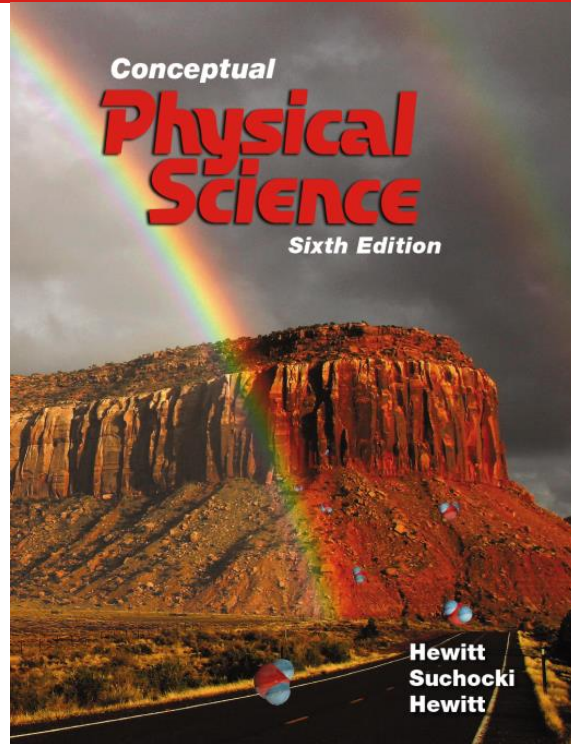


Chapter 3: Momentum and Energy



1

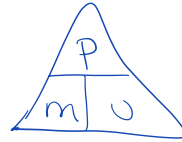
Momentum

- **Momentum**—is *inertia in motion*
 - defined as the product of mass and velocity:
momentum = mv
 - When direction is unimportant:
momentum = mass \times speed
- Momentum is a **vector quantity** (direction of momentum is the same direction of velocity). Remember to include direction (you can use **+** and **-** to indicate the direction)
- Unit of momentum is **kg.m/s = N.s**

2

Momentum: vector quantity

direction magnitude



$$P = m \times u$$

where $m = \text{mass}$, and $u = \text{velocity}$
(kg) (m/s)

$$P = mu$$

$\uparrow m$ $\uparrow P$ } directly proportional
 $\uparrow u$ $\uparrow P$ }

$$m = \frac{P}{u}$$

$$u = \frac{P}{m}$$

$m \uparrow$ $u \downarrow$ } inversely proportional
 $m \downarrow$ $u \uparrow$ }

$$P = mu$$

Momentum

- Momentum (continued)
 - high mass or high velocity \Rightarrow high momentum
 - high mass and high velocity \Rightarrow higher momentum
 - low mass or low velocity \Rightarrow low momentum
 - low mass and low velocity \Rightarrow lower momentum

3

Momentum

- Momentum of an object changes when either its mass changes, its velocity changes or both change.
- Double mass means double momentum, triple mass means triple momentum, and half mass means half momentum and so on.
- Double speed means double momentum, triple speed means triple momentum, and half speed means half momentum and so on.
- Double mass and double speed means 4 times momentum
- Double mass and half speed means same momentum



$$2m \quad \frac{v}{2} =$$

$$2m \times \frac{v}{2} = mv = P$$

$$\begin{aligned} 2m, 2v \\ P=mv \\ P=2kg \cdot 2m/s \\ =4N \cdot s \end{aligned}$$

4

Momentum

magnitude ONLY

Example: What is the speed of a 14 kg object having 122 kg.m/s of momentum?

$$p = 122 \text{ kg m/s}$$

$$m = 14 \text{ kg}$$

$$p = m v$$

$$\frac{122 \text{ kg} \cdot \text{m/s}}{14 \text{ kg}} = \frac{14 \text{ kg} \cdot v}{14 \text{ kg}}$$

$$v = 8.71 \text{ m/s}$$

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Momentum

CHECK YOUR NEIGHBOR

A moving object has

- A. momentum.
- B. energy.
- C. speed.
- D. All of the above.

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Momentum

CHECK YOUR ANSWER

A moving object has

- A. momentum.
- B. energy.
- C. speed.
- D. **All of the above.**

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Momentum

CHECK YOUR NEIGHBOR

When the speed of an object is doubled, its momentum

- A. remains unchanged in accord with the conservation of momentum.
- B. **doubles.**
- C. quadruples.
- D. decreases.

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Momentum

CHECK YOUR ANSWER

When the speed of an object is doubled, its momentum

A. remains unchanged in accord with the conservation of momentum.

B. **doubles.**

$$P = m \cdot v$$

C. quadruples.

$$P = 2m \cdot v$$

D. decreases.

$$2P$$

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Conservation of Momentum

- In every case, the momentum of a system cannot change unless it is acted on by external forces.
- A system will have the same momentum both before and after any interaction occurs. When the momentum does not change, we say it is **conserved**.
- Law of conservation of momentum:
 - In the absence of an external force, the momentum of a system remains unchanged.
 - Equation form:

$$(\text{total momentum})_{\text{before}} = (\text{total momentum})_{\text{after}}$$

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conservation of momentum

$$P_{\text{system before}} = P_{\text{system after}}$$

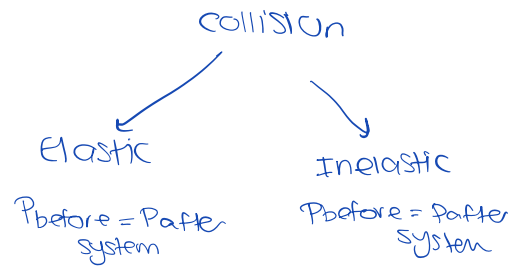
$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

• Inelastic collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v_1' = v_2' = v'$$

combine



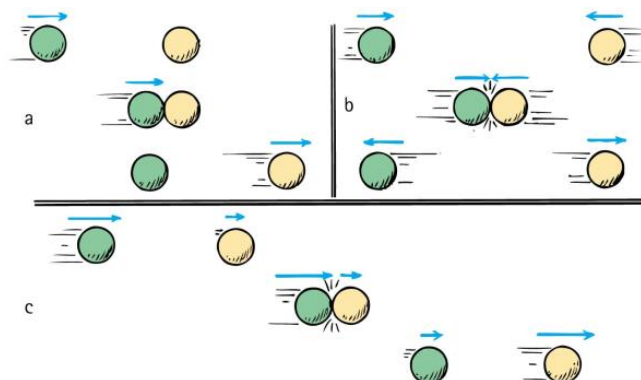
Conservation of Momentum

- Collisions
 - When objects collide in the absence of external forces,
net momentum before collision = net momentum after collision
 - *Examples:*
 - Elastic collisions
 - Inelastic collisions

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Conservation of Momentum

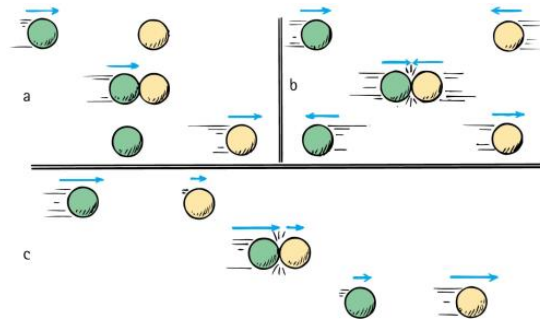
- Elastic collision
 - is defined as a collision whereupon objects collide without permanent deformation or the generation of heat (that is, total kinetic energy is conserved (not lost)). Elastic balls *bounce*!



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Conservation of Momentum

- In Figure (a)
 - moving green ball hits yellow ball, initially at rest
 - green ball comes to rest, and yellow ball moves away with a velocity equal to the initial velocity of the green ball
- In Figures (a) through (c)
 - momentum is simply transferred from one ball to the other.



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Conservation of Momentum

Example: A 4 kg ball is moving to the right at 7 m/s when it collides with a 9 kg ball that is moving initially to the left at 12 m/s. If the larger ball continues in the same direction after collision at 2 m/s, what is the speed of the smaller ball after collision?

$$m_1 = 4 \text{ kg} \quad m_2 = 9 \text{ kg}$$

$$\text{before } v_1 = +7 \text{ m/s} \quad v_2 = -12 \text{ m/s}$$

$$\text{after } v_1' = ? \quad v_2' = -2 \text{ m/s}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(4 \times 7) + (9 \times (-12)) = 4 \times v_1' + 9 \times (-2)$$

$$= -52 = 4v_1'$$

$$v_1' = 15.5 \text{ m/s}$$

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$$\begin{array}{l} \swarrow \\ \text{mag} \\ = 15.5 \text{ m/s} \end{array} \quad \begin{array}{l} \searrow \\ \text{dir} \\ (\text{left}) \end{array} \rightarrow \text{VERY IMP}$$

Conservation of Momentum

- Inelastic collision
 - is defined as a collision whereupon colliding objects become tangled or coupled together, generating heat (that is, total kinetic energy is not conserved). (Inelastic collisions are often *sticky*.)



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Conservation of Momentum CHECK YOUR NEIGHBOR

Freight Car A is moving toward identical Freight Car B that is at rest. When they collide, both freight cars couple together. Compared with the initial speed of Freight Car A, the speed of the coupled freight cars is

- A. the same.
 B. half.
 C. twice.
 D. None of the above.

$$m_A v_A + m_B v_B = (m_A + m_B) v^1$$

$$m v_A + m \times 0 = (m + m) v^1$$

$$\frac{m v_A}{2m} = \frac{(2m) v^1}{2m}$$

$$v^1 = \frac{v_A}{2}$$

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Conservation of Momentum

CHECK YOUR ANSWER

Freight Car A is moving toward identical Freight Car B that is at rest. When they collide, both freight cars couple together. Compared with the initial speed of Freight Car A, the speed of the coupled freight cars is

- A. the same.
- B. half.**
- C. twice.
- D. None of the above.

Explanation:

After the collision, the mass of the moving freight cars has doubled. Can you see that their speed is half the initial velocity of Freight Car A?

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Conservation of Momentum

Example: A 5 kg ball of clay moving at 18 m/s undergoes a collision with a 4 kg ball of clay at rest. If the two balls stick together as a result of the collision, what is the speed of the two balls after collision?

$$\begin{aligned} m_1 &= 5 \text{ kg} \\ v_1 &= 18 \text{ m/s} \\ m_2 &= 4 \text{ kg} \\ v_2 &= 0 \text{ m/s} \end{aligned}$$

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= (m_1 + m_2) v' \\ v_1' = v_2' = v' & \qquad \qquad \qquad = (5)(18) + (4)(0) \\ & \qquad \qquad \qquad = (90) + (0) \\ & \qquad \qquad \qquad = 90 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \end{aligned}$$

$$90 \text{ kg} \cdot \frac{\text{m}}{\text{s}} = (5 + 4) \cdot v'$$

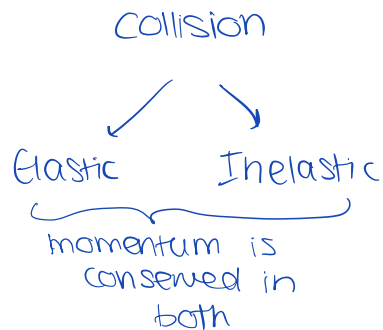
$$\frac{90 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{9 \text{ kg}} = \frac{9 \text{ kg} \cdot v'}{9 \text{ kg}}$$

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$10 \frac{\text{m}}{\text{s}} = v'$

- magnitude: 10 m/s
- direction: to the right

conservation of momentum



P_{system} ^{is} conserved in BOTH elastic and inelastic collision

In an elastic collision:

- kinetic energy in the system is conserved whereby

$$KE_{\text{before}} = KE_{\text{after}}$$

In an inelastic collision:

- kinetic energy in the system is NOT conserved

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \left. \vphantom{m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'} \right\} \text{used when dealing w/ elastic collisions}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v' \quad \left. \vphantom{m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'} \right\} \text{used when dealing with w/ inelastic collisions}$$

↓ inelastic / combine / stuck / tangled / coupled after collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

Conservation of Momentum

$$v_1 = v_2 = 0$$

Example: A 30 kg girl and a 25 kg boy face each other on frictionless roller blades. The girl pushes the boy, who moves away at a speed of 6 m/s. What is the resulting girl's speed?

$$v_1' = ?$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(30)(0) + (25)(0) = (30)(v_1') + (25)(6)$$

$$(30)(0) + (25)(0) = (30)(v_1') + (25)(6)$$

$$0 = (30)(v_1') + 150$$

$$\frac{-150}{30} = \frac{30(v_1')}{30}$$

$$v_1' = -5 \text{ m/s} \quad \begin{array}{l} \rightarrow \text{magnitude: } 5 \text{ m/s} \\ \rightarrow \text{direction: to the left} \end{array}$$

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Energy and Work ⊗ VERY IMPORTANT included in Major II, Final

- When you push a crate across a floor, you're doing work. The work done by you, by definition, is

$$\text{Work} = \text{force applied} \times \text{distance}$$

$$(J) W = F \times d$$

N x m

- Note that this formula applies **when the force is in the direction of the motion.**
- If the force is **opposite** to the direction of motion (e.g. friction), then the work done by that force is negative ($W = -F \times d$)
- If the force is **perpendicular** to the direction of motion (for example, the support force), then the work done by that force is **zero**
- Work is a **scalar quantity**. (doesn't have a direction)
- The unit of work is **N.m \equiv Joule (J)**

x work by friction is always -ve
x work by normal is always 0

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Extra Practice $\left[\begin{array}{ll} m_1 = 2 \text{ kg} & m_2 = 3 \text{ kg} \\ v_1 = 3 \text{ m/s} & v_2 = -4 \text{ m/s} \end{array} \right]$

1) object of mass $m_1 = 2 \text{ kg}$ moving with a speed 3 m/s to the right collides with another object of mass 3 kg moving to the left by 4 m/s

a) find the speed of the objects if they are stuck after collision?

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

whereby $v_1' = v_2' = v'$

$$(2)(3) + (3)(-4) = (2 + 3) v'$$

$$6 + (-12) = (5) v'$$

$$\frac{-6}{5} = \frac{(5) v'}{5}$$

$$v' = -1.2 \text{ m/s}$$

→ magnitude = 1.2 m/s

→ direction = to the left

b) Find the speed of the first object if the second object rebounds by 3 m/s

→ $v_1' = ?$ ⊗ not sure

$$m_1 = 2 \text{ kg}$$

$$m_2 = 3 \text{ kg}$$

↙ v_2 initially = -4 m/s

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(2)(3) + (3)(-4) = (2)(v_1') + (3)(3)$$

$$6 + (-12) = 2(v') + 9$$

$$-6 = 2(v') + 9$$

$$\rightarrow \frac{-15}{2} = \frac{2(v')}{2} = -7.5 \text{ m/s}$$

Energy and Work

He may expend energy when he pushes on the wall, but if the wall doesn't move, no work is done on the wall. Energy expended becomes thermal energy.

The word work, in common usage, means physical or mental exertion. Don't confuse the physics definition of work with the everyday notion of work.

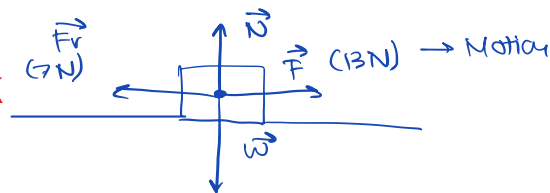
↙ magnitude
= 7.5 m/s
direction
= to the left



FIGURE 3.15
He may expend energy when he pushes on the wall, but if the wall doesn't move, no work is done on the wall. Energy expended becomes thermal energy.

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Energy and Work



Example: A box is moving to the right under the effect of a horizontal force of 13 N. The opposing force of friction is 7 N. During a movement of 25 m, calculate $W = F \times d$, $W = 13 \times 25 = 325 \text{ J}$

- a) the work done by the force (Ans. 325 J)
- b) the work done by friction $W_{fr} = -F_r \times d = -7 \times 25 = -175 \text{ J}$ (Ans. -175 J)
- c) the work done by gravity (Ans. 0)
- d) the work done by the support force (Ans. 0)
- e) The net work done on the object (Ans. 150 J)

↪ $W_{gr} = \text{perpendicular to motion} = 0$

↪ $W_{N} = \text{perpendicular to motion} = 0$

$$\rightarrow W_{net} = 325 + (-175) + 0 + 0 = 150 \text{ J}$$

22

Example: A box is moving to the right under the effect of a horizontal force of 45 N. The opposing force of friction is 10 N. During a movement of 15 m, calculate

- a) the work done by the force $W_{\vec{F}} = F \times d = 45 \times 15 = 675 \text{ J}$
- b) the work done by friction $W_{\vec{F}_r} = -F \times d = -10 \times 15 = -150 \text{ J}$
- c) the work done by gravity $W_{\vec{w}} = \text{perpendicular to motion} = 0$
- d) the work done by the support force $W_{\vec{N}} = \text{perpendicular to motion} = 0$
- e) The net work done on the object
- $$W_{\text{net}} = 675 + (-150) + 0 + 0 = 525 \text{ J}$$

east

1) object of mass $m_1 = 4 \text{ kg}$ moving with a speed 5 m/s to the East collides with another object of mass 8 kg moving to the west by 2 m/s

a) find the speed of the objects if they are stuck after collision?

$$m_1 = 4 \text{ kg}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$(4)(5) + (8)(-2) = (4 + 8) v'$$

$$20 + (-16) = (12) v'$$

$$\frac{4}{12} = \frac{(12) v'}{12}, \quad v' = 0.33 \text{ m/s}$$

b) Find the speed of the first object if the second object rebounds by 1 m/s

$$v_2' = 1 \text{ m/s}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(4)(5) + (8)(-2) = (4)(v_1') + (8)(1)$$

$$20 + (-16) = (4)(v_1') + 8$$

$$4 = 4(v_1') + 8$$

$$\frac{-4}{4} = \frac{4(v_1')}{4}$$

$$v_1' = -1 \text{ m/s}$$

• magnitude: 1 m/s

• direction: to the west

Energy and Work

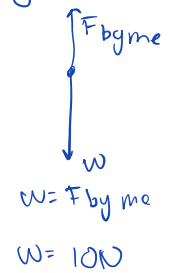
⊗ constant speed
(same direction)
 $F_{net} = 0$

$$W = F \times d = W = (m \cdot g) \times d$$

Example: what is the work done by me in lifting a 1 kg box 1 m off the ground at constant speed? $(1 \text{ kg} \cdot 10) \times 1 \text{ m} = 10 \text{ Joules}$

Example: what is the work done by me in lowering a 1 kg box 1 m down to the floor at constant speed? $(1 \text{ kg} \cdot 10) \times 1 \text{ m} = -10 \text{ Joules}$

Example: I carry a box to the right across the floor at constant speed. Which of the following shows the direction of the force I apply to the box?



- (A) (B) (C) no force at all
- (D) (E)

Example: how much work is done by me in carrying the box to the right at constant speed?



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Energy and Work

Example: Work is done in lifting a barbell. How much work is done in lifting a twice-as-heavy barbell the same distance?

- a) The same
- b) half as much
- c) twice as much
- d) Not enough information



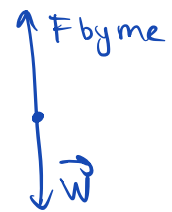
Example: A student carries a load of 500 N for a 100 m distance at constant speed. The work done by him is

- a) 5 J
- b) 50000 J
- c) 0 J
- d) 0.2 J
- e) 0.5 J

↑ to the right

→ motion

↓ perpendicular



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Energy and Work

- In general, **energy** is the property of a system that enables it to do *work*.
- There are many forms of energy.
- We will discuss two forms of energy:
 1. **Potential Energy = Stored Energy**
 2. **Kinetic Energy = Energy of Motion**

↳ anything that is moving

Formula

$$K.E. = \frac{1}{2} m v^2 = \text{measured in Joules}$$

↳ J

↓ ↓ m/s
kg

Formula

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Potential Energy (PE)

- Potential energy (also referred to as stored energy) is the ability of a system to do work due to its position or internal structure.
- Examples are energy stored in a spring, fuel, food, wood, batteries,
- Potential energy is a **scalar quantity** with the same units as work, **joules (J)**.
- Here we will focus only on the **gravitational potential energy**
- Gravitational potential energy is the potential energy due to an **elevated position**. The work done to lift the object at constant speed is equal to the potential energy stored in the object.
- To lift an object at a constant speed, a force equal to its weight (mg) is applied to move it to height (h)

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$$K.E. = \frac{1}{2} m v^2$$

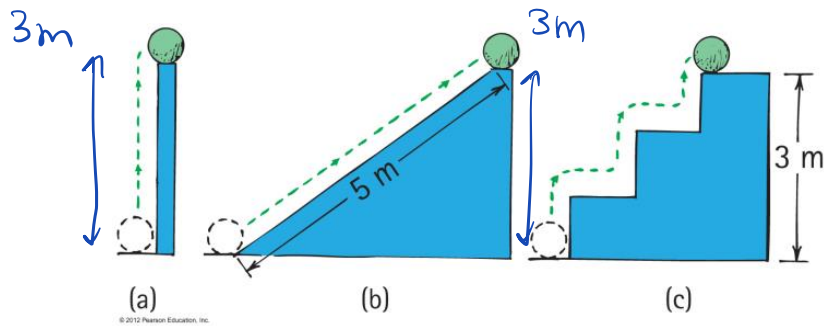
| m | v | KE |
|----------------|---------------|---------------------------------|
| 2m | v | 2KE |
| m | 2v | (2) ² KE |
| 3m | 4v | 3(4) ² KE |
| $\frac{1}{5}m$ | $\frac{1}{2}$ | $\frac{1}{5}(\frac{1}{2})^2$ KE |

Potential Energy

- Therefore, the potential energy is:

$$PE = mgh$$

- Note: The work done against gravity depends on the height and does not depend on the path taken.



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Potential Energy

Example: What is the gravitational potential energy of a 3 kg box placed at the top of a 25 m high building.

$$GPE = mgh$$

- $m = 3 \text{ kg}$
- $g = 10 \text{ m/s}^2$
- $h = 25$

$$= 750 \text{ J}$$

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Kinetic Energy (KE)

- **Kinetic energy:** A moving object has kinetic energy. is the energy of an object in motion. In other words, it is the energy possessed by an object due to its velocity.
- The kinetic energy (KE) of an object of mass m moving with speed v is defined as

$$KE = \frac{1}{2}mv^2$$

- Kinetic energy is a form of energy, and its unit is **Joule (J)** (same as work). K.E. is a **scalar** quantity (any energy is scalar).

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Kinetic Energy

- Kinetic energy is **directly proportional to the mass** and **directly proportional to the square of velocity**. For example:

$$2m \rightarrow 2K.E$$

$$2v \rightarrow 4K.E$$

$$3m \rightarrow 3K.E$$

$$3v \rightarrow 9K.E$$

$$5m \rightarrow 5K.E$$

$$5v \rightarrow 25K.E$$

$$\frac{1}{2}m \rightarrow \frac{1}{2}K.E$$

$$\frac{1}{3}v \rightarrow \frac{1}{9}K.E$$

and so on

and so on

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Kinetic Energy

$$K.E. = \frac{1}{2} m v^2$$

Example: An object of mass $m = 1.5 \text{ Kg}$, moves with a velocity of 10 m/s

- a) Calculate the kinetic energy of this object $\frac{1}{2} (1.5) (10)^2 = 75 \text{ J}$
 b) What happens to its KE if its velocity is doubled? $\frac{1}{2} (1.5) (20)^2 = 300 \text{ J}$
 c) What happens to its KE if its velocity is tripled? $\frac{1}{2} (1.5) (30)^2 = 675 \text{ J}$ (quadrupled)

$$\frac{1}{2} (1.5) (30)^2$$

$$\frac{1}{2} (1.5) (3 \cdot 10)^2 = \frac{1}{2} (1.5) (300) = 675 \text{ J}$$

increases by 9

Example: What is the speed of a running woman of mass 60 kg having 1920 J of kinetic energy?

$$K.E. = \frac{1}{2} 60 \cdot v^2$$

$$1920 = \frac{1}{2} 60 \cdot v^2$$

$$\frac{1920}{30} = \frac{30 \cdot v^2}{30}$$

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$$v^2 = 64$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$v = 8 \text{ m/s}$$

Kinetic Energy

$$K.E. = 124 \text{ J}$$

Example: A bicycle has a kinetic energy of 124 J . What kinetic energy would the bicycle have if it had ...

- a) Twice the mass and was moving at the same speed?

$$\hookrightarrow 2m \rightarrow 2 K.E. = 2 \times 124 = 248 \text{ J}$$

- b) The same mass and was moving with twice the speed?

$$\hookrightarrow m \quad 124 \times 4 = 496 \text{ J} \quad (2)^2 v = 4v$$

- c) One-half the mass and was moving with twice the speed?

$$\hookrightarrow \frac{1}{2} m \quad K.E. \text{ doubles} = 124 \times 2 = 248 \text{ J} \quad \hookrightarrow 2v^2 = 4v$$

- d) The same mass and was moving with one-half the speed?

$$\hookrightarrow m \quad 124 \times \frac{1}{4} = 31 \text{ J} \quad \hookrightarrow \left(\frac{1}{2} v\right)^2 = \frac{1}{4} v^2$$

- e) three times the mass and was moving with one-half the speed?

$$\hookrightarrow 3m \quad \hookrightarrow \left(\frac{1}{2} v\right)^2 = \frac{1}{4} v^2$$

$$\frac{3}{4} K.E. = 124 \times \frac{3}{4} = 93 \text{ J}$$

$$K.E. = \frac{1}{2} m v^2$$

↑ J
↓ kg ↓ m/s

| m | v | K.E. |
|----|-----------------|---------------------------------------|
| 2m | $\frac{1}{3} v$ | $2 \times \left(\frac{1}{3}\right)^2$ |

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Kinetic Energy

$$m_{\text{man}} = 80 \text{ kg}$$

$$m_{\text{horse}} = 320 \text{ kg}$$

Example: Consider an 80 kg man and 320 kg horse both running along a road with the same kinetic energy. The man must run

- With the same speed as the horse.
- 4 times as fast as the horse.
- Twice as fast as the horse.
- 16 times as fast as the horse.

$$KE_{\text{man}} \text{ 4 times}$$

$$KE_{\text{horse}}$$

$$4 KE_{\text{man}} = KE_{\text{horse}}$$

$$4 \times \frac{1}{2} m_{\text{man}} \times v_{\text{man}}^2 = \frac{1}{2} m_{\text{horse}} \times v_{\text{horse}}^2$$

$$\frac{4 \times 80 \times v_{\text{man}}^2}{80} = \frac{320 \times v_{\text{horse}}^2}{80}$$

⊗

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$$KE_{\text{man}} = KE_{\text{horse}}$$

$$\frac{1}{2} m_{\text{man}} \times v_{\text{man}}^2 = \frac{1}{2} m_{\text{horse}} \times v_{\text{horse}}^2$$

$$\frac{80 \times v_{\text{man}}^2}{80} = \frac{320 \times v_{\text{horse}}^2}{80}$$

$$v_{\text{man}} = 2 v_{\text{horse}}$$

$$v_{\text{man}} = 2 v_{\text{horse}}$$

Mechanical Energy (ME)

- The sum of the kinetic and potential energies of an object is called the mechanical energy

$$ME = KE + PE$$

$$\frac{1}{2} m v^2 + mgh$$

Example: A 78-kg skydiver has a speed of 62 m/s at an altitude of 870 m above the ground.

- Determine the kinetic energy possessed by the skydiver. $\frac{1}{2} m v^2 = \frac{1}{2} 78 (62)^2 = 149,916$
- Determine the potential energy possessed by the skydiver. $(78)(10)(870) = 678,600 \text{ J}$
- Determine the total mechanical energy possessed by the skydiver.

$$ME = KE + PE$$

$$149,916 + 678,600 = 828,516 \text{ J}$$

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Work-Energy Theorem

- **Work-Energy Theorem:** The **net work** done on an object is **equal to the change in the kinetic energy** of the object.

$$\Sigma W = \Delta KE$$
$$\Sigma W = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Example: How much work is required to increase the kinetic energy of an object from 360 J to 1800 J?

Example: The speed of a 900 kg car is increased from 90 km/h to 126 km/h. Calculate the work done on the car. (270 kJ)

Note: 90 km/h = 25 m/s, 126 km/h = 35 m/s.

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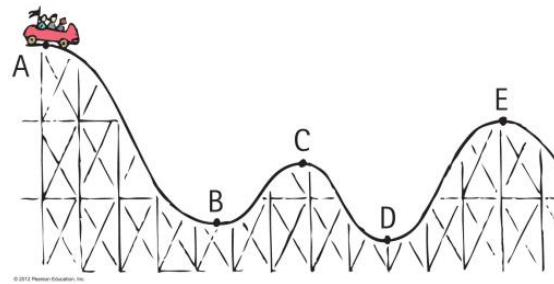
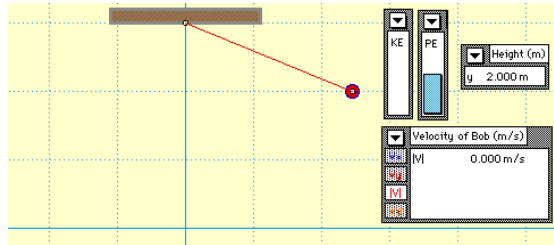
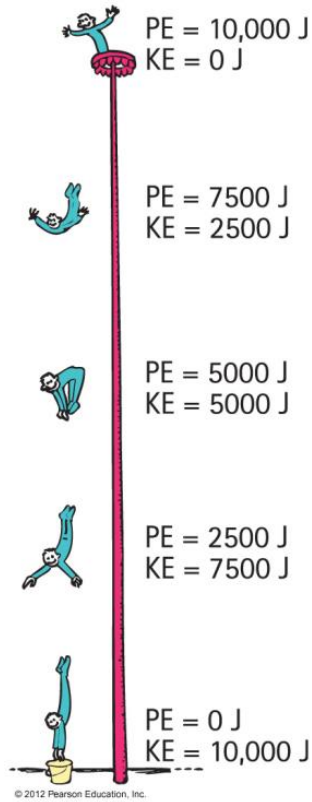
Conservation of Energy

- Principle of Conservation of Energy: **Energy cannot be created or destroyed, it may be transformed from one form into another or transferred from one object to another, but the total amount of energy never changes.**
- If the only force doing work is the **force of gravity**, then the **mechanical energy of the object is conserved:**

$$PE_i + KE_i = PE_f + KE_f$$

- Examples include:
 - Free-fall (no air resistance)
 - Pendulum (no frictional forces)
 - Roller coaster (no engine or friction)

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Conservation of Energy

Example: If air resistance is negligible, the sum of potential and kinetic energies of a freely falling body _____

- a) increases
 - b) decreases
 - c) becomes zero
 - d) remains the same** (ME is conserved when air resistance is negligible)
- $w = mg \Rightarrow m = \frac{w}{g} = \frac{40}{10} = 4 \text{ kg}$

Example: A 40 N object is released from a height of 10 m. Just before it hits the ground, its kinetic energy, in joules is:

- a) 400**
- b) 3920
- c) 2800
- d) 4000

$$KE_1 = PE_2$$

$$KE_2 = PE_1$$

$$ME_1 = ME_2$$

$$KE_1 + PE_1 = KE_2 + PE_2$$

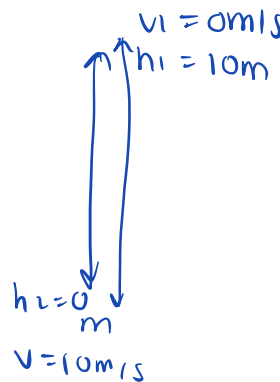
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$$0 + PE_1 = KE_2 + PE_2$$

$$0 + mgh_1 = KE_2 + 0$$

$$0 + 4(10)(10) = KE_2 + 0$$

$$400 (PE) = 400 (KE)$$



a) what is the speed of the athlete of P.E.
= 4000J and mass_{athlete} = 60kg

$$ME = 10,000$$

$$10,000 = KE + PE$$

$$10,000 = KE + 4000$$

$$\downarrow 6000 = KE = \frac{1}{2}(60)(v)^2$$

$$\frac{6000}{30} = \frac{30v^2}{30}$$

$$200 = v^2$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$v = 14.1 \text{ m/s}$$

b) what is the height of athlete of the KE = 7800J
and mass_{athlete} = 60kg

$$ME = PE + KE$$

$$101000 = PE + 7800$$

$$- 7800 \quad \quad - 7800$$

$$2200 = PE$$

$$PE = mgh$$

$$2200 = (60)(10)(h)$$

$$\frac{2200}{600} = \frac{600h}{600}$$

$$h = 3.67 \text{ m}$$

⊕ only force doing work is weight
Mechanical Energy is conserved (constant)

$$ME_1 = ME_2$$

$$KE_1 + PE_1 = KE_2 + PE_2$$

c) what is the speed of the athlete at a height of 10m / $m_{\text{athlete}} = 60\text{kg}$

$$ME = 10,000\text{J}$$

$$ME = KE + PE \rightarrow PE = mgh$$
$$= (60)(10)(10) = 6000\text{J}$$

$$10,000 = KE + 6000$$

$$KE = 4000\text{J}$$

$$4000 = \frac{1}{2} m v^2$$

$$4000 = \frac{1}{2} (60) (v^2)$$

$$\sqrt{\frac{4000}{30}} = \frac{30 v^2 \sqrt{\quad}}{30}$$

$$v^2 = 133.33$$

$$v = 11.5\text{ m/s}$$

⊕ at what height + speed we have same kinetic and potential (KE = PE)

RECAP

$$ME = KE + PE$$

$$ME = \frac{1}{2}mv^2 + mgh$$

No friction / no air resistance and the only force doing work weight

ME is conserved (constant)

$$ME_1 = ME_2$$

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

(cont.)

d) $h = ??$ $v = ??$ $KE = PE?$

$$KE = PE$$

$$PE = mgh$$

$$KE = \frac{1}{2}mv^2$$

$$5000 = 60 \times 10 \times h$$

$$5000 = \frac{60}{2}v^2$$

$$\frac{5000}{600} = \frac{600 \times h}{600}$$

$$\frac{5000}{30} = v^2$$

$$h = \frac{5000}{600} \text{ m}$$

$$v = \sqrt{\frac{5000}{3}} \text{ m/s}$$

NO friction

$m = 500\text{kg}$ and is released from rest at A

$$h_A = 15\text{m}$$

$$v_A = 0\text{m/s}$$

a) calculate M.E. at A?

$$ME = KE + PE$$

$$\frac{1}{2}mv^2 + mgh$$

$$\frac{1}{2}(500)(0)^2 + (500)(10)(15)$$

$$ME = 75000\text{J}$$

b) calculate v_B if $h_B = 5\text{m}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(500)v^2$$

$$PE = mgh = (500)(10)(5) = 25000$$

$$ME = 75000\text{J}$$

$$75000 = \frac{1}{2}(500)v^2 + 25000$$
$$-25000 \quad \quad \quad -25000$$

$$\frac{50000}{250} = \frac{250v^2}{250}$$

$$200 = \frac{v^2}{1}$$

$$v_B = \sqrt{200} \approx 14.14\text{m/s}$$

c) calculate h_c if $v_c = 10 \text{ m/s}$

$$ME_c = 75000 \text{ J}$$

$$KE_c = \frac{1}{2} mv^2 = \frac{1}{2} (500)(10)^2 = 25000 \text{ J}$$

$$PE_c = mgh = (500)(10)h = 5000h$$

$$ME_c = KE_c + PE_c$$

$$75000 = 25000 + 5000h$$
$$-25000 \quad -25000$$

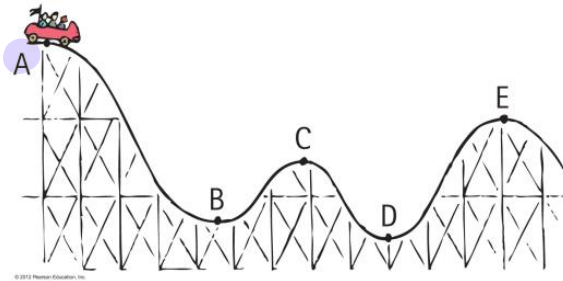
$$\frac{50000}{5000} = \frac{5000h}{5000}$$

$$h_c = 10 \text{ m}$$


Conservation of Energy

Example: The roller coaster ride starts from rest at point A. Rank these quantities from greatest to least at each point:

- a) Speed
- b) KE
- c) PE



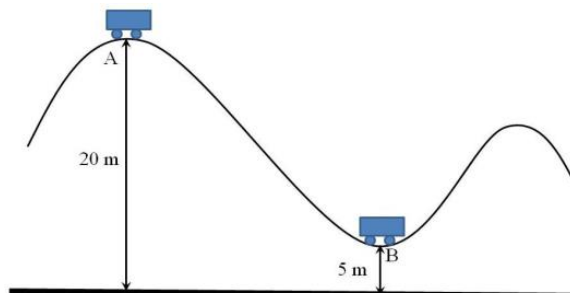
| | Maximum at | Minimum at |
|-------|------------|------------|
| speed | D | A |
| KE | D | A |
| PE | A | D |

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Exercises

Example: An isolated object has 500 J of gravitational potential energy and 300 J of kinetic energy at a certain location. If its kinetic energy is increased to 700 J, what is its new gravitational potential energy?

Example: The shown roller coaster has a mass of 200 kg including passengers. If its speed at point A is 4 m/s, calculate its speed at point B, neglecting air resistance and friction. (Ans. 17.8 m/s)



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1)

$$PE_1 = 500J$$

$$KE_1 = 300J$$

$$PE_2 = ??$$

$$KE_2 = 700J$$

$$ME_1 = ME_2$$

$$PE_1 + KE_1 = PE_2 + KE_2$$

$$500 + 300 = PE_2 + 700$$

$$800 = PE_2 + 700$$

$$PE_2 = 100J$$

2)

$$\text{mass} = 200\text{kg}$$

$$\text{speed}_A = 4\text{m/s}$$

$$\text{speed}_B = ??$$

$$ME_A = ME_B$$

$$PE_A + KE_A = PE_B + KE_B$$

$$mgh_A + \frac{1}{2}mv_A^2 = mgh_B + \frac{1}{2}mv_B^2$$

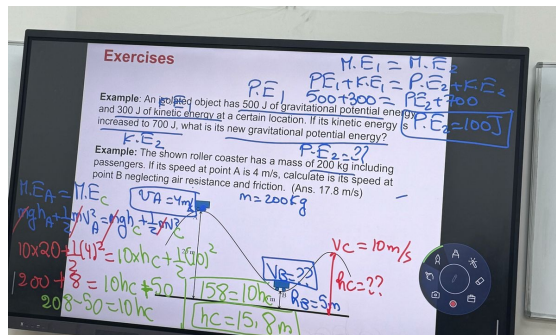
$$(200)(10)(20) + \frac{1}{2}(200)(4)^2 = (200)(10)(5) + \frac{1}{2}(200)(v)^2$$

$$401000 + 1600 = 10000 + 100v^2$$

$$411600 = 10000 + 100v^2$$

$$\frac{311600}{100} = \frac{100v^2}{100}, v^2 = 3116, v = 17.7 \approx 17.8\text{m/s}$$

thus the speed at point B is equivalent to 17.8 m/s



Sources of Energy

- The sun, which provides solar energy, is practically the source of all our energy, except nuclear energy.
 - Petroleum, Coal, Natural gas, and Wood originally came from the sun since fuel are created by (photosynthesis).
 - Sunlight evaporates water, which later falls as rain, rainwater flows in to dams where it's directed to generator turbines.
 - The energy of wind can be used to turn generator turbines within specially equipped windmills.(and wind is caused by unequal warming of Earth's surface).
- Nuclear fuel – Uranium and plutonium
 - It is the most concentrated source of usable energy and for the same weight of fuel, nuclear reactions release about one million times more energy than do chemical or food reactions.

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Sources of Energy

The ultimate source of energy for wind power, fossil fuel, and rain is

- a) Nuclear
- b) Matter itself
- c) Solar
- d) None of the above

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