Scaler is just adding #s

Vectors

mention the direction

Chapter 3

Units of Chapter 3

x & y is either one of them

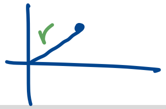
Scalars Versus Vectors

- **Scalar**: number with units   Just numbers
  - Examples: time, temperature

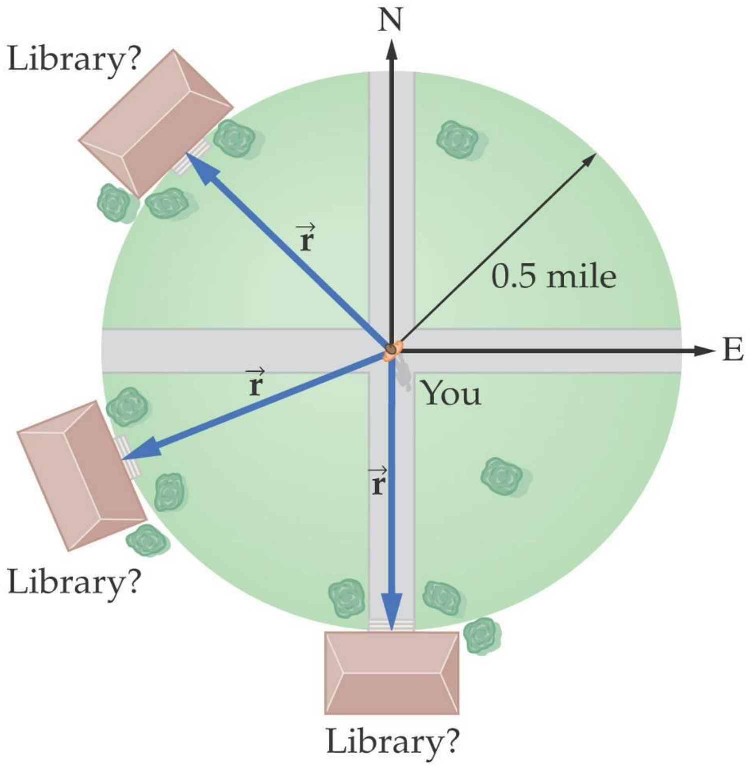
displace velocity, acceleration   # 0.5 mile   south of west

- **Vector**: quantity with **magnitude and direction**
  - How to get to the Library? You need to know how far and in which direction
  - Examples: position, displacement, velocity
  - $\vec{r}$ represents a vector of magnitude (number) r or $|\vec{r}|$

يمثل متجهًا للحجم

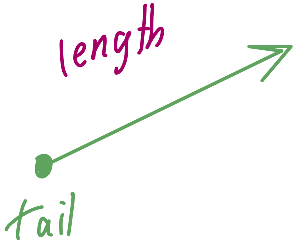


The position in 2 Dimension



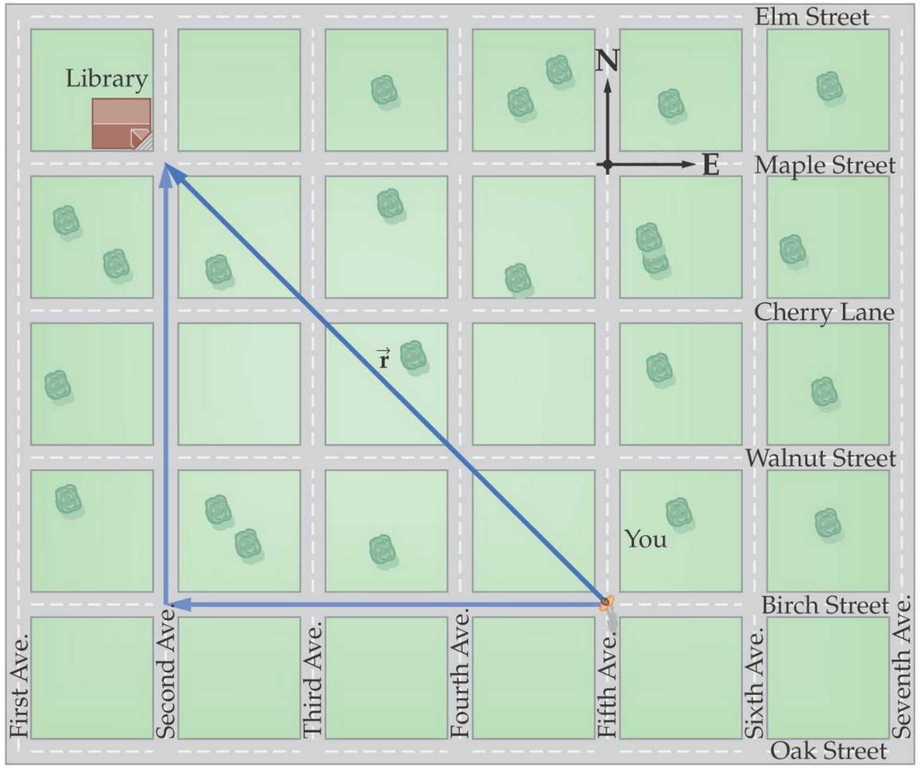
Describe Vector either by component or magnitude & Direction

Vector Components

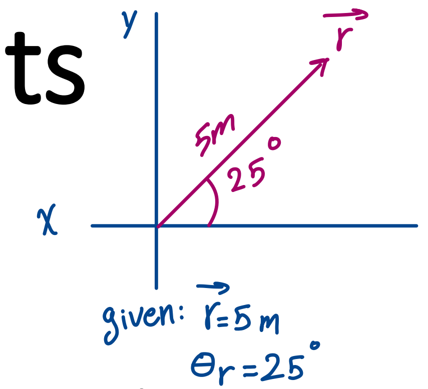


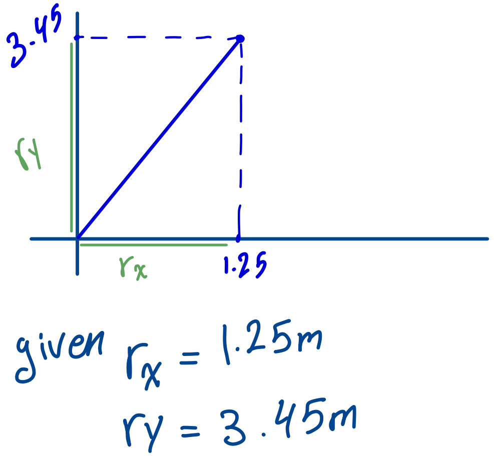
Know Direction by head.

- Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line!
- Adding the components of $\vec{r}$ leads to the same vector $\vec{r}$] means vector OR with Bold font r **r**
- To describe a vector, one needs:
  - It's length and direction (angle), OR
  - It's components (here two components; along the x-axis and the y-axis). For 3D vectors you need 3 components.



Vector Components

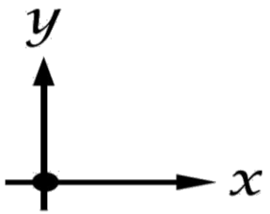


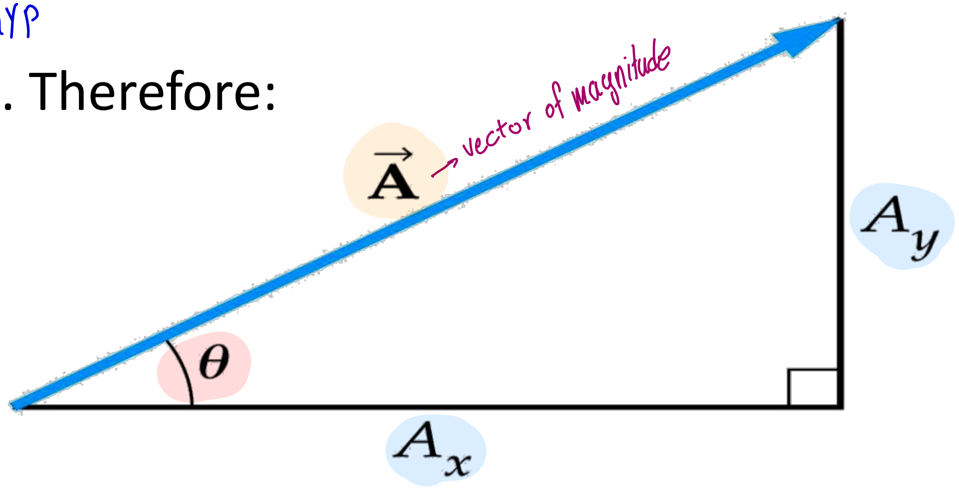


I. If you know the length (magnitude) and the angle, you can calculate the components using trigonometry.

Note that $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$. Therefore:

- $A_x = A\cos\theta$
- $A_y = A\sin\theta$

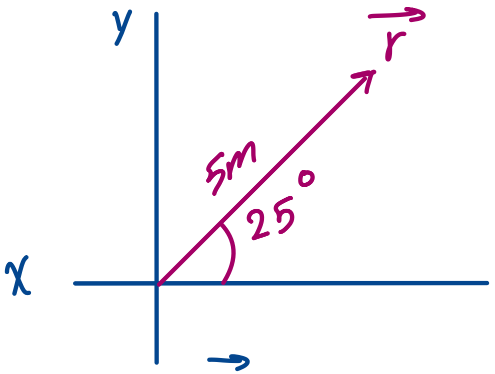




II. On the other hand, if you know the components you can calculate the length and the angle: (Pythagorean theorem)

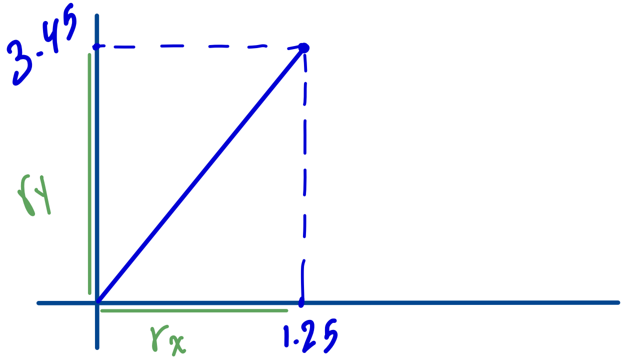
$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{|A_y|}{|A_x|}$$

To present vector



given: $\vec{r} = 5\,m$

$\theta_r = 25°$



given $r_x = 1.25m$

$r_y = 3.45m$

$$hyp^2 = adj^2 + opp^2$$

$$|r|^2 = r_x^2 + r_y^2$$

$$|r| = \sqrt{r_x^2 + r_y^2}$$

$$\tan\theta = \frac{opp}{adj} = \frac{r_y}{r_x}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

find

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad (1)$$

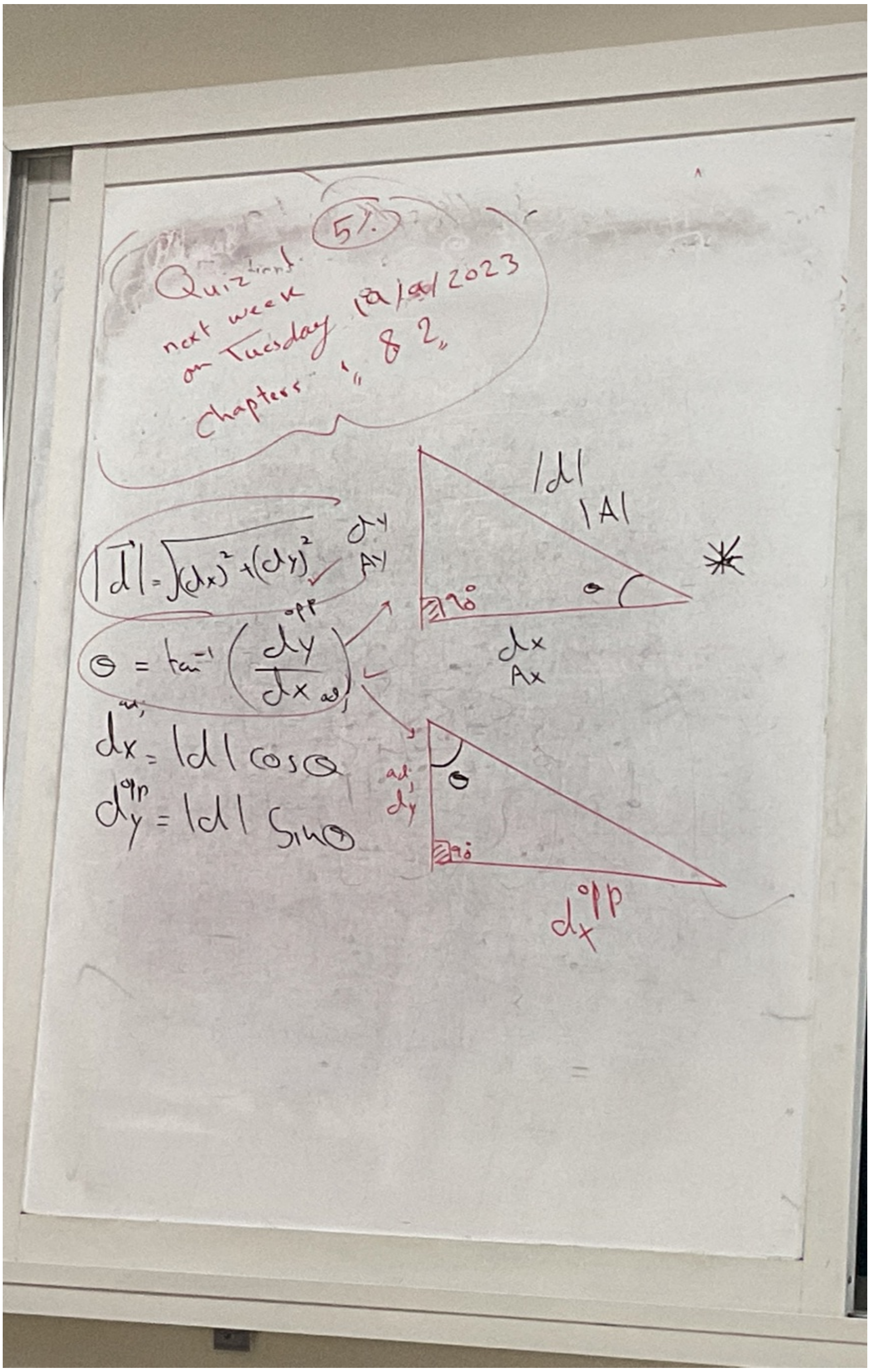
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad (2)$$

By magnitude

$$A_x = |A| \cdot \cos\theta \quad (3)$$

$$A_y = |A| \sin\theta \quad (4)$$

By component

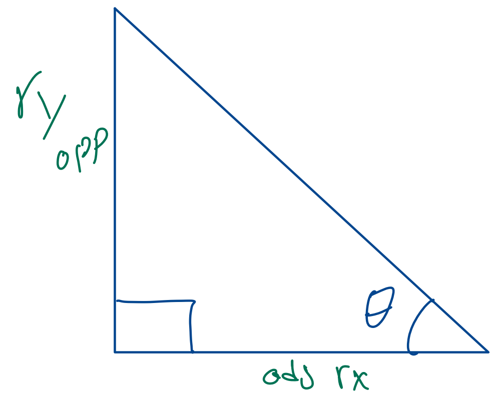


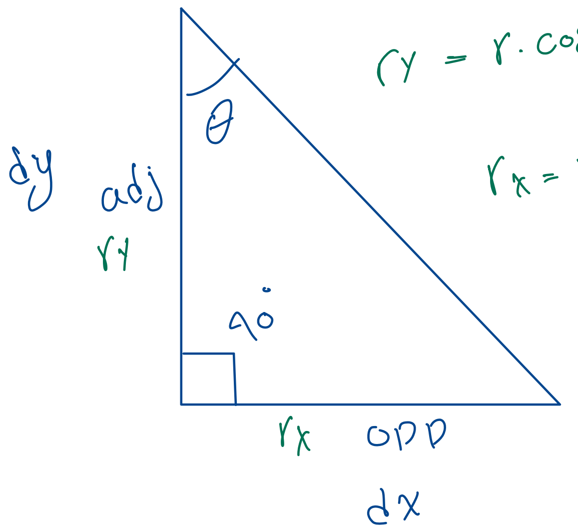
5%
Quiz
next week
on Tuesday
Chapters 1 & 2
|d| = √((dx)² + (dy)²)
θ = tan⁻¹ (dy / dx)
dx = |d| cos θ
dy = |d| sin θ
|d|
|A|
dy
Ay
dx
Ax
opp
adj
90°
θ

$$|\vec{d}| = \sqrt{dx^2 + dy^2}$$

$r_y = r \cdot \sin\theta$

$r_x = r \cdot \cos\theta$

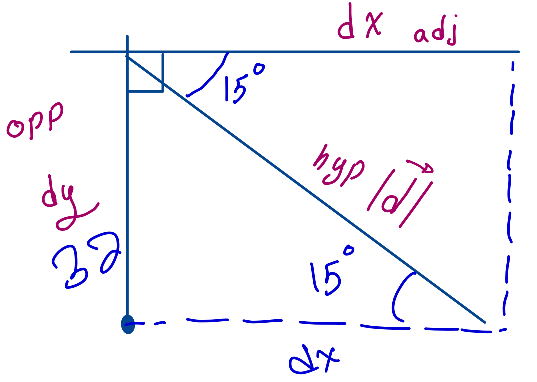




$r_y = r \cdot \cos\theta$

$r_x = r \cdot \sin\theta$

Problems



$\tan\theta = \frac{dy}{dx}$

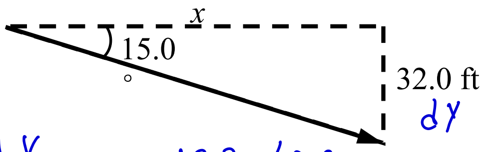
$dx = \frac{dy}{\tan\theta} = \frac{32}{\tan 15} = 119.4 = 119$ ft (3 sf)

Problem 5
The Press box at a baseball park is 32.0 ft above the ground. A reporter in the press box looks at an angle of 15.0° below the horizontal to see second base. What is the horizontal distance from the press box to second base?

$A_x = A\cdot\cos\theta$

$A_y = A\sin\theta$

$A = \frac{A_y}{\sin\theta} = 123.63$ ft



$A_x = 123.63 \times \cos 15 = 119.4 = 119$ ft

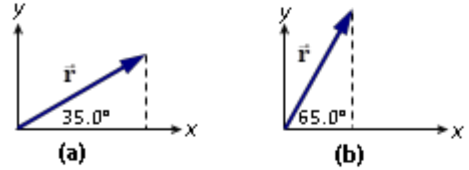
Answer: 119 ft

Problem 8
Find the x and y components of a position vector **r** of magnitude r = 75 m, if its angle relative to the x axis is (a) 35.0° and (b) 65.0°.

Answer: (a) r_x

(a) $r_x = |r|\cos\theta_a = 75 \times \cos 35 = 61$ m

$r_y = |r|\sin\theta_a = 75 \times \sin 35 = 43$ m



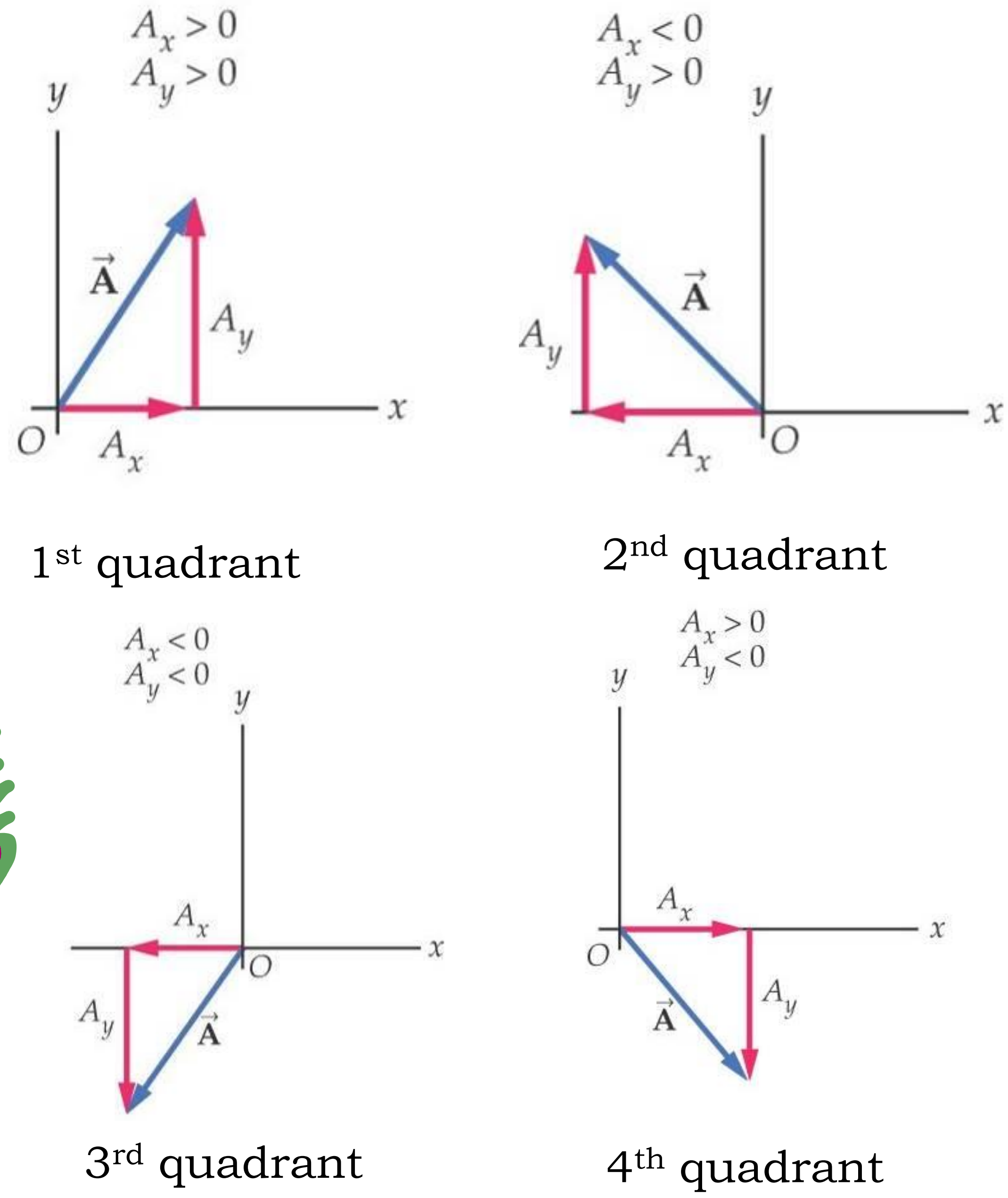
(a) $r_x = 61\,m$; $r_y = 43\,m$ (b) $r_x = 32\,m$; $r_y = 68\,m$

multiple choic

& problems.

Vector Components

- Direction angle: the full angle with the positive *x*-axis counterclockwise. The signs of components will come automatically if you use the **full angle.** You can use the small angle (with +ve/−ve axes) but be careful with the signs.
- Signs of vector components.



$r_y = r\sin\theta = (150\text{ m})\sin(20.0°) = 51\text{ m}$ = 0.14 km

Problems

Direction:

$\theta = \tan^{-1}\left(\frac{340}{680}\right) =$ $\frac{opp}{adj}$

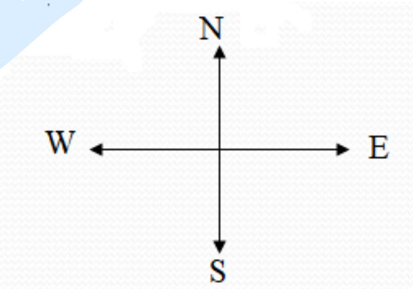
$|\vec{d}| = \sqrt{dx^2 + dy^2}$ displacement

$= \sqrt{680^2 + 340^2} = 760\text{ ft}$

Problem 14

You drive a car 680 ft to the east, then 340 ft to the north. (a) What is the magnitude of your displacement? (b) estimate the direction of your displacement with a numerical calculation.

The Sign of component Show the quarter/



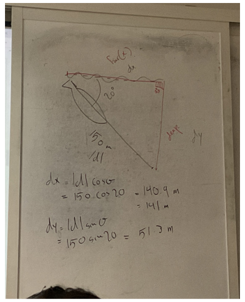
Answer: $r = 760\ ft$ $\qquad \theta = 27^0\ north\ of\ east$

North of East

To Know the place of θ

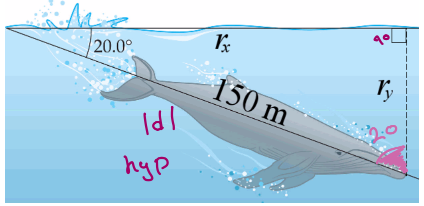
Problem 17

A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal. If the whale continues in a straight line for 150 m, (a) how deep it is, and (b) how far has it travelled horizontally?



Answer: The depth is given by r_y: $\qquad r_y = r\sin\theta = (150\text{ m})\sin(20.0°) = \boxed{51\text{ m}}$

(b) The horizontal travel distance is given by r_x: $\qquad r_x = r\cos\theta = (150\text{ m})\cos(20.0°) = \boxed{140\text{ m}} = 0.14\text{ km}$

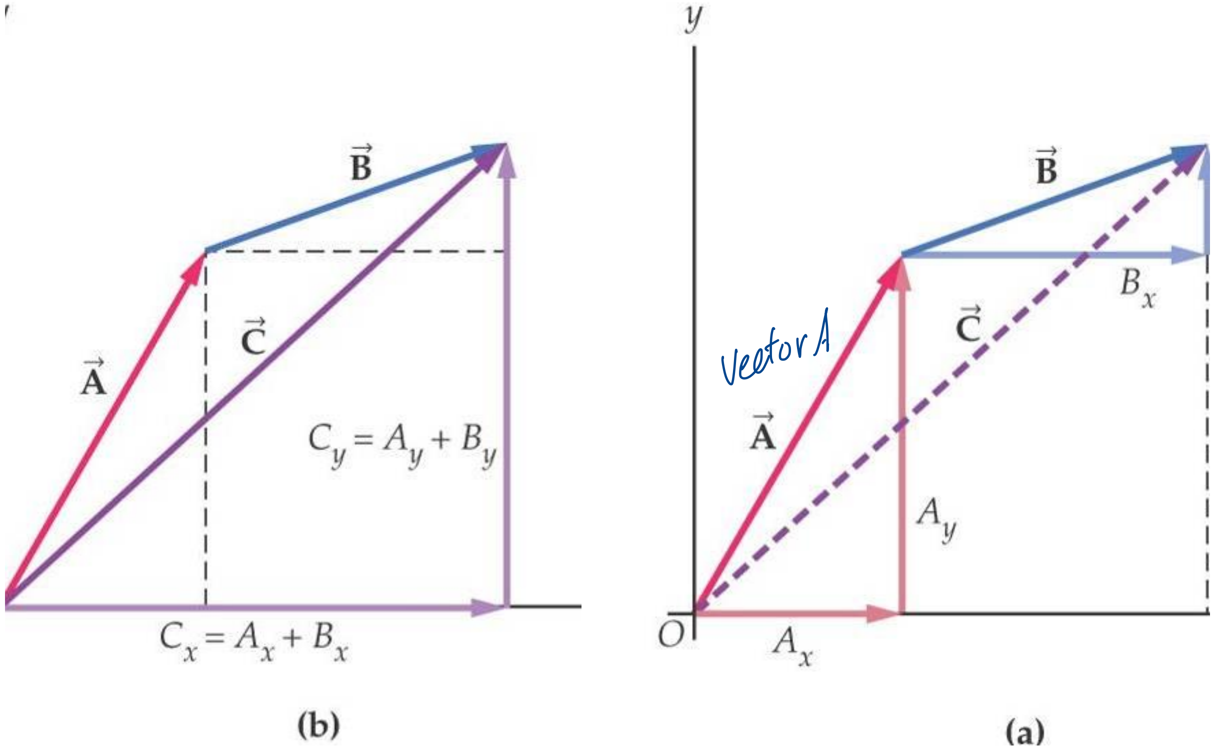


$dx = 150 \cdot \sin 20$

$dy = 150 \cdot \cos 20$

Adding Vectors Analytically/Mathematically

- Vectors are added by components:
- If $\vec{C} = \vec{A} + \vec{B}$ then $C_x = A_x + B_x$ and $C_y = A_y + B_y$



3-3 Adding and Subtracting Vectors

Adding Vectors Using Components:

1. Find the components of each vector to be added.
2. Add the *x*- and *y*-components separately.
3. Find the resultant vector.

Example

$|\vec{C}| = \sqrt{Cx^2 + Cy^2}$

$\sqrt{6.25^2 + 5.66^2} = 8.43$ m

- The vector $\vec{A}$ has a magnitude of 5.0^o m and the direction angle of 60^o;the vector $\vec{B}$ has a magnitude of 4.00m and a direction angle of 20^o.

$C_x = A_x + B_x$

$C_y = A_y + B_y$

1) Find (a) magnitude of resultant vector '$\vec{C} = \vec{A} + \vec{B}$

(b) and, its direction angle

$Ax = A \cdot \cos\theta = 5 \cdot \cos 60 = 2.5$ m

$Ay = A \cdot \sin\theta = 5 \cdot \sin 60 = 4.3$ m

$Bx = B \cdot \cos\theta = 4 \cdot \cos 20 = 3.75$ m

$By = B \cdot \sin\theta = 4 \cdot \sin 20 = 1.36$ m

2) Find (a) magnitude of resultant vector $\vec{D} = \vec{A} - \vec{B}$

(b) and, its direction angle

$Cx = 2.5 + 3.75 = 6.25$ m

$Cy = 4.3 + 1.36 = 5.66$ m

1st found The component

Solution: Ax = 2.50m, Ay = 4.33m , Bx= 3.76m, By= 1.37m

1) Cx = 6.26m, Cy= 5.70m

a1) |C| =8.47m, a1) ϴ= 42.3^o

2) Dx = -1.26 m, Dy= 2.96 m

a2)|D| = 3.22 m , a2) ϴ= - 67^o = 113^o

+180 = 113^o

$|\vec{A}| = 5.0\,m$, $\theta_A = 60^\circ$

$|\vec{B}| = 4.0\,m$, $\theta_B = 20^\circ$

$|\vec{C}| = ?$ $\theta_C = ?$

$$\boxed{\vec{C} = \vec{A} + \vec{B}}$$

(A) $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right)$

$C_x = A_x + B_x$, $C_y = A_y + B_y$

$A_x = |A| \cos\theta = 5 \cos 60 = 2.5\,m$

$A_y = |A| \sin\theta = 5 \sin 60 = 4.3\,m$

$B_x = |B| \cos\theta = 4 \cos 20 = 3.8\,m$

$B_y = |B| \sin\theta = 4 \sin 20 = 1.4\,m$

$C_x = 2.5 + 3.8 = 6.3\,m$

$C_y = 4.3 + 1.4 = 5.7\,m$

$|C| = \sqrt{(6.3)^2 + (5.7)^2} = 8.5\,m$

$\theta = \tan^{-1}\left(\frac{5.7}{6.3}\right) = 42^\circ$

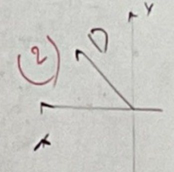
$|\vec{D}| = ?$ $\theta_D = ?$ $\vec{D} = \vec{A} - \vec{B}$

$|\vec{D}| = \sqrt{D_x^2 + D_y^2}$, $\theta_D = \tan^{-1}\left(\frac{D_y}{D_x}\right)$

$D_x = A_x - B_x$, $D_y = A_y - B_y$

$D_x = 2.5 - 3.8 = -1.3\,m$

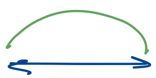
$D_y = 4.3 - 1.4 = 2.9\,m$

(2) 

$|D| = \sqrt{(-1.3)^2 + (2.9)^2} = 3.2\,m$

$\theta_D = \tan^{-1}\left(\frac{2.9}{-1.3}\right) = -66^\circ + 180 = 114^\circ$

$90^\circ < \theta < 180^\circ$

negative vectors 

are opp by 180

but, same length

equal

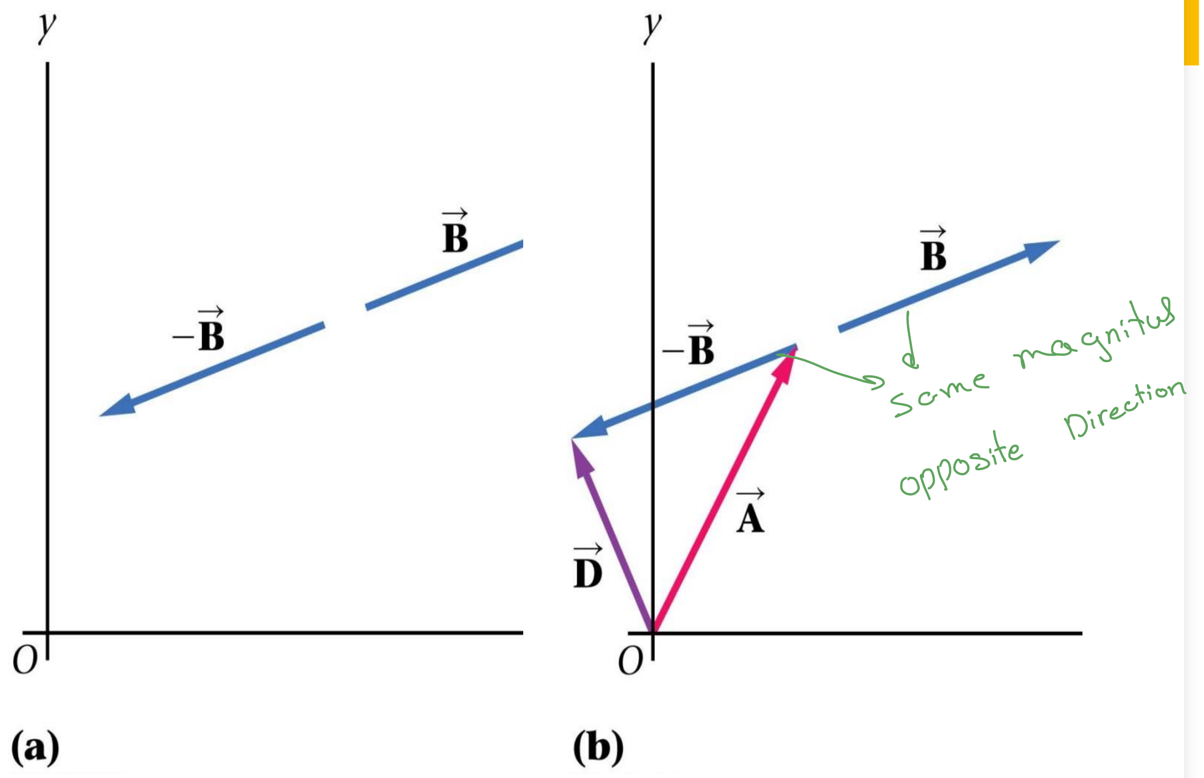
same length & Direction

Subtracting Vectors

- Subtracting vectors works similar to adding vectors.

$\vec{D} = \vec{A} - \vec{B}$ is the same as $\vec{D} = \vec{A} + (-\vec{B})$.

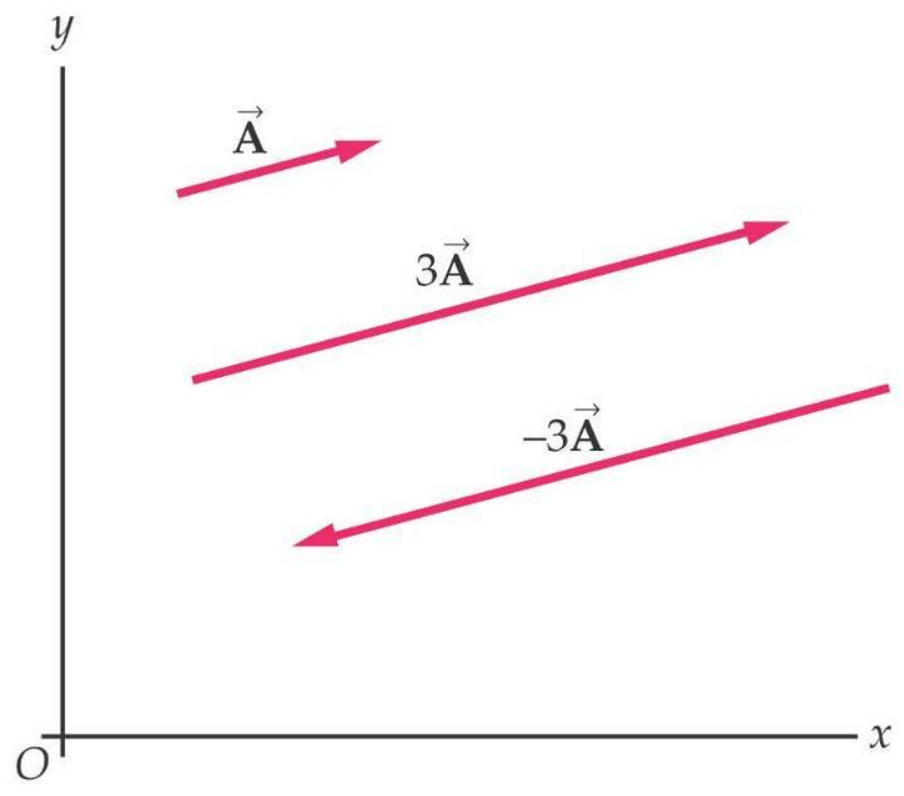
Note that $-\vec{B}$ **(negative of vector B)** has the same magnitude as $\vec{B}$, but with opposite direction



Multiplying Vectors by Scalars

Multiplying vectors

- The scalar multiplier changes the vector length, and the scalar sign can change the direction.
- This allows vectors to be factored into a unit vector and a multiplier. The unit vector specifies *only* the direction, while the multiplier provides the magnitude and units of the vector.



Note for multiples

We have 4 type of multiplication

multiply num with vector or ÷

positive : only change in length
θ ← negative : change length & Direction by 180°
(increas * - Decreas ÷)

① No $\times \vec{A}$ — Change of the Magnitut (length)

if the coefficient is positive change in length

if the coefficient is negative # chang in length & Direction by 180° (θ)

multiply unit vector with num (scaler)

② unit vector × No — changes scaler to vector component

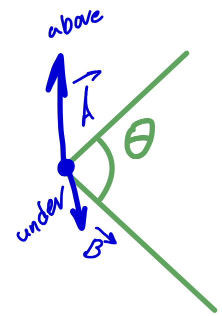
③ $\vec{A} \cdot \vec{B}$ (dott)
vector · vector

④ $\vec{A} \times \vec{B}$ (cross)
vector X Vector

magnitued by فيثاغورس

$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$ = scaler

result = just #



$\vec{A} \times \vec{B} = |A||B| \sin\theta \, \hat{n}$

result = vector < magnitude, Direction

magnitued of A & B

$\hat{n}$ Normal (vertical) عامودي
→ unit vector
→ up or under, out or iside

Righ hade resistance

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

information

② $\hat{i} = \hat{x}$ → unit vector = 1 in magnitued

$\hat{j} = \hat{y}$

$\hat{k} = \hat{z}$

$|\hat{x}|$, $\hat{y}$ & $\hat{z}$ — its length one unit

direction: + positive: +x, +y, +z

(scaler) $A_x = (1.23\text{ m})\,\hat{x}$ — Know it is vector

scaler $= A_y = (5.2\text{ m})\,\hat{y}$ — Know it is vector

There is No D-analysis

make scaler component as vector component
multiply by unit Vector

unit vector notation:

$\vec{A} = (1.23\text{m})\,\hat{x} + (5.3\text{ m})\,\hat{y}$

length in x — length in y

(+ = &)

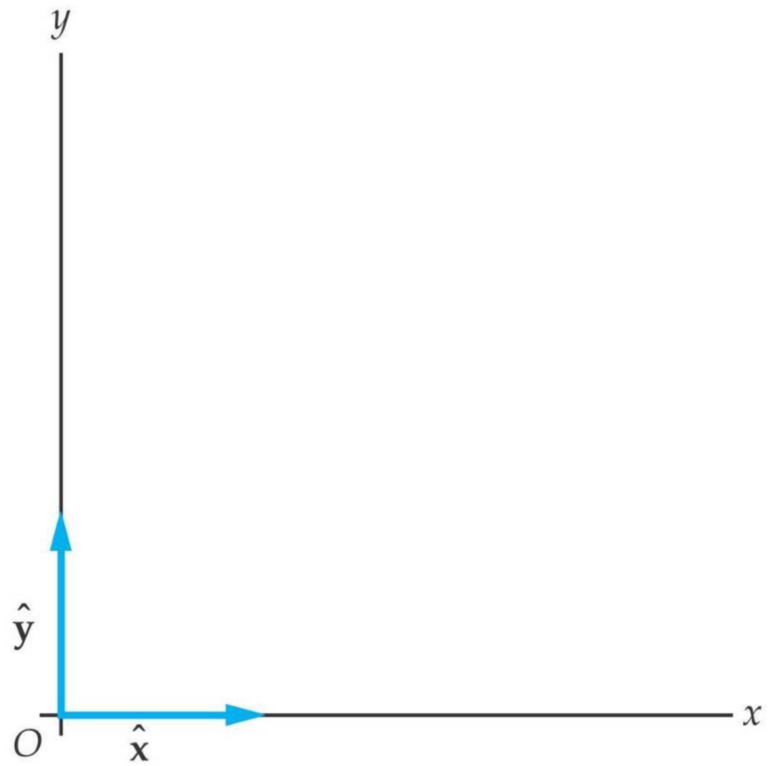
Representing Vectors Mathematically

- The vector A with an *x*-component of A_x and a y-component of A_y is written mathematically as:

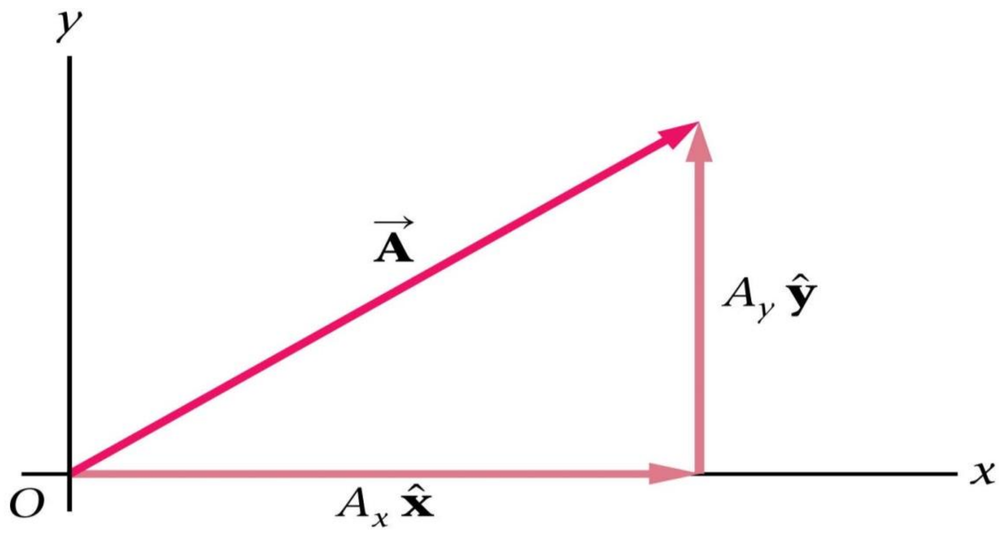
$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

- Here: $\hat{x}$ and $\hat{y}$ are the units' vectors.
- Unit vectors are dimensionless vectors of unit length (magnitude of one).

$\hat{x}$ points along the *x*-axis and $\hat{y}$ points along the *y*-axis. They are used to indicate the directions of the components.



Adding vectors using the unit vector notation



$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y}$$
$$\vec{D} = \vec{A} - \vec{B} = (A_x - B_x)\hat{x} + (A_y - B_y)\hat{y}$$

This works for any number of vectors: add the x-components together and the y-components together

1D

$\Delta x = x_f - x_i$

$\Delta y = y_f - y_i$

2D

$\Delta \vec{r} = \vec{r_f} - \vec{r_i}$

$(r_{f_x} - r_{i_x}) + (r_{f_y} - r_{iy})$

$= \Delta \vec{r_x} + \Delta \vec{r_y}$

$$\vec{r}_i = (r_{x_i})\hat{x} + (r_{y_i})\hat{y}$$
$$\vec{r}_f = (r_{x_f})\hat{x} + (r_{y_f})\hat{y}$$

1D

$$\Delta x = x_f - x_i$$
$$\Delta y = y_f - y_i$$

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$\vec{a} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i}$$

2D

$$\vec{\Delta r} = \vec{r_f} - \vec{r_i}$$
$$= (r_{f_x} - r_{ix}) \; \& \; (r_{f_y} - r_{iy})$$
$$= \vec{\Delta r_x} + \vec{\Delta r_y}$$

$$\vec{V}_{avg} = \frac{\Delta r}{\Delta t} \text{ (Displacement)} = \left(\frac{r_{f_x} - r_{ix}}{\Delta t}\right) \& \left(\frac{r_{f_y} - r_{iy}}{\Delta t}\right)$$

$$\vec{a} = \frac{\Delta v}{\Delta t} = (\vec{V_x})\hat{x} \; \& \; (\vec{V_y})$$
$$= \frac{\Delta V_x}{\Delta t} + \frac{\Delta V_y}{\Delta t}$$
$$= \frac{V_{fx} - V_{ix}}{\Delta t} + \frac{V_{fy} - V_{iy}}{\Delta t} = a_x + a_y$$

$$\Delta r = r_f - r_i$$
$$\left(\frac{r_{fx} - r_{ix}}{T}\right) + \left(\frac{r_{fy} - r_{iy}}{T}\right)$$

$$r_i = (2.00\,m)\hat{x} \; \& \; (3.5\,m)\hat{y}$$
$$r_f = (-3.00\,m)\hat{x} + (5.50\,m)\hat{y}$$
$$\Delta t = 3.00\,s$$

$$\vec{V}_{avg} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i}$$

$$\vec{V}_{avg} = \left(\frac{-3-2}{3}\right)\hat{x} + \left(\frac{5.50 - 3.5}{3}\right)\hat{y}$$
$$= \left(\frac{-5\,m}{3\,s}\right)\hat{x} + \left(\frac{2\,m}{3\,s}\right)\hat{y}$$
$$= (-1.67\,m/s)\hat{x} + (0.667\,m/s)\hat{y}$$

$$|\vec{V}| = \sqrt{(V_x)^2 + (V_y)^2}$$

$$\theta_v = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

Displacement Vector

- The **position** vector $\vec{r}$ points from the origin to the location in question.

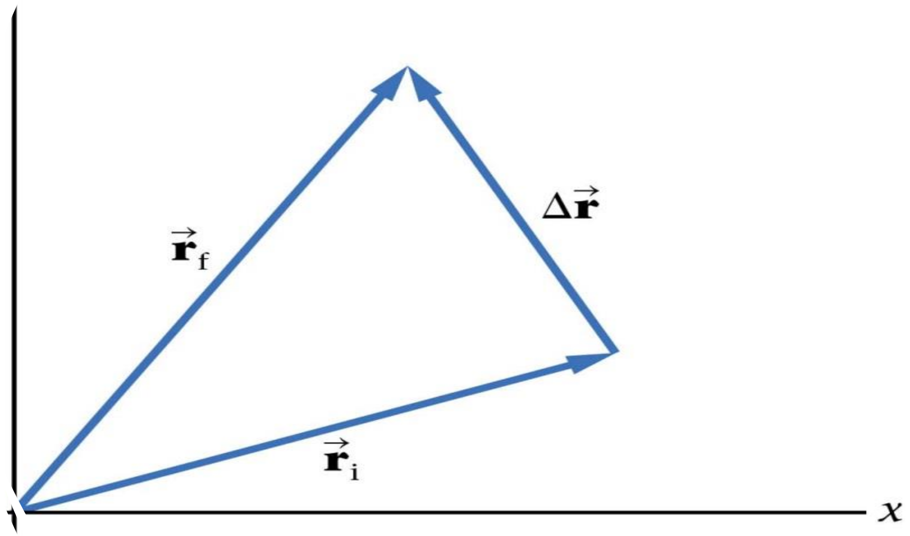
- The **displacement** vector $\Delta\vec{r}$ points from the original/initial position to the final position:

$|\vec{r}|\sqrt{r_x^2 - r_y^2}$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$(r_{fx} - r_{ix}) + (r_{fy} - r_{iy})$

$r_x + r_y$

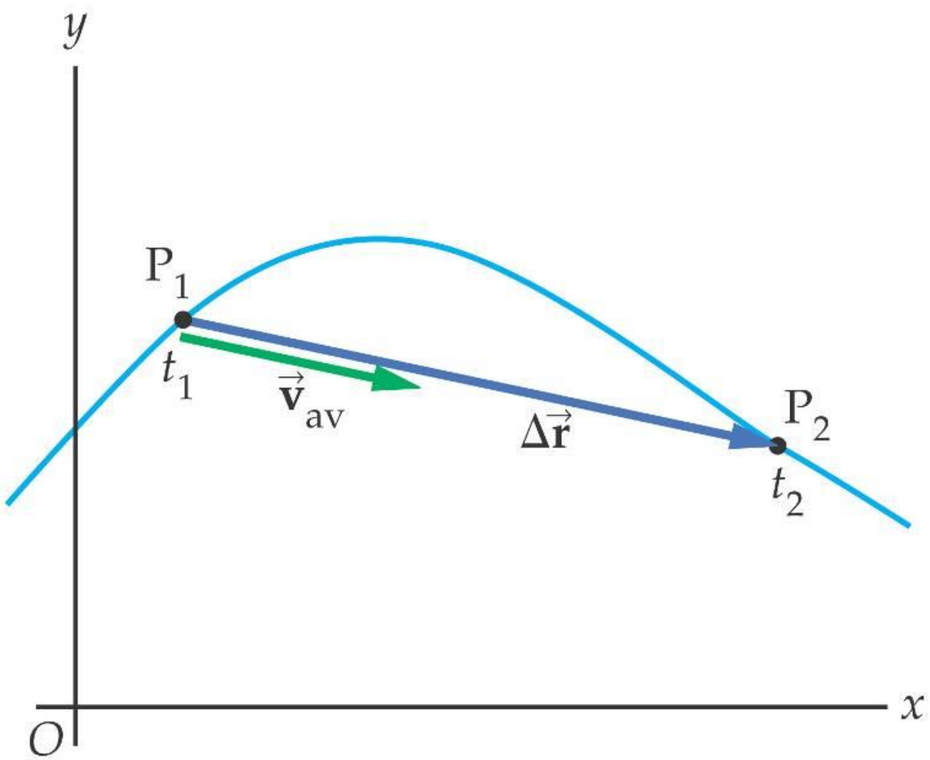


Velocity

Average Velocity:

- In one dimension: $v_{av} = \frac{\Delta x}{\Delta t}$
- In general: $\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$

$\vec{v}_{av}$ is in the same direction as $\Delta \vec{r}$



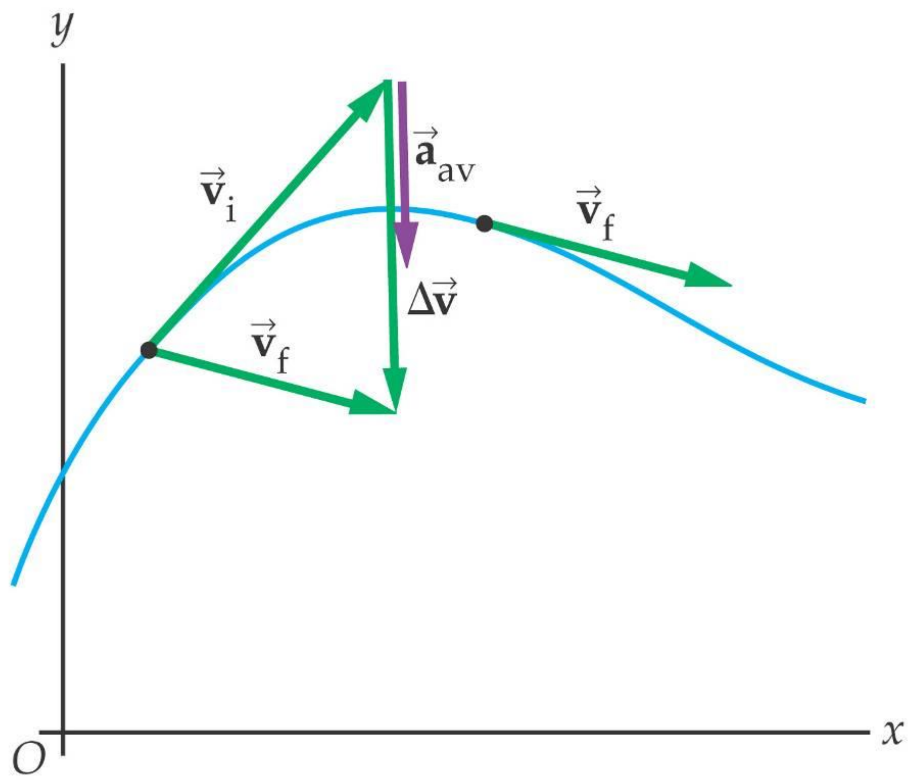
Acceleration

- Average Acceleration:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} \qquad \frac{V_{fx} - V_{ix}}{T} + \frac{V_{fy} - V_{iy}}{T}$$

$\vec{a}_{av}$ is in the same direction as

the change in velocity $\Delta\vec{v}$



Problems

$\vec{V} = \frac{\Delta r}{\Delta T} = \frac{r_f - r_i}{T_f - T_i} = \frac{r_{fx} - r_{ix}}{T} + \frac{r_{fy} - r_{iy}}{T}$ $\quad |V| = \sqrt{V_x^2 + V_y^2}$ $\quad \sqrt{(1.6)^2 + 0.66^2}$

$= \frac{-3-2}{3} + \frac{5.5-3.5}{3} = -1.6\,\hat{x} + 0.66\,\hat{y}$ $\quad V_x + V_y$ $\quad \tan^{-1}\left(\frac{V_y}{V_x}\right)$ $\quad \tan^{-1}\left(\frac{0.66}{-1.6}\right) = -22^\circ$ $\quad 157^\circ$

Exercise 6: A dragonfly is observed initially at the position $\vec{r_i} = (2.00\text{ m})\,\hat{x} + (3.50\text{ m})\,\hat{y}$. Three seconds later it is at the position $\vec{r_f} = (-3.00\text{ m})\,\hat{x} + (5.50\text{ m})\,\hat{y}$. What was the dragonfly's average velocity during this time? $T = 3\,s$

Exercise:7 A sailboat has coordinates (130 m, 205 m) at $t_1 = 0.0$ s. Two minutes later its position is (110 m, 218 m). Find its average velocity. $1\,min = 60\,sec$, $2 = 120\,sec$

$\vec{V} = \frac{\Delta r}{T}$ $\quad \Delta r = r_f - r_i = r_{fx} - r_{ix} + r_{fy} - r_{iy}$

$(110 - 130) + (218 - 205) = -20 + 13$ (r_x, r_y)

$\vec{V} = \frac{r_{fx} - r_{ix}}{T} + \frac{r_{fy} - r_{iy}}{T} = \frac{-20}{120} + \frac{13}{120} = -0.1\bar{6} + 0.108$ $\quad V_x + V_y$

$-10 + 6.5$

$|\vec{V}| = \sqrt{V_x^2 + V_y^2} = \sqrt{0.16^2 + 0.10^2} = 0.1886\,m$

$\tan^{-1}\left(\frac{0.10}{-0.16}\right) = -32^\circ + 180 = 147.9 = 148^\circ$

for knowledge

Scalar or Dot Product

إلحق رحلتي

- Dot (or Scalar) Product:

$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$$

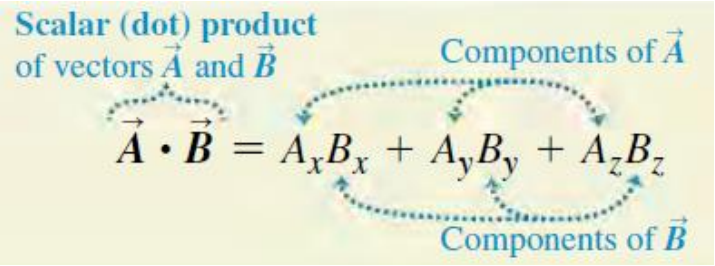
  - θ is the angle between the two vectors.
  - The result is a scalar.

- (note: $\hat{x} \cdot \hat{x} = 1, \hat{x} \cdot \hat{y} = 0, \ldots\ldots$)

The following laws are valid

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$



Vector or Cross Product

- Cross product defined as:

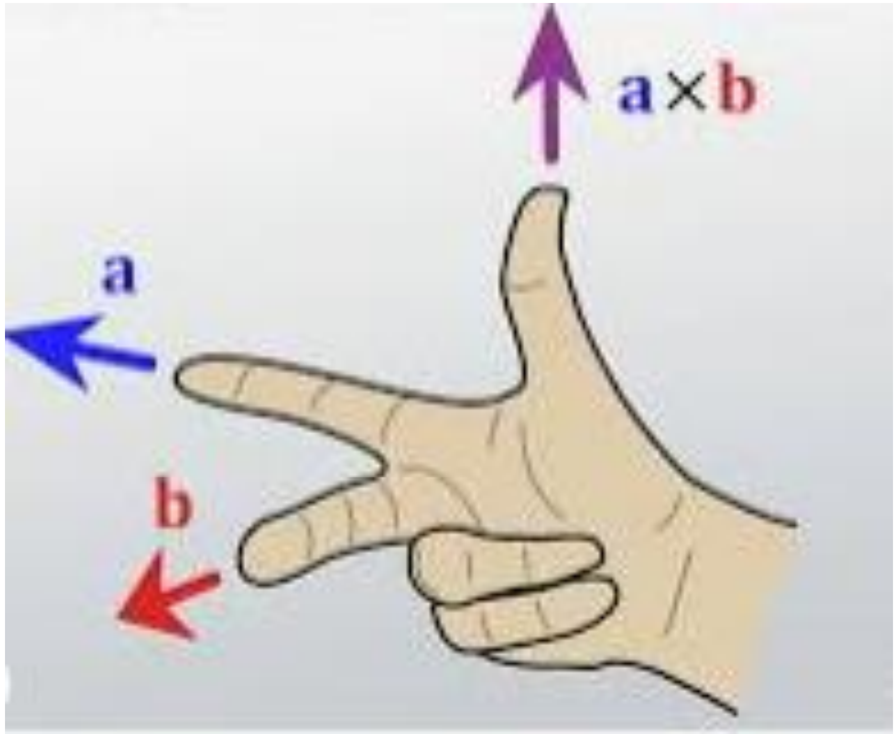
$$\vec{A} \times \vec{B} = |A|\,|B| \sin\theta\, \hat{n}$$

  - θ is the angle between the two vectors.
  - The result is a vector, perpendicular to the plane in which the two vectors exist. (for example: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{x} = 0$)
  - Where n is a unit vector indicating the direction of $\vec{A} \times \vec{B}$.

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

The following Laws are Valid:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$
$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$



Exercises

Exercise 3-7: Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{v} = (3.7\ \mathrm{m/s})\hat{x} + (-1.3\ \mathrm{m/s})\hat{y}$.

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{3.7^2 + (-1.3)^2} = 3.92 = 4\ \mathrm{m/s}$$

$$\tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-1.3}{3.7}\right) = -19.36^\circ$$ not sure

$|\hat{x}| = 1$ + x axis always positive

$|\hat{y}| = 1$ + y axis

$\vec{A} \cdot \vec{B} = |A| \cdot |B| \cdot \cos\theta$ (maybe scaler)

$\vec{A} \times \vec{B} = |A||B| \sin\theta \, \hat{n}$ (vector)

Right hand to check the Direction of vector to know if Direction is in or out paper