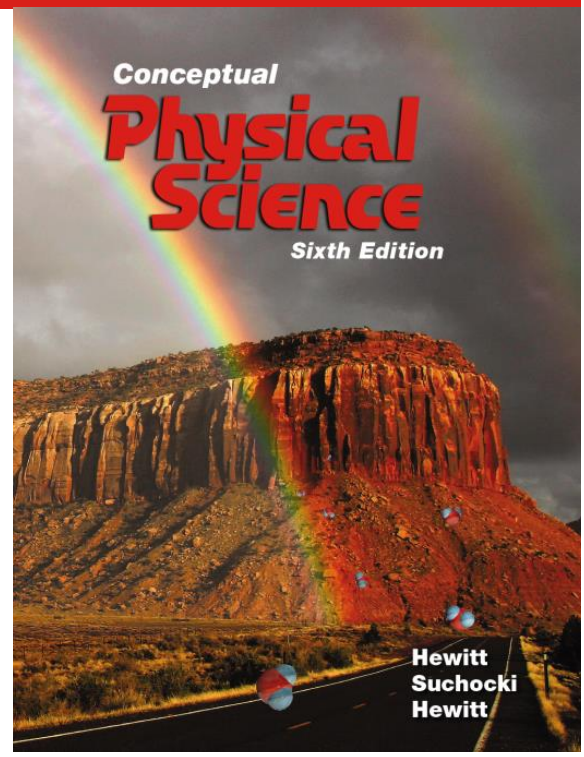
Chapter 4 Lecture



Chapter 4: Gravity, Projectiles, and Satellites

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The Universal Law of Gravity

- Newton discovered that gravity is *universal*.
- Every mass pulls on every other mass.
  - Check the cavendish experiment https://youtu.be/MbucRPiL92Q?t=450

* any two masses in universe attract each other

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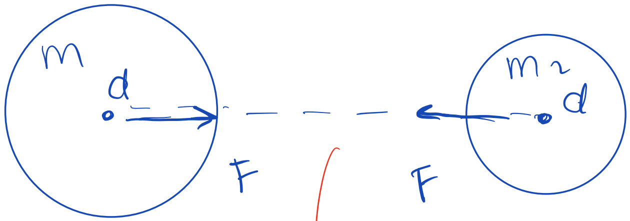
newtons law of gravitation

$m \uparrow \quad F \uparrow$

$d \uparrow \quad F \downarrow$



2 on 1 , 1 on 2

have the same magnitude due to action reaction

$$F = \frac{G\, m_1 m_2}{d^2} \quad , \quad G = 6.67 \times 10^{-11}$$

$F \to N$, $m_1 m_2 \to kg$, $d \to m$

× derive unit of G

$$N = \frac{G\ kg \times kg}{m^2} = \frac{x\ kg^2}{m^2} = N$$

$$\frac{N\ m^2}{kg^2} = \frac{kg^2\ x}{kg^2}$$

$$x = \frac{N m^2}{kg^2}$$

$$F = \frac{G m_1 m_2}{d^2}$$

$m \uparrow \quad F \uparrow$

$d \uparrow \quad F \downarrow$

m_1	m_2	d	F	
$2m_1$	m_2	d	$2F$	
$3m_1$	$5m_1$	d	$3 \times 5\, F$	$= 15F$
m_1	m_2	$2d$	$\left(\frac{1}{2}\right)^2 F$	$= \frac{1}{4} F$
m_1	m_2	$\frac{d}{4}$	$\left(\frac{4}{1}\right)^2 F$	$= 16F$
$\frac{1}{4} m_1$	$16 m_2$	$\frac{3}{2} d$		

$\rightarrow \frac{1}{4} \times 16 \times \left(\frac{2}{3}\right)^2 F$

m_1	m_2	d	F
$3m_1$	$\frac{1}{9}m_2$	$3d$	$3 \times \frac{1}{9} \times \left(\frac{1}{3}\right)^2 F = \frac{1}{27}F$
$5m_1$	$10m_2$	$\frac{2d}{5}$	$5 \times 10 \times \left(\frac{5}{2}\right)^2 F = \frac{625}{2}F$
$\frac{1}{2}m_1$	$\frac{1}{4}m_2$	$8d$	$\frac{1}{2} \times \frac{1}{4} \times \left(\frac{1}{8}\right)^2 F = \frac{1}{512}$
$\frac{1}{9}m_1$	$3m_2$	$\frac{1}{6}d$	$\frac{1}{9} \times 3 \times (6)^2 F = 12F$

The Universal Law of Gravity

- Newton's Law of Universal Gravitation
  - Every body in the universe attracts every other body with a mutually attracting force.
  - For two bodies, this force is directly proportional to the product of their masses and inversely proportional to the square of the distance separating them,

$$F = G\frac{m_1 m_2}{d^2}$$

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The Universal Gravitational Constant, *G*

- *G* is the proportionality constant in Newton's law of gravitation.
- *G* has the same magnitude as the gravitational force between two 1-kg masses that are 1 meter apart:

  6.67×10^{-11} N.

  So $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

  $F = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}(m_1 \times m_2)/d^2$

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Newton's Law of Universal Gravity

$$F = G\frac{m_1 m_2}{d^2}$$

- The greater m_1 and $m_2 \Rightarrow$ the greater the force of attraction between them.
- The greater the distance of separation d, the weaker is the force of attraction—weaker as the inverse square of the distance between their centers.

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The Inverse-Square Law: Gravity and Distance
CHECK YOUR NEIGHBOR

The force of gravity between two planets depends on their

A. masses and distance apart.
B. planetary atmospheres.
C. rotational motions.
D. All of the above.

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The Inverse-Square Law: Gravity and Distance
CHECK YOUR ANSWER

The force of gravity between two planets depends on their

A. masses and distance apart.
B. planetary atmospheres.
C. rotational motions.
D. All of the above.

Explanation:

The equation for gravitational force,

$$F = G\frac{m_1 m_2}{d^2}$$

cites only masses and distances as variables. Rotation and atmospheres are irrelevant.

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The Inverse-Square Law: Gravity and Distance
CHECK YOUR NEIGHBOR

If the masses of two planets are each somehow doubled, the force of gravity between them

A. doubles.
B. quadruples.
C. reduces by half.
D. reduces by one quarter.

$F = G\frac{m_1 m_2}{d^2}$

$F = G\frac{2m_1 2m_2}{d^2}$

$F = G\frac{4m_1 m_2}{d^2}$

$F = 4F$

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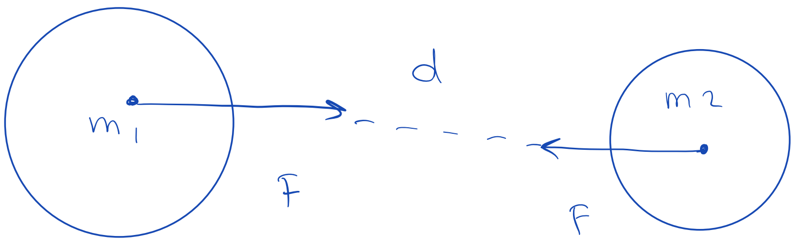

$$F = G\,\frac{m_1 m_2}{d^2}$$

kg

N

m

whereby $G = 6.67 \times 10^{-11}$

The Inverse-Square Law: Gravity and Distance
CHECK YOUR ANSWER

If the masses of two planets are each somehow doubled, the force of gravity between them

A. doubles.

B. quadruples.

C. reduces by half.

D. reduces by one quarter.

Explanation:

Note that both masses double, so double × double = quadruple.

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The Universal Law of Gravity
CHECK YOUR NEIGHBOR

If the mass of one planet is somehow doubled, the force of gravity between it and a neighboring planet would

A. doubles.

B. quadruples.

C. reduces by half.

D. reduces by one quarter.

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The Universal Law of Gravity
CHECK YOUR ANSWER

If the mass of one planet is somehow doubled, the force of gravity between it and a neighboring planet would

A. doubles.

B. quadruples.

C. reduces by half.

D. reduces by one quarter.

Explanation:

Let the equation guide your thinking:

$$F = G\,\frac{m_1 m_2}{d^2}$$

Note that if one mass doubles, the force between them doubles.

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Example

- Note: Gravitational force is weak when masses are small.

Example: Calculate the gravitational attractive force between two people of masses 90 kg and 50 kg when they are 4 m apart.

$F = G\,\frac{m_1 m_2}{d^2}$, whereby $m_1 = 90\,kg$, $m_2 = 50\,kg$, $d = 4\,m$, $G = 6.67\times10^{-11}$

$$F = (6.67\times10^{-11})\,\frac{(90)(50)}{(4)^2} = 1.8759\ldots\times10^{-8}$$

$$\boxed{\approx 1.88\times10^{-8}\,N}$$

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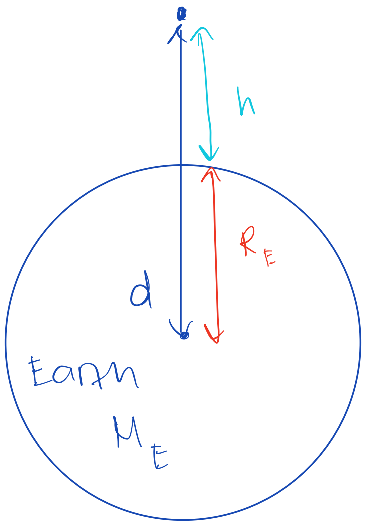


$$d = R_E + h$$

$$F = \frac{G M_E \cdot m}{(R_E + h)^2}$$

Example

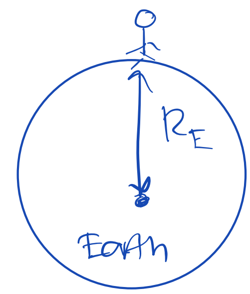
Calculate the gravitational attractive force between an 80 kg man and Earth when he is at earth surface. Given that mass of Earth is 6×10^{24} kg and the radius of Earth is 6380 km.

$m = 80\,kg$ $\qquad G = 6.67 \times 10^{-11}$

$M_E = 6 \times 10^{24}\,kg$

$R_E = 6380 \times 10^{3}\,m$

$h = 0$



$$F = (6.67 \times 10^{-11}) \frac{(80)(6 \times 10^{24})}{(6380000)^2}$$

$= 786.54$

$\boxed{\approx 787\,N}$

The Universal Law of Gravity

- Note that the gravitational force in the previous example is equal to the weight of the man when calculated using $w = mg$ (with small difference due to approx. values)
- We can write

$$\underset{F}{W} = \cancel{m_{\text{man}}}\, g = G \frac{m_{\text{Earth}} \cancel{m_{\text{man}}}}{d^2}$$

- Therefore:

$$g_{\text{Earth}} = G \frac{m_{\text{Earth}}}{{R_{Earth}}^2}$$

  - Where *d* is set to equal the radius of Earth (R_{Earth})
- The above formula can be used to calculate the acceleration of gravity on anther planet, given its radius and mass

$$F = \frac{G\, M_E \times \cancel{m}}{(R_E + h)^2} = w = \cancel{m} \cdot g$$

$$g_{Earth} = \frac{G\, M_E \rightarrow kg}{(R_E + h)^2 \rightarrow m}$$

$\swarrow \frac{m}{s^2}$

$$g_{planet} = \frac{G\, M_{planet} \rightarrow kg}{(R_{planet} + h)^2 \rightarrow m}$$

$\swarrow \frac{m}{s^2}$

Calculate the gravitational attractive force between a ~~90~~ kg man and Earth when he is above the surface of Earth at h. Given that = 5250m mass of Earth is 6×10^{24} kg and the radius of Earth is 6380 km.

$m + M_E$

$90 + (6 \times 10^{24})$

$m = 90kg$, $M_E = 6 \times 10^{24}$ kg

$R_E = 6380$ km

$= 6380 \times 10^3$ m

$h + R_E$

$5250 + 6380$

$h = 5250m$

$d = R_E + h$

$= (6380 \times 10^3) + 5250 = 6385250$ m

$G = 6.67 \times 10^{-11}$

$$F = G \frac{m \cdot M_E}{(h + R_E)^2}$$

$$F = G \frac{m \cdot M_E}{(R_E + h)^2}$$

$$F = (6.67 \times 10^{-11}) \frac{(90)(6 \times 10^{24})}{(6385250)^2}$$

$$\boxed{F = 883.4 N}$$

Example

gravitational force = weight

Example:

a) Find the weight of a 70 kg man when he is at 2500 km ($\times 10^3$) above earth surface.

- $G = 6.67 \times 10^{-11}$
- $m = 70 kg$
- $M_E = 6 \times 10^{24} kg$
- $R_E = (6380 km \times 10^3) m$

- $h = (2500 km) \times 10^3$

$$F = \frac{(6.67 \times 10^{-11})(70)(6 \times 10^{24})}{((6380 km \times 10^3) + (2500 km \times 10^3))^2} = 355.26 N$$

b) use your answer to part (a) to calculate the acceleration of gravity at that height. (in two different methods)

$w = mg$

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$F = ma$ / $W = mg$

$$\frac{355 N}{70} = \frac{(70) a}{70} \qquad a = 5.07 m/s^2$$

2nd method

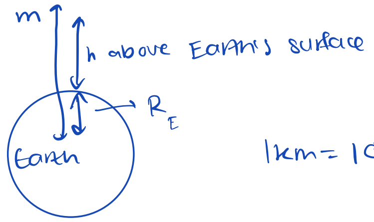
$$g = \frac{GM_E}{(R_E + h)^2} = \frac{(6.67 \times 10^{-11})(6 \times 10^{24})}{((6380 \times 10^3) + (2500 \times 10^3))^2} =$$

$5.07 m/s^2 \approx 5 m/s^2$

(acceleration of gravity)

Summary

$$F = \frac{G m_1 m_2}{d^2}$$



$1km = 10^3 m$

$$F = \frac{G\, m_E \times \cancel{m}}{(R_E + h)^2} = \cancel{m} g$$

$$g_E = \frac{G m_E}{(R_E + h)^2} \quad , \quad g_{planet/star} = \frac{G M_{planet}}{(R_{planet} + h)^2}$$