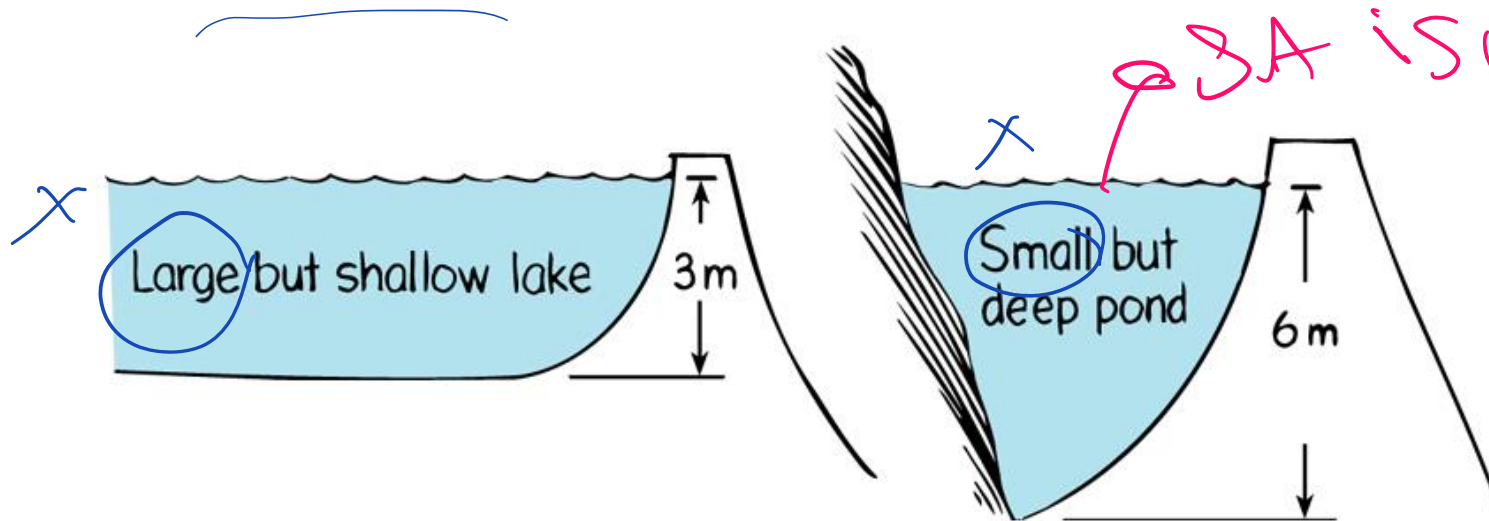
Liquid Pressure

- Liquid pressure on a certain point is exerted equally in all directions, not only downward
- Examples:
 - your ears feel the same amount of pressure under water no matter how you tip your head
 - bottom of a boat is pushed upward by water pressure
 - pressure acts upward when pushing a beach ball under water



Example

- Which exerts more pressure on a dam: a 3 meter deep lake or a 6 meter deep small pond?



$$P_{\text{due to liquid}} = \frac{\text{kg}}{\text{m}^3} (10 \text{ m/s}^2) (\text{m})$$
$$P_{\text{due to liquid}} = \rho \times g \times h$$

Example

$$\rho = 0.6 \frac{\text{g}}{\text{cm}^3} \times 1000 = 600 \frac{\text{kg}}{\text{m}^3}$$

$$600 \times 10 \times 3$$

$$6000 \times 3 = 18000 \text{ Pa}$$

- A rectangular swimming pool of length 40 m and width 20 m is 3 m deep and filled with oil of density 0.6 g/cm^3 , what is the pressure at the bottom of the pool?

$$\text{Density} = 0.6 \frac{\text{g}}{\text{cm}^3} \times 1000 = 600 \frac{\text{kg}}{\text{m}^3} = \rho_{\text{oil}}$$

$$P_{\text{oil}} = 600 \frac{\text{kg}}{\text{m}^3} \times g \times h = 600 \frac{\text{kg}}{\text{m}^3} \times 10 \times 3 \text{ m}$$

$$\downarrow P_{\text{oil}}$$

$$P_{\text{oil}} = 18000 \text{ Pa}$$

$$6000 \times 3$$

$$4 - 1.5 = 2.5 \text{ m}$$

Examples

$$2.5 \times 10 \times 1000$$

$$25 \times 1000$$

$$25000 \text{ Pa}$$

⊗ VERY IMPORTANT EXERCISES

- Find the water pressure at the bottom of the 50 meter water tower given that the weight density for water is 9800 N/m^3 .

$$P_w = \rho_w \times g \times h = 9800 \times 50$$

$$\text{kg} \times 10 = \text{N}$$

$$= \rho \times g$$

$$\text{or } 980 \times 10 \times 50 = 490,000 \text{ Pa} = 4.9 \times 10^5 \text{ Pa}$$

- A 1.5 m thick layer of oil floats above the surface of a swimming pool. What is the liquid pressure 4 m deep? Given that the density of water is 1000 kg/m^3 and the density of oil is 600 kg/m^3 . (34 kPa)

$$P_{\text{liquid}} = P_{\text{oil}} + P_{\text{water}}$$

$$1.5 \times 10 \times 600$$

$$P_{\text{liquid}} = \rho_{\text{oil}} \times g \times h_{\text{oil}} + \rho_w \times g \times h_{\text{water}}$$

$$1.5 \times 6000$$

$$P_{\text{liquid}} = 600 \times 10 \times 1.5 + 1000 \times 10 \times 2.5 = 34,000 \text{ Pa}$$

$$P_{\text{water}} + P_{\text{oil}} = P_{\text{liquid}} = 34 \text{ kPa}$$

$$4 - 1.5 = 2.5 \text{ m}$$

$$\rho_{\text{oil}} = 600 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

Extra Exercise

1) calculate the depth of water where the pressure due to water is **150 kPa**

$$P_{\text{liquid}} = \rho g h \rightarrow \text{depth}$$

↓ ↓
density gravity

$$P_{\text{liquid}} = \rho (10) h$$

$$150 \text{ kPa} = \rho (10) h$$

$$\frac{150,000}{10} = \frac{\rho (10) h}{10}$$

$$15000 = \rho h$$

$$= 1000 \times 15$$

$$\rightarrow h = 15 \text{ m}$$

b)

$$\text{pressure} = 2.5 \text{ kPa}$$

$$\times 1000 = 2500 \text{ Pa}$$

$$\text{depth} = 12.5 \text{ m}$$

$$\text{Pressure} = \rho g h$$

$$2500 = \rho (10) (12.5)$$

$$\frac{2500}{125} = \frac{\rho (125)}{125} \rightarrow \rho = 20 \frac{\text{kg}}{\text{m}^3}$$

$$h_{\text{oil}} = 1.5 \text{ m}$$

$$h_{\text{water}} = 4 - 1.5 = 2.5 \text{ m}$$

$$\rho_w = \frac{1 \text{ g}}{\text{cm}^3} \times \frac{1000 \text{ kg}}{\text{m}^3}$$

$$\frac{0.001 \times 1000}{10^6}$$

$$\frac{1 \text{ g}}{\text{cm}^3} \times \frac{1000 \text{ kg}}{\text{m}^3}$$

$$\frac{1000 \text{ kg}}{\text{m}^3} = \text{density}$$

$$150 \text{ kPa} = 150000 \text{ Pa}$$

$\times 1000 = \text{pressure}$

$$P_{\text{liquid}} = \rho \times g \times h$$

$$150000 = 1000 \times 10 \times h$$

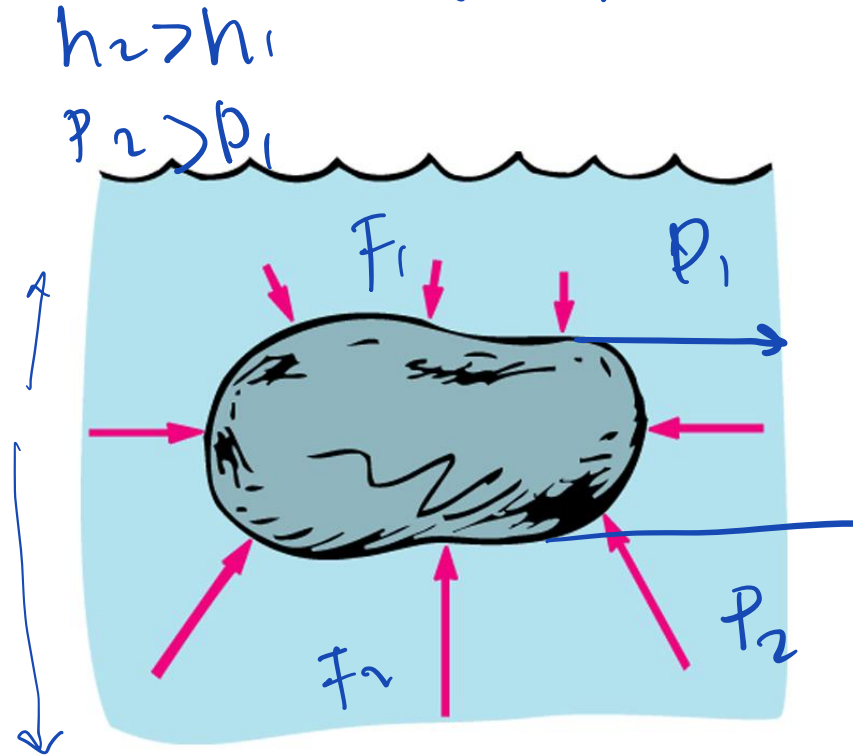
$$\frac{150000}{10000} = \frac{10000 h}{10000}$$

$$\boxed{h = 15 \text{ m}}$$

Buoyancy in Liquids

Volume of displaced =
submerged/immersed
volume

- Buoyancy: is the apparent loss in weight of submerged objects.
 - It is easier to lift a boulder in water than out of water
- Buoyancy exists because the pressure against the bottom of the object is greater since it is deeper. There is a net upward force which is called the buoyant force.



↑ pressure liquid = ρgh ↑

$$\frac{F}{A} \rightarrow F_2 > F_1$$

high density = more float

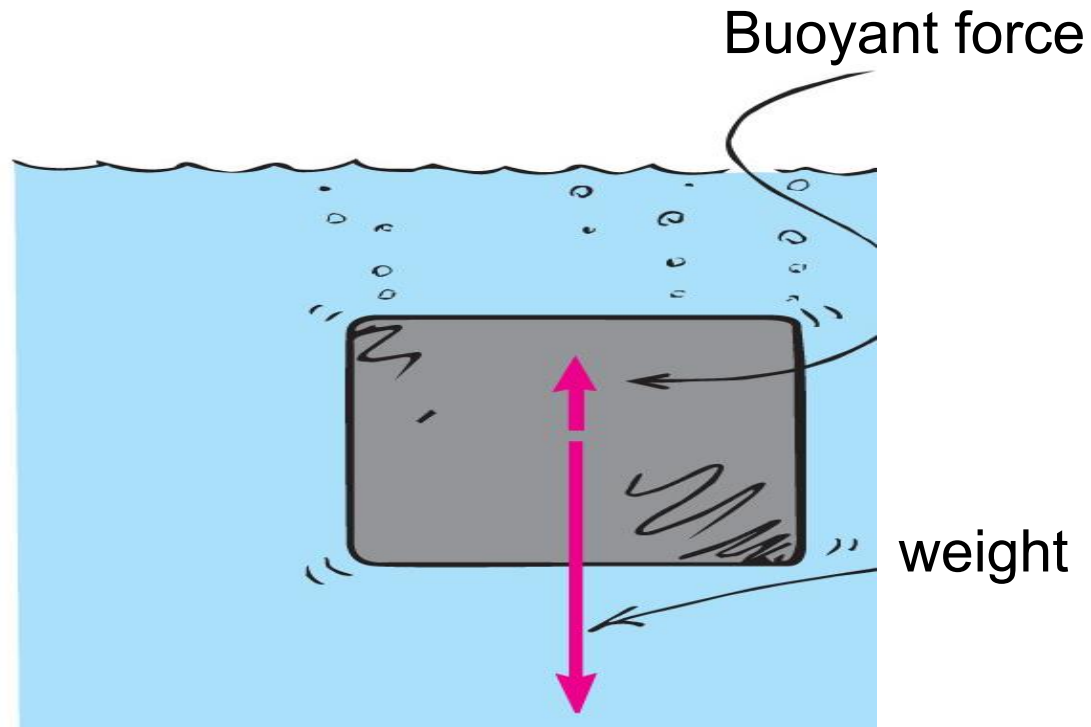
$F_{\text{Buoyant force}} / F_{\text{upthrust force}}$ is

$$V_{\text{displaced liquid}} = V_{\text{immersed / submerged object}}$$

Buoyancy in Liquids

upwards

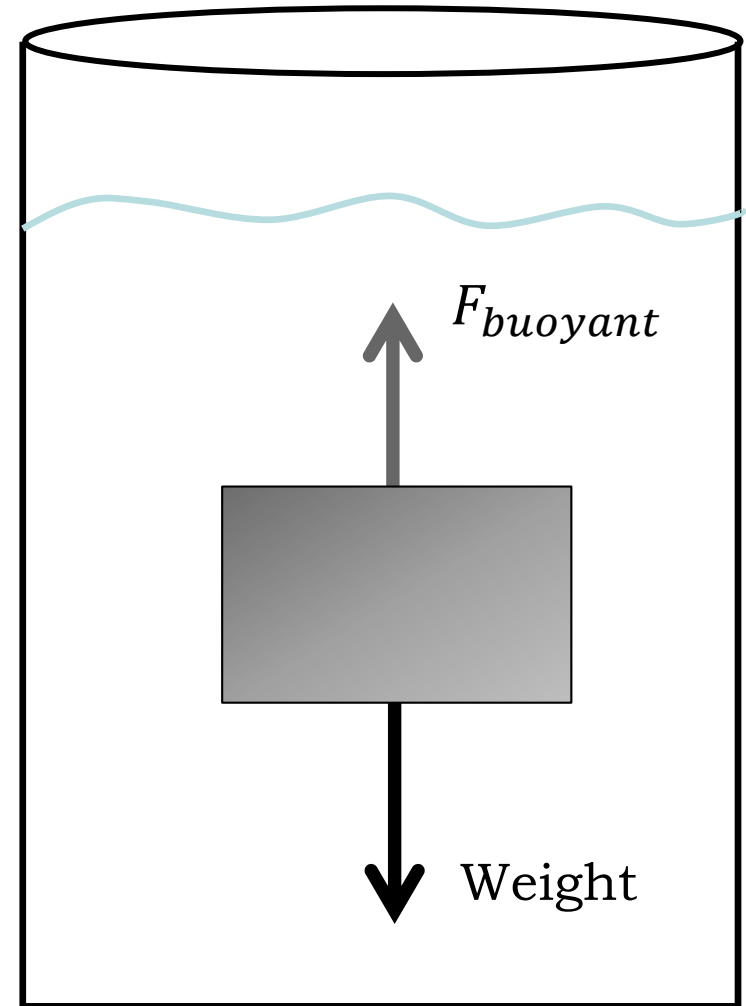
- The buoyant force is an upward force exerted by a fluid, that opposes the weight of an immersed object



Buoyancy in Liquids

- If the buoyant force is less than the weight of the object it will **sink** (completely submerged)
- If it is equals to its weight, the object will remain at any level (also submerged)
- If the buoyant force is grater than the weight of the completely submerged objects, it will **float** (it becomes partially submerged)

But how much is the buoyant force?

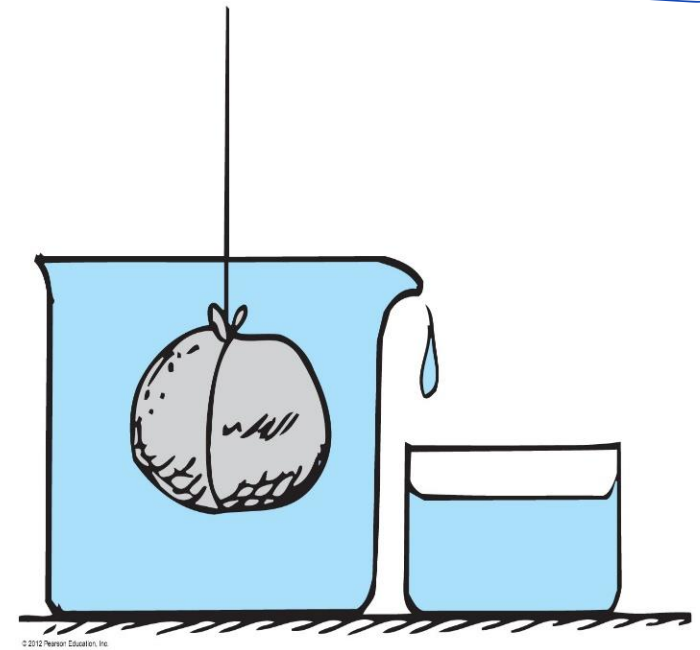
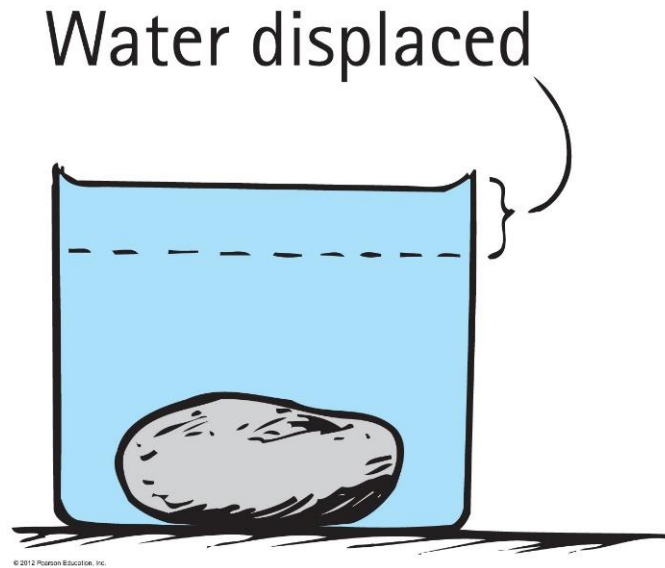


Archimedes' Principle

- **Archimedes' Principle:** *An immersed object is buoyed up by a force that is equal to the weight of the fluid it displaces*
 - immersed means partially or completely submerged
- $F_{\text{buoyant}} = \text{weight of displaced fluid}$
- This applies to any object in a fluid (liquids and gases), partially or completely submerged.

Archimedes' Principle

- How much fluid is displaced? What is **displaced** fluid?



- How much liquid is displaced?
 - A *completely submerged* object displaces a volume of liquid equal to its own volume. (This can be used for determining the volume of **irregularly shaped objects**)

Archimedes' Principle

$\rho_{\text{liquid}} > \rho_{\text{object}} \rightarrow$ partially submerged

Completely submerged objects: $\rho_{\text{object}} > \rho_{\text{liquid}} \rightarrow$ completely submerged

- When the density of the object is greater than the density of the liquid it is completely submerged in the liquid.
- The volume of the displaced liquid equals the volume of the immersed body.
 $V_{\text{displaced}} = V_{\text{immersed}}$
- The weight of the object in liquid equals its weight outside water reduced by the buoyant force.

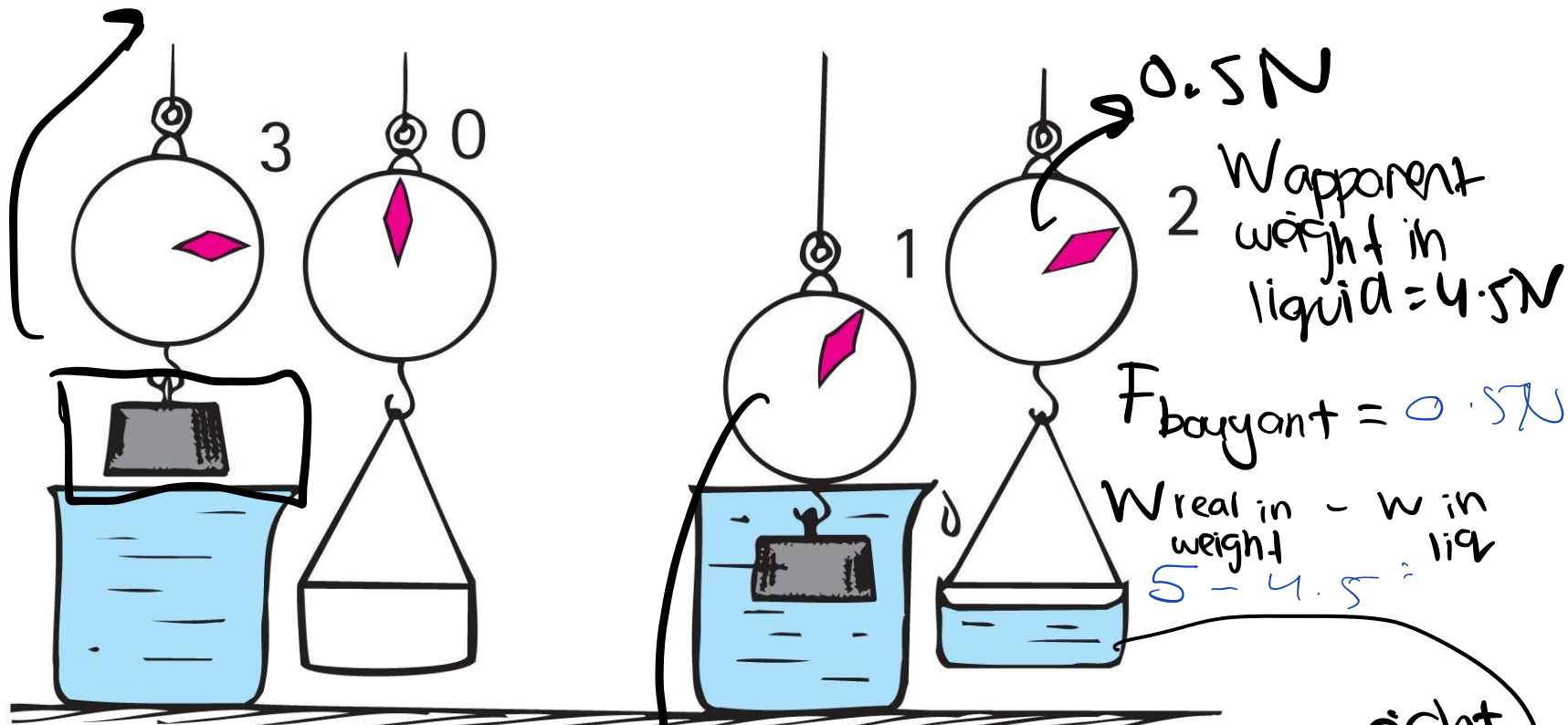
Weight in liquid = weight out of liquid – buoyant force

Weight in liquid = weight out of liquid – weight of displaced liquid

- Weight in liquid is also called the apparent weight.
- The weight of object outside of liquid is the actual weight = mg
 $W_{\text{apparent}} \neq mg$, whereas $W_{\text{actual}} = mg$

Example

real weight = $w = w_r = 5\text{ N}$



- The weight of the block in water is less than outside.
- The loss in its weight in water equals the buoyant force, which equals the weight of the displaced water (= 2 N)

$0.5\text{ N} = \text{volume of displaced}$

$4.5\text{ N} = \text{apparent weight}$

liquid

RECAP

$V_{\text{displaced liquid}} = V_{\text{immersed / submerged obj}}$

$W_{\text{displaced liquid}} = F_B \rightarrow \text{displaced liq.}$

$$F_B = \rho_L \times V_{\text{immersed / sub obj}} \times g$$

$\downarrow N$

$\downarrow \frac{kg}{m^3}$

$\downarrow m^3$

$\downarrow m/s^2$

$\underbrace{\hspace{10em}}_{\text{mass}} \times g = W (N)$

$$F_B = \underbrace{\rho_L \times V_{\text{immersed}}}_{m} \times g$$

$$\left(\begin{array}{l} W = m \times g \\ m = \rho \times V \end{array} \right)$$

$$\underbrace{m \times g}_{\rho_L \times V_{\text{immersed}}}$$

$$F_B = W_{\text{real in air}} - W_{\text{apparent in liquid}}$$

a) $m_o = \rho_o \times V_o$

$$m_o = 3000 \times 0.05 = 150 \text{ kg}$$

b) $w = m \times g = 150 \times 10 = 1500 \text{ N}$

Example

$$3 \text{ g/cm}^3 \times 1000$$

An object of volume 0.05 m^3 and density 3 g/cm^3 is immersed in water of density 1 g/cm^3 , find:

$$= \frac{3000 \text{ kg}}{\text{m}^3}$$

- The mass of the object

$$\text{Mass} = \text{density} \times \text{volume} = (3 \times 1000) \times 0.05 = 150 \text{ kg}$$

- The weight of the object

$$W = mg = 150 \times 10 = 1500 \text{ N}$$

- The volume of the displaced water

$$v_{\text{displaced}} = v_{\text{object}} \quad \begin{array}{l} \text{g/cm}^3 \times 1000 \\ 3000 \\ 0.05 \\ \hline 3000 \end{array} \quad \begin{array}{l} = \rho \\ \times \\ \frac{\text{m}}{0.05} \end{array}$$

$$\text{Volume of displaced water} = \text{volume of the object} = 0.05 \text{ m}^3$$

- The mass of the displaced water

$$\text{Mass} = \text{density} \times \text{volume} = (1 \times 1000) \times 0.05 = 50 \text{ kg}$$

- The weight of the displaced water

$$W = mg = 50 \times 10 = 500 \text{ N}$$

$$F_B = \underbrace{\rho \times V}_{m} \times g$$

$$m = \rho \times V$$

Example

An object of volume 0.05 m^3 and density 3 g/cm^3 is immersed in water of density 1 g/cm^3 , find:

$$3000 = \frac{m}{0.05} = 150 \text{ kg}$$

- The mass of the object

$$\text{Mass} = \text{density} \times \text{volume} = (3 \times 1000) \times 0.05 = 150 \text{ kg}$$

- The weight of the object

$$W = mg = 150 \times 10 = 1500 \text{ N}$$

- The volume of the displaced water

$$\text{Volume of displaced water} = \text{volume of the object} = 0.05 \text{ m}^3$$

- The mass of the displaced water

$$\text{Mass} = \text{density} \times \text{volume} = (1 \times 1000) \times 0.05 = 50 \text{ kg}$$

- The weight of the displaced water

$$W = mg = 50 \times 10 = 500 \text{ N}$$

$$W_{\text{displaced liquid}} = F_B$$

$$F_B = \rho_L \times V_{\text{im/sub}} \times g$$

$$W = m \times g$$

$$W = \underbrace{\rho \times V}_m \times g$$

$$58 \times 10 = 580$$

Example continued ..

- Why the object sinks?

It sinks because the weight of displaced water for the fully submerged object is less than the weight of the object. In other words, the density of the object is higher than the density of water.

- Find the buoyant force on the object

$$F_B = W_{\text{displaced}} = 500 \text{ N}$$

The buoyant force = weight of the displaced water = 500 N

$$W_{\text{real}} - W_{\text{displaced}} = W_{\text{in fluid}}$$

- Find the weight of the object inside water

$$\begin{aligned} \text{weight inside water} &= \text{weight outside} - \text{buoyant force} \\ &= 1500 - 500 = 1000 \text{ N} \end{aligned}$$

Archimedes' Principle

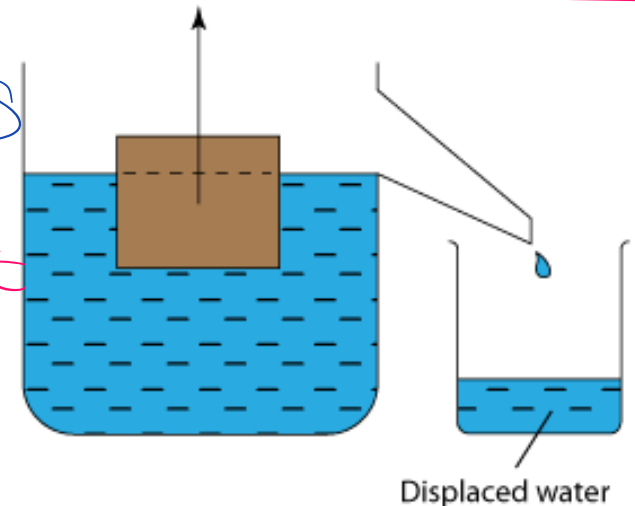
Partially submerged objects (floating objects):

- When the density of the object is less than the density of the liquid it floats (partially submerged)
- The volume of the displaced liquid equals the volume of the submerged part of the object.
- Principle of floatation: A floating object displaces a weight of fluid equal to its own weight

Buoyant force = weight of displaced fluid = weight of the object

- Weight of the object inside liquid is Zero.

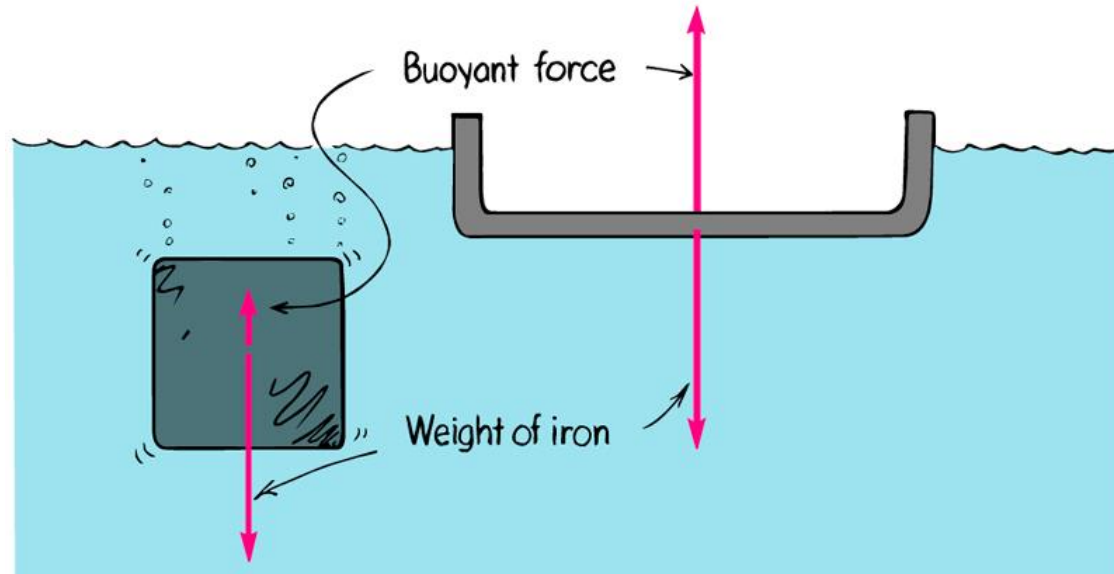
∴ apparent weight



Archimedes' Principle

more volume immersed = more Buoyance force

- How can an iron ship float?
- An iron block sinks, but the same quantity of iron floats when shaped like a bowl. Why?
 - Hint: think about the volume & weight of the displaced water in the two cases.



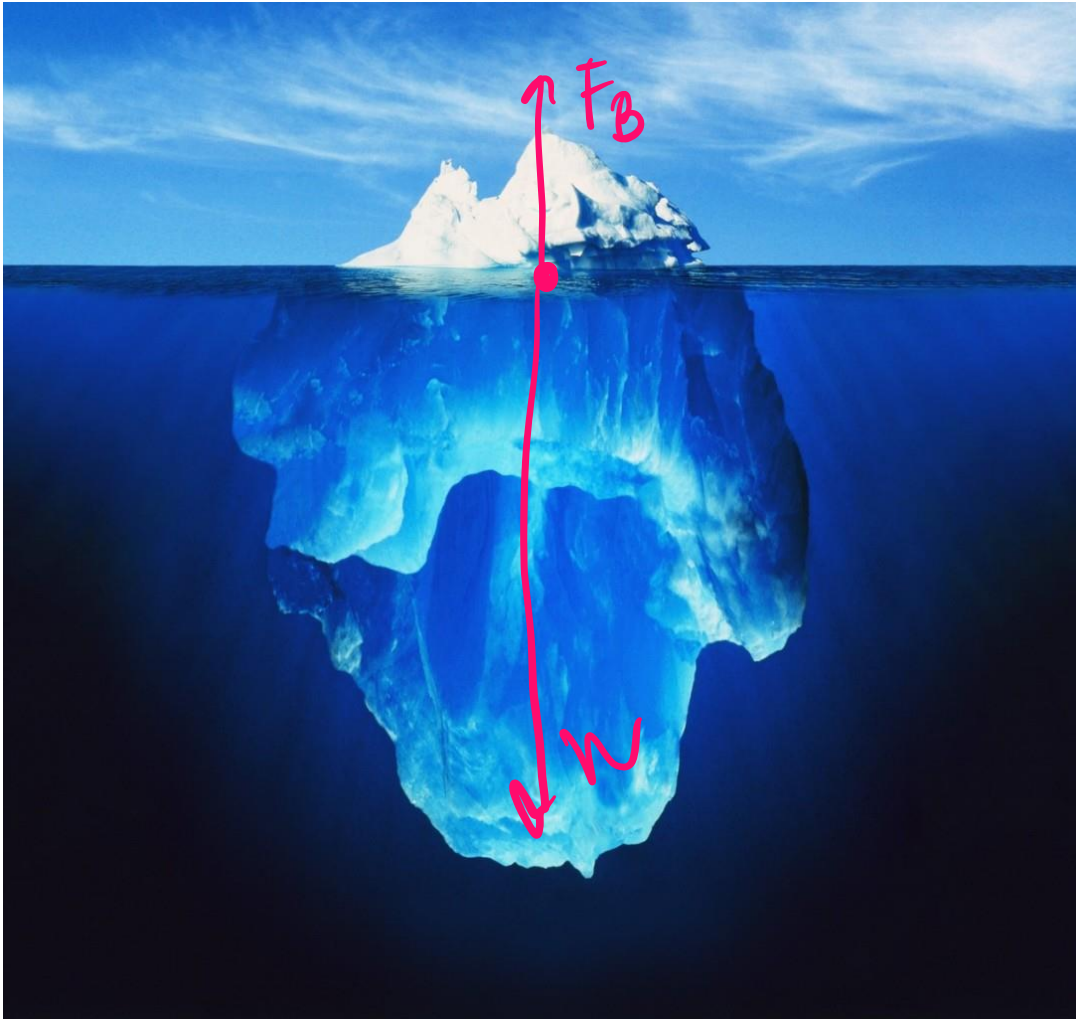
More load → heavier → more displaced water → more buoyant force

Archimedes' Principle

- An unloaded ship (left) versus the same ship when it is loaded (right). Note that the loaded ship experience more buoyant force because more water is displaced (larger volume is under water)



The tip of the iceberg



$$F_B = W \text{ (floating)}$$



The density of ice is about 90% the density of water. Hence, 90% of the iceberg is submerged, so that the weight of the displaced water is the same as the weight of the iceberg

Examples

$$W_{obj} = F_{\text{buoyant}} \text{ (equilibrium)}$$

$$F_B = \rho_L \times V_{\text{im/sub}} \times g$$

$$\rightarrow \times 10 = 10 \text{ (m/s}^2\text{)}$$

- What weight of water is displaced by a 100000 kg floating ship?
What is the buoyant force acting on this ship?

$$m \quad \downarrow \quad W_{\text{liquid}} = F_{\text{buoyant}}$$

$$F_B = \rho_L \times V_{\text{im/sub}} \times g = 100,000 \times 10 = 1,000,000 \text{ N}$$

$$10^6 \text{ N}$$

- A 3 kg object placed in water displaces 1.2 kg of water.

- What is the buoyant force on this object

$$F_B = W_{\text{displaced}}$$

$$F_B = \rho_L \times V \times g \rightarrow 1.2 \text{ kg} \times 10 \text{ m/s}^2 = \boxed{12 \text{ N}}$$

$$1.2 \times 10 = 12 \text{ N}$$

$$= F_B = W_{\text{displaced liquid}}$$

- What is the apparent weight of the object

$$W_{\text{air}} - W_{\text{displaced water}} = 30 - 12 = 18 \text{ N}$$

- Does the object sink or float? Why?

$$F_B = 12 \text{ N}$$

$$W_{\text{obj}} > W_{\text{displaced}}$$

$$W_{\text{obj}} = 30 \text{ N}$$

= sinks

$$\uparrow F_B = 18 \text{ N}$$

$$\downarrow W_{\text{obj}} = 30 \text{ N}$$

Examples

density + volume of obj is given \rightarrow mass!
 2000 cm^3
 V_{obj}

$$W_{\text{displaced}} = W_{\text{wood}}$$

$$\text{mass}_{\text{obj}} = \text{mass}_{\text{fluid}}$$

- A piece of wood of volume 2000 cm^3 and density 0.4 g/cm^3 is floating in water of density 1 g/cm^3 . $\rightarrow \rho_{\text{water}}$

$$2000 \times 0.4$$

- a) What is the volume of the part of wood under water?

Weight of displaced water = weight of wood \rightarrow find missing values

$$\text{Mass of wood} = \text{volume} \times \text{density} = 2000 \times 0.4 = 800 \text{ g} = 0.8 \text{ kg}$$

$$\text{Weight of wood} = mg = 0.8 \times 10 = 8 \text{ N} = \text{Weight of displaced water}$$

$$\text{Mass of displaced water} = 8/10 = 0.8 \text{ kg} = 800 \text{ g} \quad 2000 \times 0.4 = 800 \rightarrow 0.8 \text{ kg}$$

$$\text{Volume of displaced water} = \text{mass} / \text{density} = 800/1 = 800 \text{ cm}^3$$

$$\text{Volume the immersed part of wood} = 800 \text{ cm}^3$$

$$\text{OR } \underbrace{(\rho_{\text{wood}} V_{\text{wood}})}_{\rho_{\text{water}}} = \underbrace{(\rho_{\text{water}} V_{\text{water}})}_{\rho_{\text{water}}} \Rightarrow V_{\text{water}} = V_{\text{wood}} (\rho_{\text{wood}} / \rho_{\text{water}})$$

- b) What is the volume of the part of wood above water? $V_{\text{object}} - V_{\text{below}} = V_{\text{above}}$

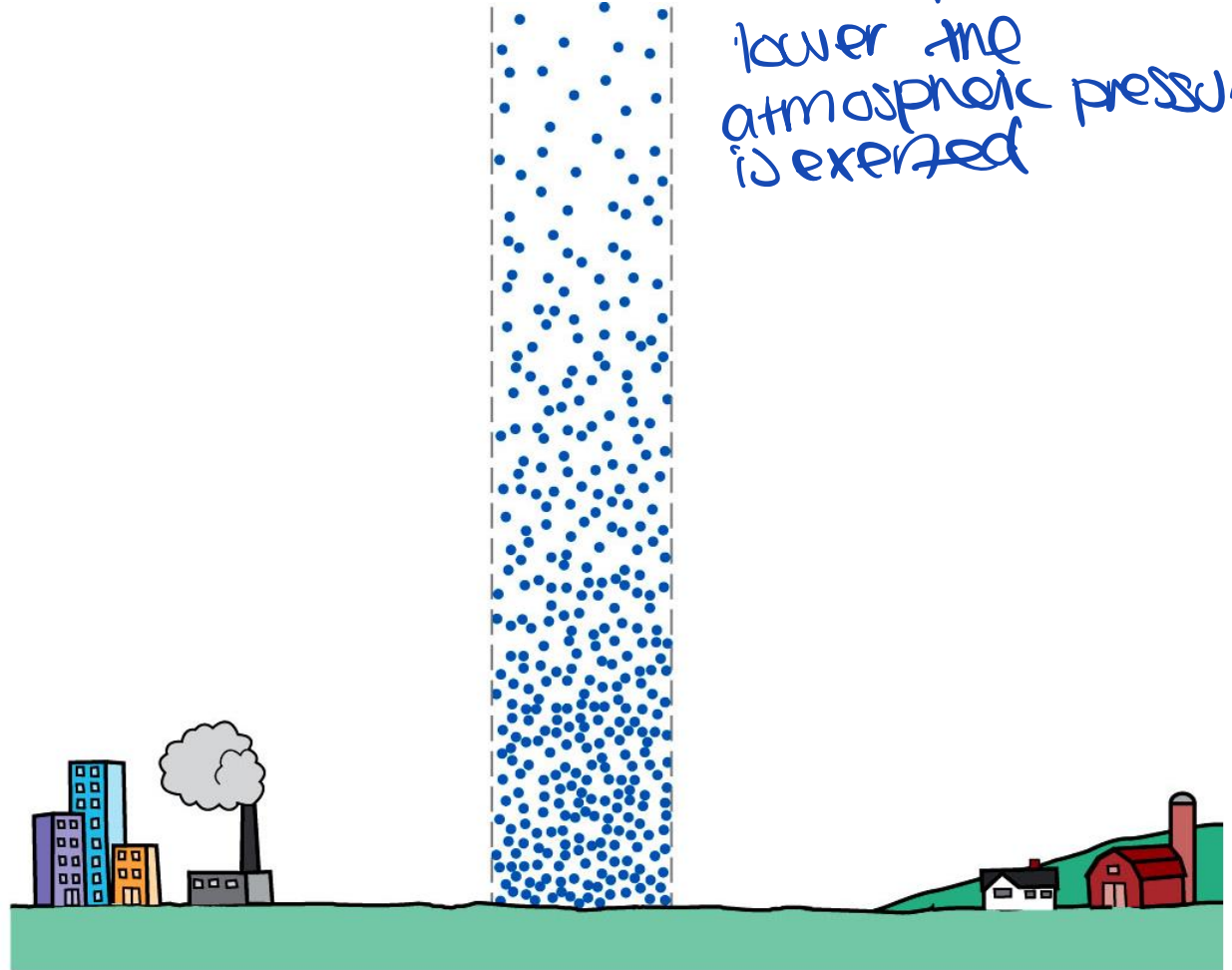
$$\text{Volume of the part of wood above water} = 2000 - 800 = 1200 \text{ cm}^3$$

Atmospheric Pressure

- Atmospheric pressure is caused by the weight of air above us.

→ due to the weight of air above me

→ the more you climb up the lower the atmospheric pressure is exerted



Atmospheric Pressure

- The average atmospheric pressure at sea level is **101.325 kPa**
- The standard atmosphere, abbreviated atm, is another unit of pressure equals to the average atmospheric pressure at sea level.
 - 1 atm = 101325 Pa
- Why doesn't the pressure of the atmosphere break windows?

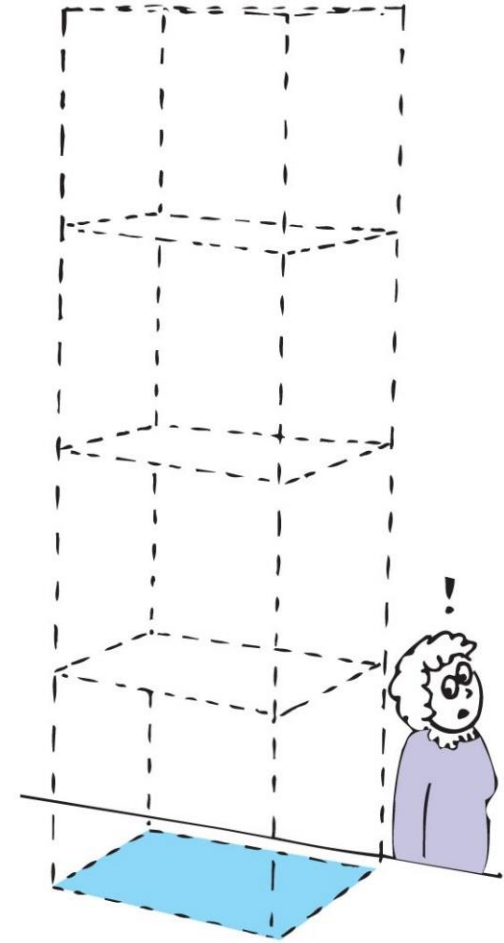


FIGURE 5.24

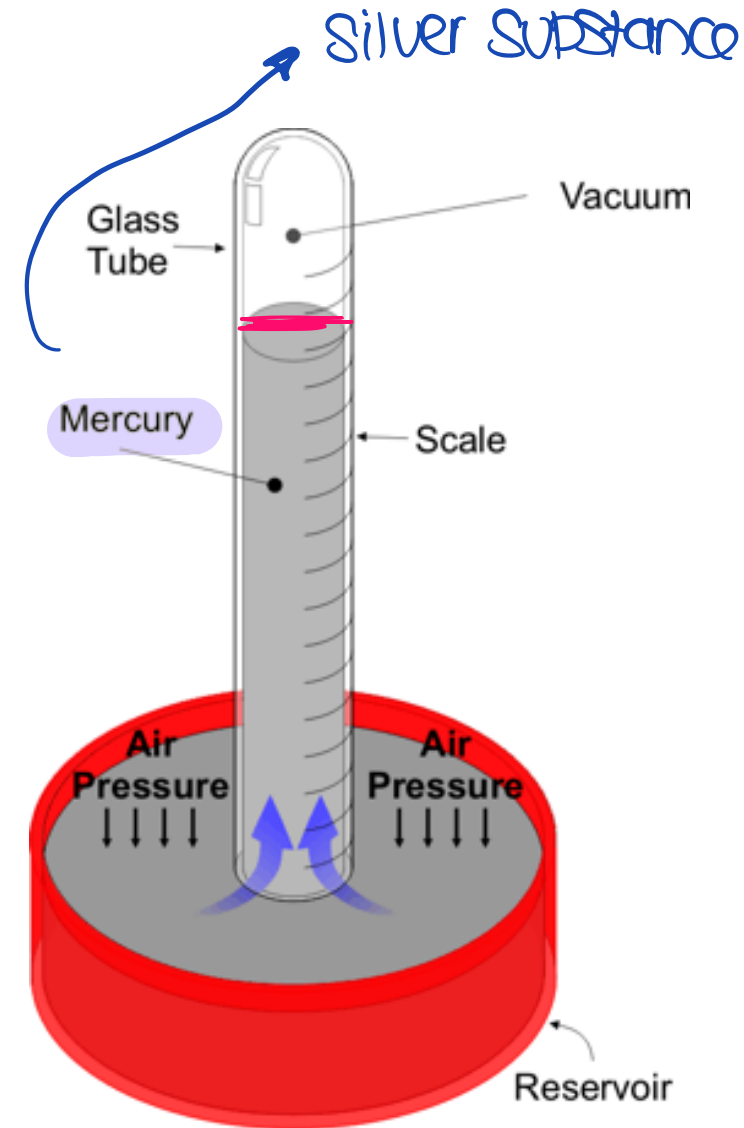
The weight of air that presses down on a 1-m² surface at sea level is about 100,000 N. So atmospheric pressure is about 10⁵ N/m², or about 100 kPa.

Atmospheric Pressure: Barometers

- The barometer is used for measuring atmospheric pressure.
- It can be created by filling a tube of mercury and tipping it upside down in a dish of mercury.
- The barometer will balance when the weight of the liquid in the tube produces the same pressure as the atmosphere outside. At sea level this will be a 76 cm high column of mercury
- What happens in a barometer is similar to what happens when you drink through a straw.
- cmHg or mmHg are also used as a units of pressure

$$1 \text{ atm} = 76 \text{ cmHg} = 101.325 \text{ kPa}$$

$$\rho_{\text{Hg}} = 13600 \frac{\text{kg}}{\text{m}^3}, \quad \rho_{\text{H}_2\text{O}} = 1000 \frac{\text{kg}}{\text{m}^3}$$



Atmospheric Pressure: Barometers

- Water could be used instead of Hg in the barometer but in this case the glass tube would be 13.6 times as long (10.3 m) which is not practical.
 - As a result, there is a 10.3 m limit on the height to which water can be lifted with vacuum pumps.
- **Exercise:** Check that a column of 10.3 m of water produces the same pressure as 76 cm of mercury, and this is equal to the atmospheric pressure at sea level. (Use the internet to find the density of Mercury and Water).

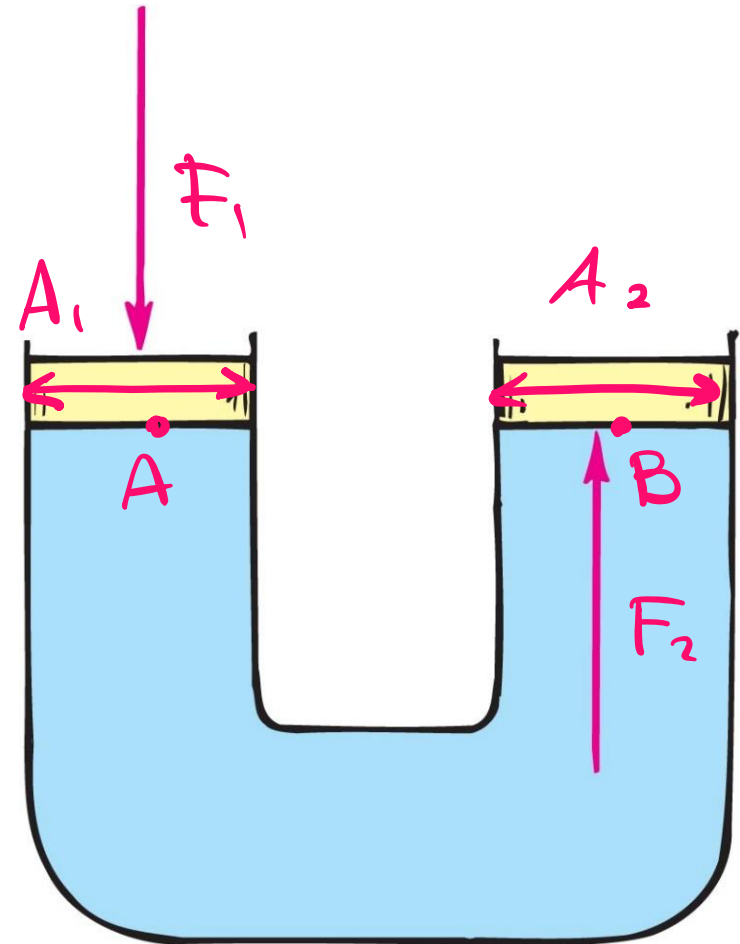
$$\begin{array}{l} \rho_{\text{Hg}} = 13,600 \frac{\text{kg}}{\text{m}^3} \\ \rho_{\text{H}_2\text{O}} = 1,000 \frac{\text{kg}}{\text{m}^3} \\ 1 \text{ cm} = 10^{-2} \text{ m} \end{array} \quad \left| \quad \begin{array}{l} P_{\text{atm}} = \rho_{\text{H}_2\text{O}} \times g \times h_{\text{H}_2\text{O}} \\ = 1000 \times 10 \times 10.3 \\ = 103,000 \text{ Pa} \\ = 103 \text{ kPa} \end{array} \quad \left| \quad \begin{array}{l} P_{\text{atm}} = \rho_{\text{Hg}} \times g \times h_{\text{Hg}} \\ = 13,600 \times 10 \times 0.76 \\ = 103,360 \text{ Pa} \end{array}$$

Pascal's Principle OR hydraulic press/jack

- Pascal's Principle: A change in pressure at any point in an enclosed fluid at rest is transmitted to all points in the fluid.

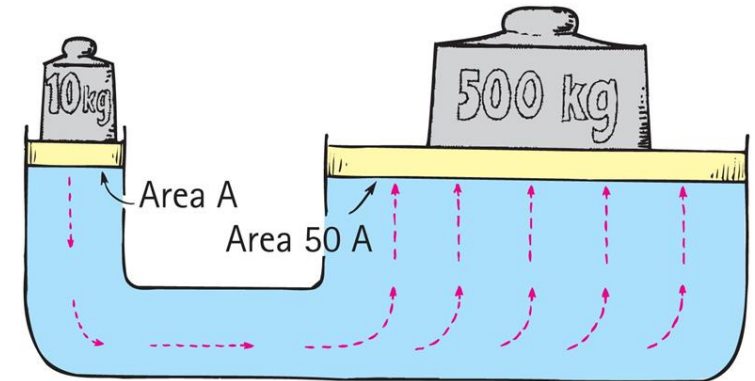
$$P_A = P_B \quad \begin{array}{l} \text{(same liquid)} \\ \text{(same pressure)} \\ \text{(same level)} \end{array}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Pascal's Principle

- Application: The lift piston (hydraulic press/hydraulic jack)
- Pressure applied to the left piston is transmitted to the right piston
- A 10-kg load on small piston (left) lifts a load of 500 kg on large piston (right)



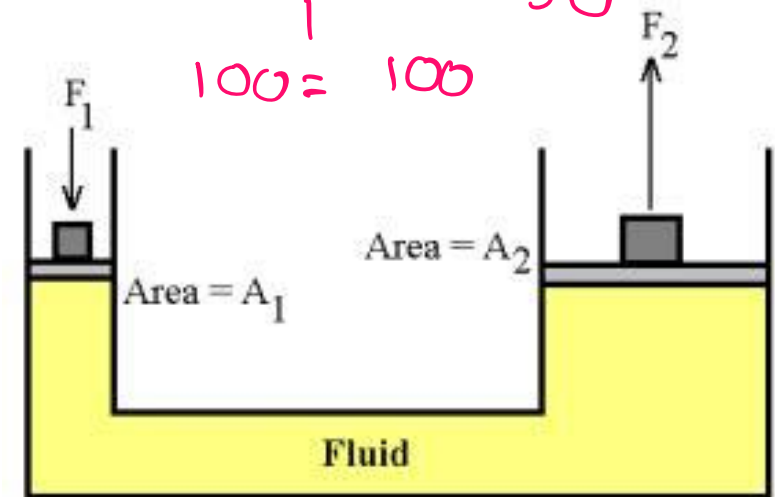
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{10 \cdot 10}{A_1} = \frac{500 \cdot 10}{A_2} \rightarrow \frac{100}{1} = \frac{5000}{50}$$

100 = 100

$$F_2 = 5000 \text{ N}$$

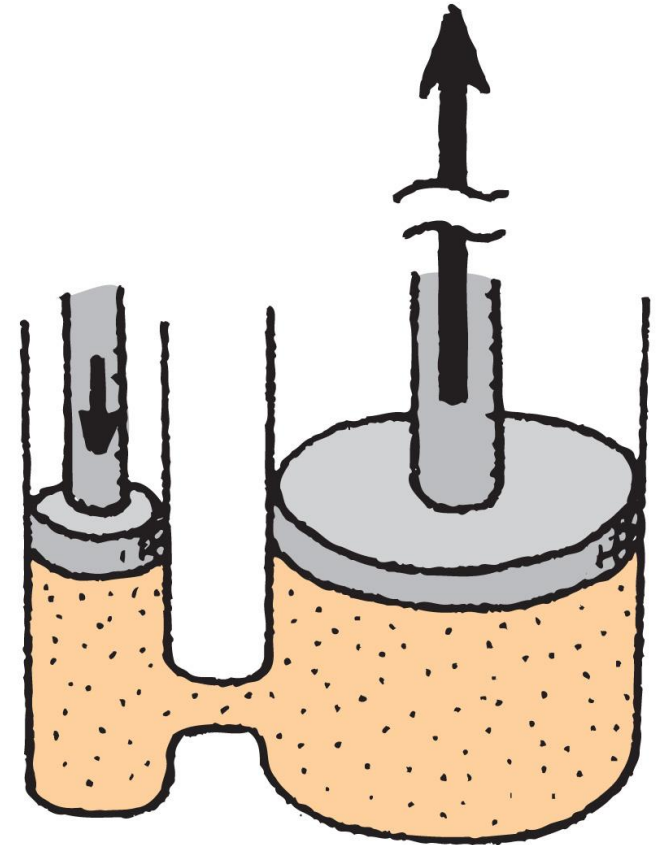
- According to Pascal's principle, since $P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



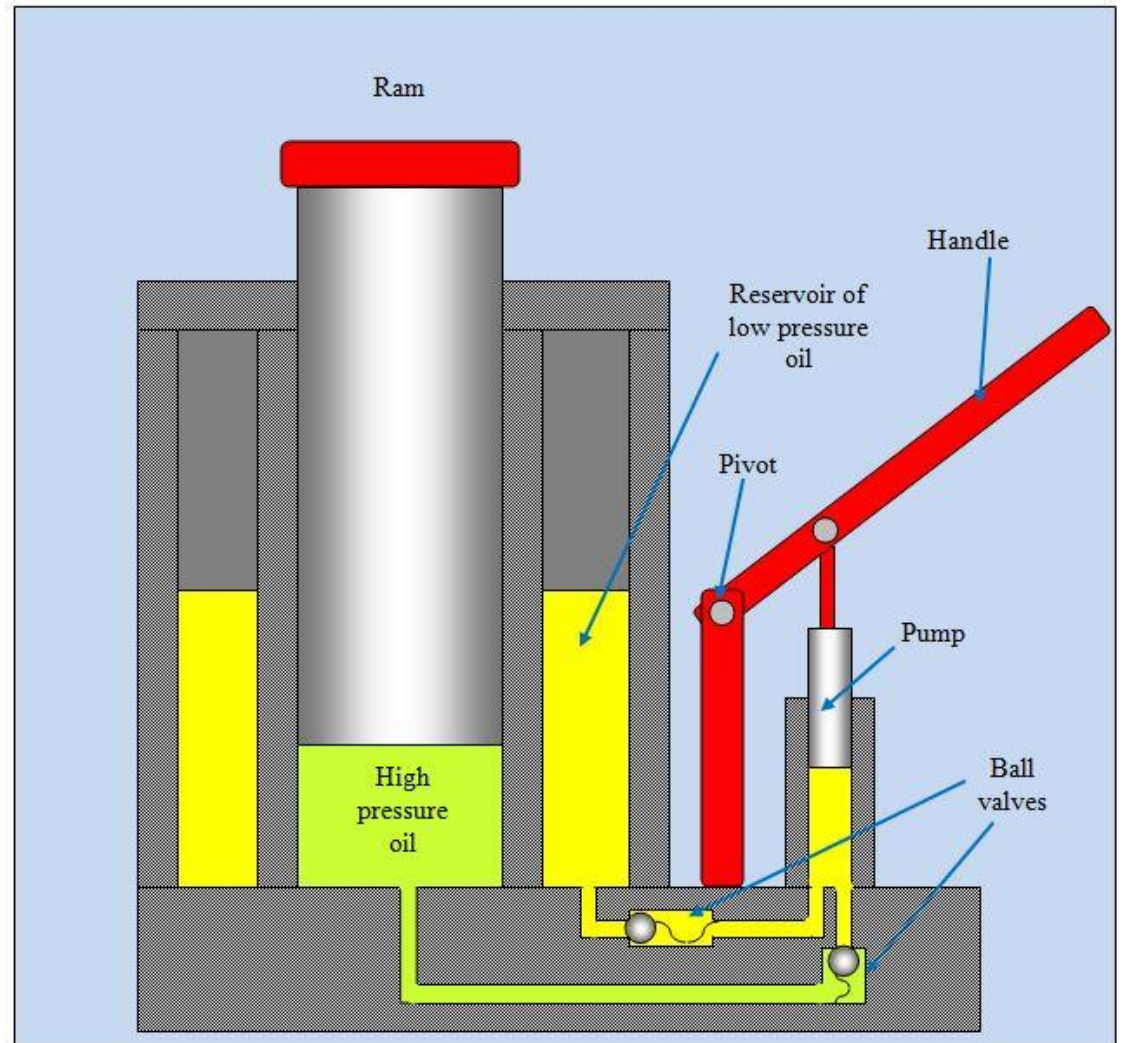
Pascal's Principle

- By increasing the area of the large piston or decreasing the area of the small piston, we can multiply force.
- This piston does not violate energy conservation because a decrease in the distance moved compensates for the increase in force.



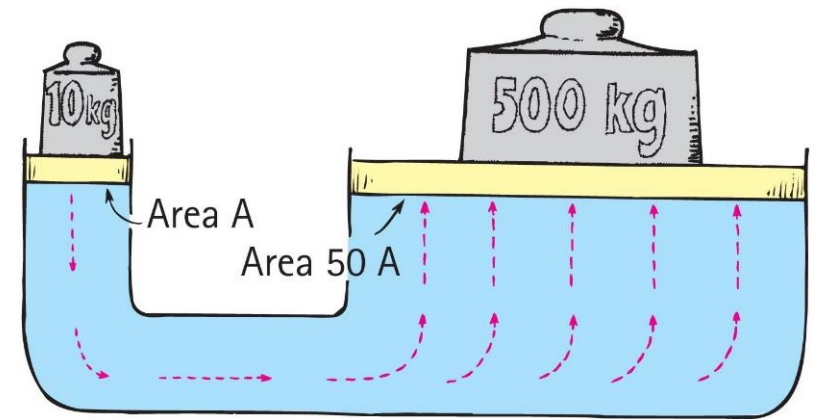
Pascal's Principle

The hydraulic jack



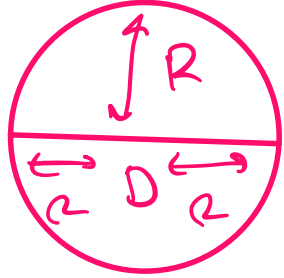
Example

- If the mass on the left piston = 10 kg and its area is 1 m^2 , and the area of the right piston = 50 m^2 . Find the force created on the right piston.



- Note that the area of the right piston is 50 times the area of the left piston, which increased the force 50 times, because the pressure is the same on both pistons.

Example



$$\text{Diameter} = 2 \text{ radius}$$
$$\text{Area} = \pi R^2$$

- The small piston of the hydraulic piston has a diameter of 2 cm and the large piston has a diameter of 6 cm. if a 15 N force is applied to the small piston, what is the force exerted by the large piston?

$$\text{Area of a circular piston} = \pi r^2$$

$$\text{Radius of the small piston} = 2/2 = 1 \text{ cm} = 0.01 \text{ m}$$

$$\text{Radius of the large piston} = 6/2 = 3 \text{ cm} = 0.03 \text{ m}$$

$$F_1/A_1 = F_2/A_2$$

$$15/(\pi (0.01)^2) = F_2 / (\pi (0.03)^2)$$

$$F_2 = 135 \text{ N}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{15 \text{ N}}{\pi R_1^2} = \frac{F_2}{\pi R_2^2}$$

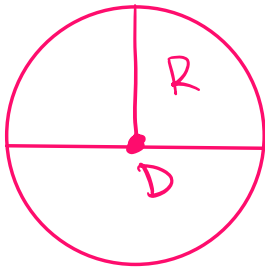


$$\pi(0.01)^2 \quad \pi(0.03)^2$$

$$\frac{15 \pi(0.03)^2}{\pi(0.01)^2} = \frac{F_2 \pi(0.01)^2}{\pi(0.01)^2}$$

$$\frac{0.0135}{0.0001} = \boxed{F_2 = 135 \text{ N}}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



$$D = 2R$$

$$R = \frac{D}{2}$$

$$A = \pi r^2$$

Extra Exercise

A small piston of diameter = 10cm and a force applied to it is 50N
If a force of 250N is applied on larger piston what is its radius?

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{50\text{N}}{\pi(5)^2} = \frac{250\text{N}}{\pi(r)^2}$$

$$50 \cdot \pi r^2 = 250 \pi 25$$
$$\frac{50\pi r^2}{50} = \frac{6250\pi}{50}$$

$$\frac{\pi r^2}{\pi} = \frac{125\pi}{\pi}$$

$$r^2 = 125$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$r = \sqrt{125}$$

$$r = 11.18 \dots \text{cm}$$