Chapter 6

APPLICATION OF NEWTON'S LAWS

Units of Chapter 6

6.1- Frictional Force

Static Friction, Kinetic Friction.

6.2- Strings and springs.

6.3- Translational Equilibrium.

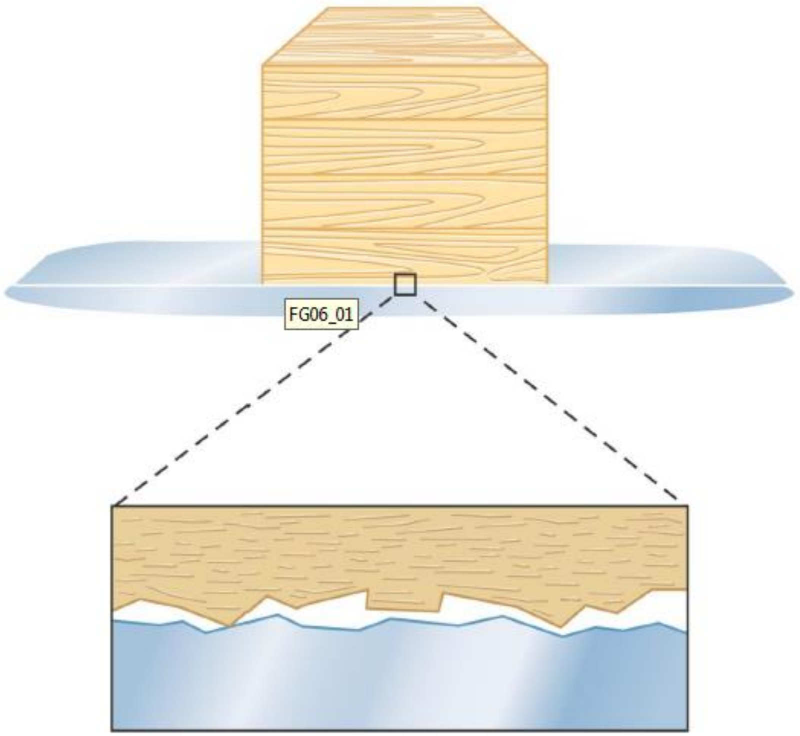
6.5- Circular Motion.

Frictional Forces

Viewed at a microscopic scale,

Even smooth surfaces have irregularities when viewed at the microscopic level. This type of roughness contributes to **Friction**.

No surface is perfectly smooth.



- On the other hand , Friction can be helpful- indispensable- in other situation.
- Ex: Walk, Run, starting or stopping a car,…etc.

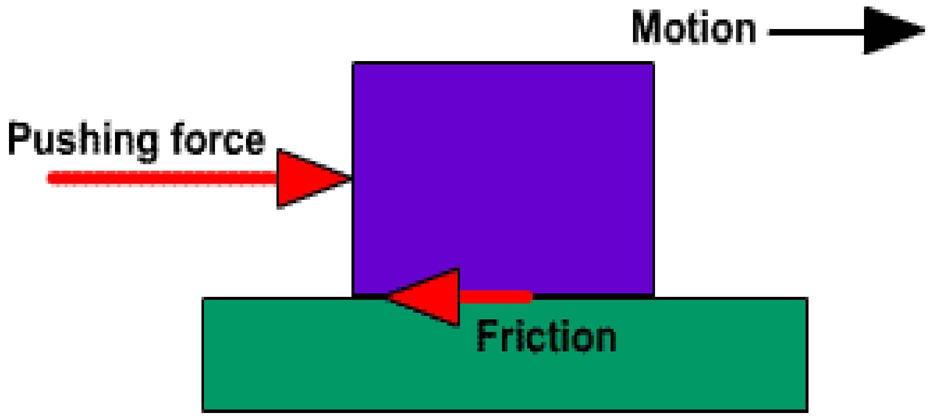
Frictional Forces

Definition: friction is the force that bodies can impart on each other when they are in contact.

The friction force on an object is opposite to its motion or impending motion relative to the surface.

The direction of Friction Force is parallel to the surface in contact and occur when:

- ❖One body slides over the other.
- ❖They cling together despite external force.



Types of Frictional Forces

1) **Static friction force**: static friction tends to keep two surfaces from moving relative to each other. (no relative motion).

Static friction is typically stronger than kinetic friction.

2) **Kinetic friction force:** kinetic friction is the friction encountered when surfaces slide against one another with a finite relative speed.

Kinetic friction acts to oppose the sliding motion at the point of contact between the surfaces.

Tow types of friction is force …measured in (N).

Static Friction (f_s)

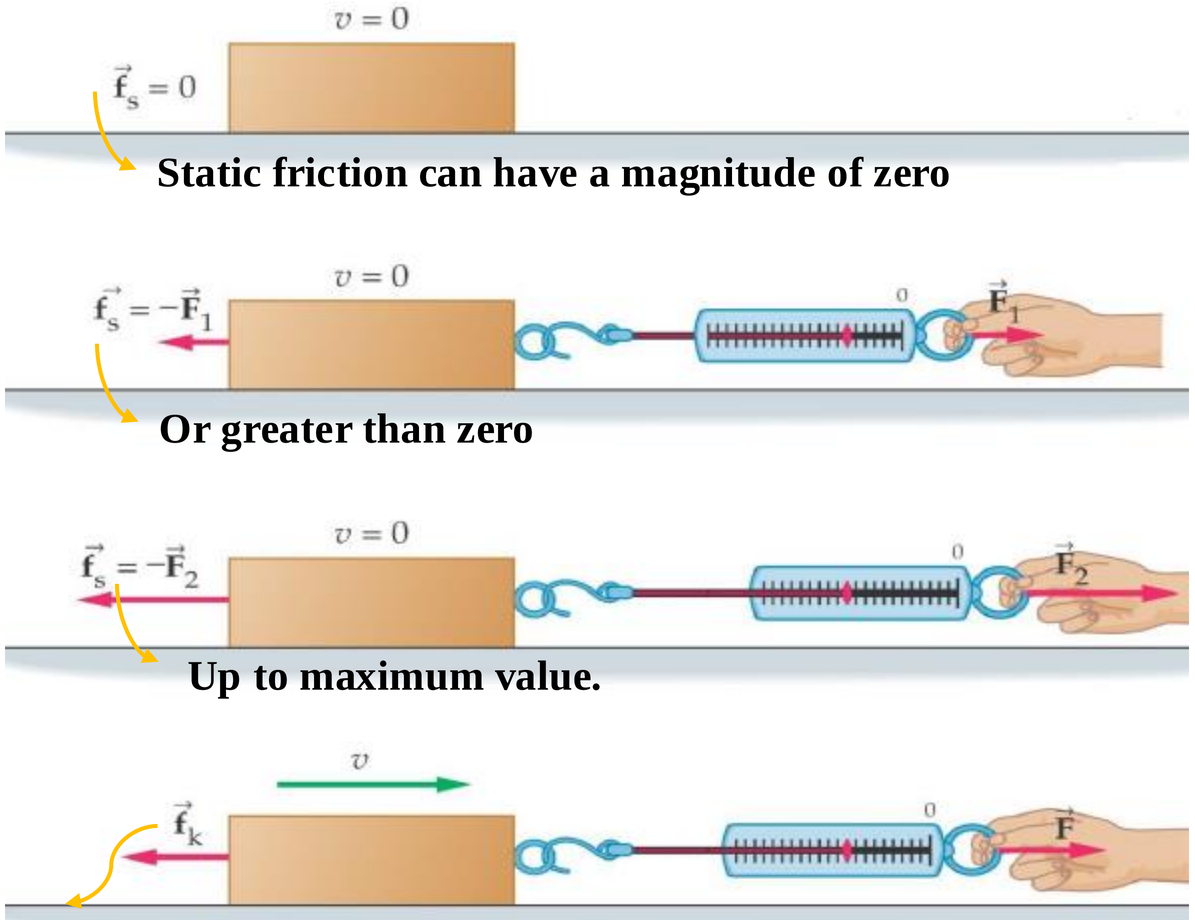
The force of static friction can have any value between zero and $f_{s,max}$. Can written mathematically:

$$\mathbf{0 \leq f_s \leq f_{s,max}}$$

Repeat exp. with double mass ⇒ Normal force is ???

Thus, the maximum force is proportional to the magnitude of normal force.

$$\boldsymbol{f_s = \mu_s N}$$



Once sliding begins, the friction is Kinetic and has value less than the maximum value of static friction.

Static Friction (f_s)

$$f_s = \mu_s N$$

The constant of proportionality μ_s is called **the coefficient of static friction.**

- Dimensionless number
- always positive
- typical values range between 0 and 1.

Static coefficient (μ_s)

TABLE 6–1 Typical Coefficients of Friction

Materials	Kinetic, μ_k	Static, μ_s
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

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Static Friction (f_s)

static friction has the following properties:

❖ It takes any value between 0 and the maximum possible force of static friction, $f_s = \mu_s N$:

$$0 \le f_s \le f_{s,max}$$

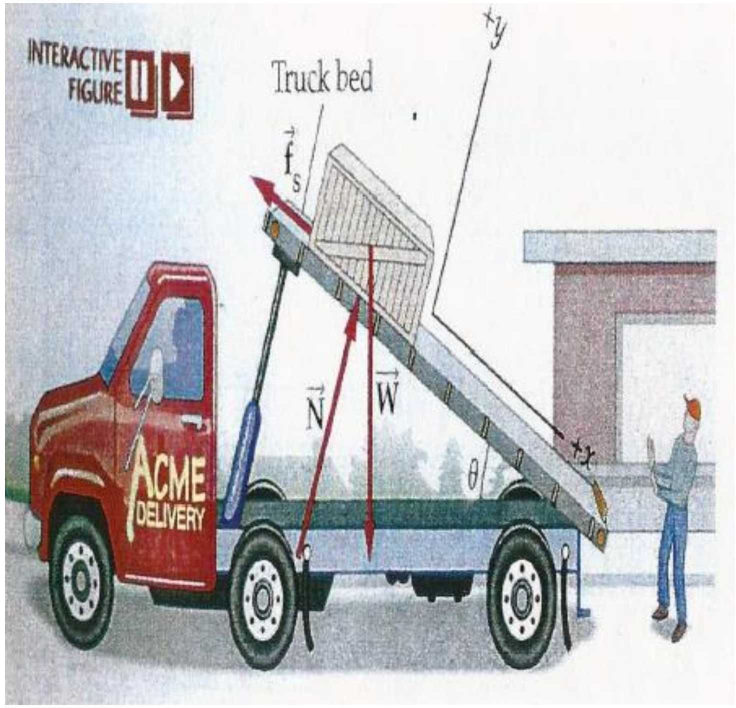
❖ Static friction is independent on the area of contact. It depends only on **Normal force** and **the nature of the surfaces.**

❖ It is parallel to the surface of contact, and in the direction that opposes relative motion.

Static Friction (f_s)

Example: A flatbed truck slowly tilts its bed upward to dispose of a 95.0kg crate. For small angles of tilt the crate stays put, but when the tilt angle exceeds 23.2^o, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?

Given: $m = 95.0kg, \quad \theta = 23.2^o, \quad \mu_s = ???$



Solution:

Free body diagram

Found N, f_s from Newton 2nd law

Found μ_s

Static Friction (f_s)

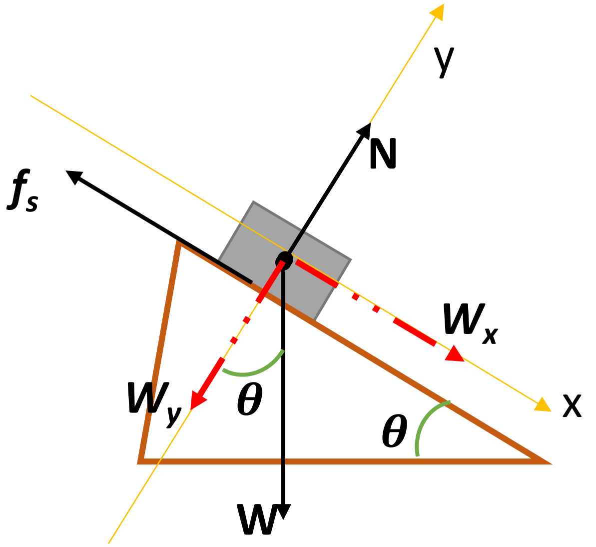
Example: A flatbed truck slowly tilts its bed upward to dispose of a 95.0kg crate. For small angles of tilt the crate stays put, but when the tilt angle exceeds 23.2°, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?

Given: $m = 95.0kg, \quad \theta = 23.2^o, \quad \mu_s = ???$

Solution:

$$\sum F_y = 0 \quad , a_y = 0.$$
$$N - W_y = 0$$
$$N = W_y = W\cos\theta = mg\cos\theta$$

$$\sum F_x = 0 \quad , a_x = 0.$$
$$W_x - f_s = 0$$
$$f_s = W_x = W\sin\theta = mg\sin\theta$$
$$\mu_s = \frac{f_s}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 23.2 = 0.429$$



Kinetic Friction (f_k)

Kinetic friction force: kinetic friction is the friction encountered when surfaces slide against one another with a finite relative speed.

Kinetic friction acts to oppose the sliding motion at the point of contact between the surfaces.

The kinetic friction force depends (proportional) on:

1. How hard the two bodies are pressed together (The normal force, N).
2. The materials from which the bodies are made (The coefficient of friction, μ_k).

$$\boldsymbol{f_k = \mu_k N}$$

Kinetic Friction (f_k)

$$f_k = \mu_k N$$

The constant of proportionality μ_k is called **the coefficient of kinetic friction.**

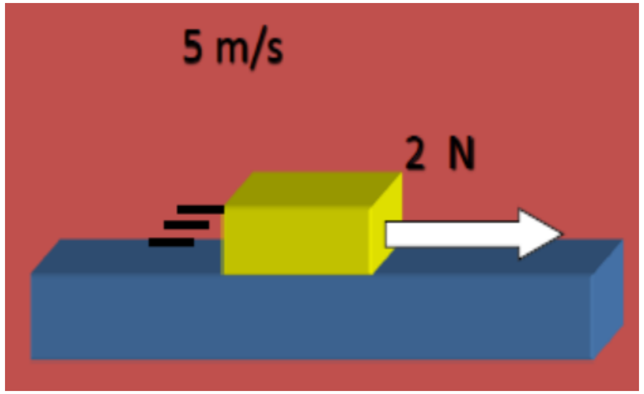
- Dimensionless number
- always positive
- typical values range between 0 and 1.
- Its value remains fairly constant as **the Velocity of the object increase**.

Kinetic Friction (f_k)

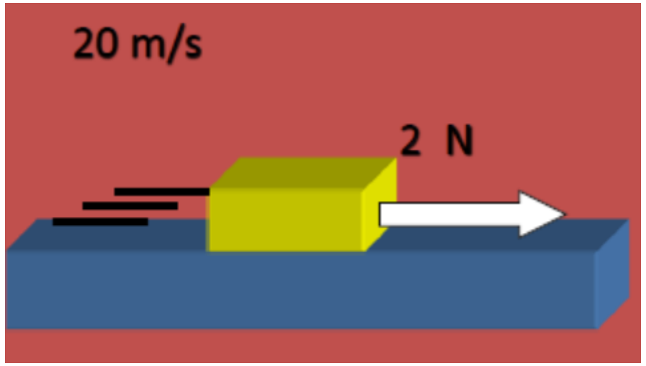
$$f_k = \mu_k N$$

Attributes that have little or no effect of :

1. Sliding speed. 2. Contact area.



The force of kinetic friction is the same at 5 m\s as it is for 20m\s. Again, we must assume that there are no chemical or mechanical changes due to speed.



Kinetic Friction (f_k)

$$f_k = \mu_k N$$

Attributes that have little or no effect of :

1. Sliding speed.
2. **Contact area.**

If the total mass pulled is constant, the same force (4N) is required to overcome friction even with twice the area of contact.

For this to be true, it is essential that **ALL** other variables be rigidly controlled.

Kinetic Friction (f_k)

Kinetic friction has the following properties:

- ❖It is proportional to the magnitude of the normal force N between the surfaces, $f_k = \mu_k N$:
- ❖Kinetic friction is independent on the area of contact. It depends only on **Normal force** and **the nature of the surfaces.**
- ❖It is parallel to the surface of contact, and in the direction that opposes relative motion.
- ❖It is independent of the relative speed of the surfaces.

Kinetic Friction (f_k)

Example: someone at the other end of table asks you to pass the salt. Feeling quiet dashing you slide the 50g salt shaker in their direction, giving it an initial speed of 1.15m\s.

a) If the shaker comes to rest with constant acceleration in 0.840m, what is the coefficient of kinetic friction between the shaker and the table?

b) How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32m\s?

Given: $m = 50g,\quad v_o = 1.15m\backslash s,\quad v = 0\ m\backslash s,\quad \Delta x = 0.84\ m$

Solution:

Kinetic Friction (f_k)

Example: someone at the other end of table asks you to pass the salt. Feeling quiet dashing you slide the 50g salt shaker in their direction, giving it an initial speed of 1.15m\s.

a) If the shaker comes to rest with constant acceleration in 0.840m, what is the coefficient of kinetic friction between the shaker and the table?

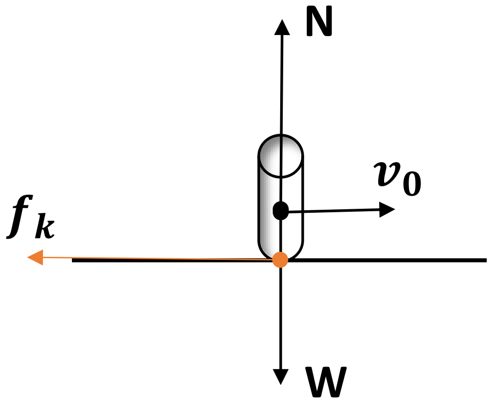
Given: $m = 50g,\quad v_o = 1.15m\backslash s,\quad v = 0\ m\backslash s,\quad \Delta x = 0.84\ m,\quad \mu_k = ???,$

Solution:

$$v^2 = v_o^2 + 2a_x \Delta x$$

$$0 = (1.15)^2 + 2 \times a_x \times 0.84$$

$$a_x = -0.787 m/s^2$$



$$\sum F_x = ma_x$$

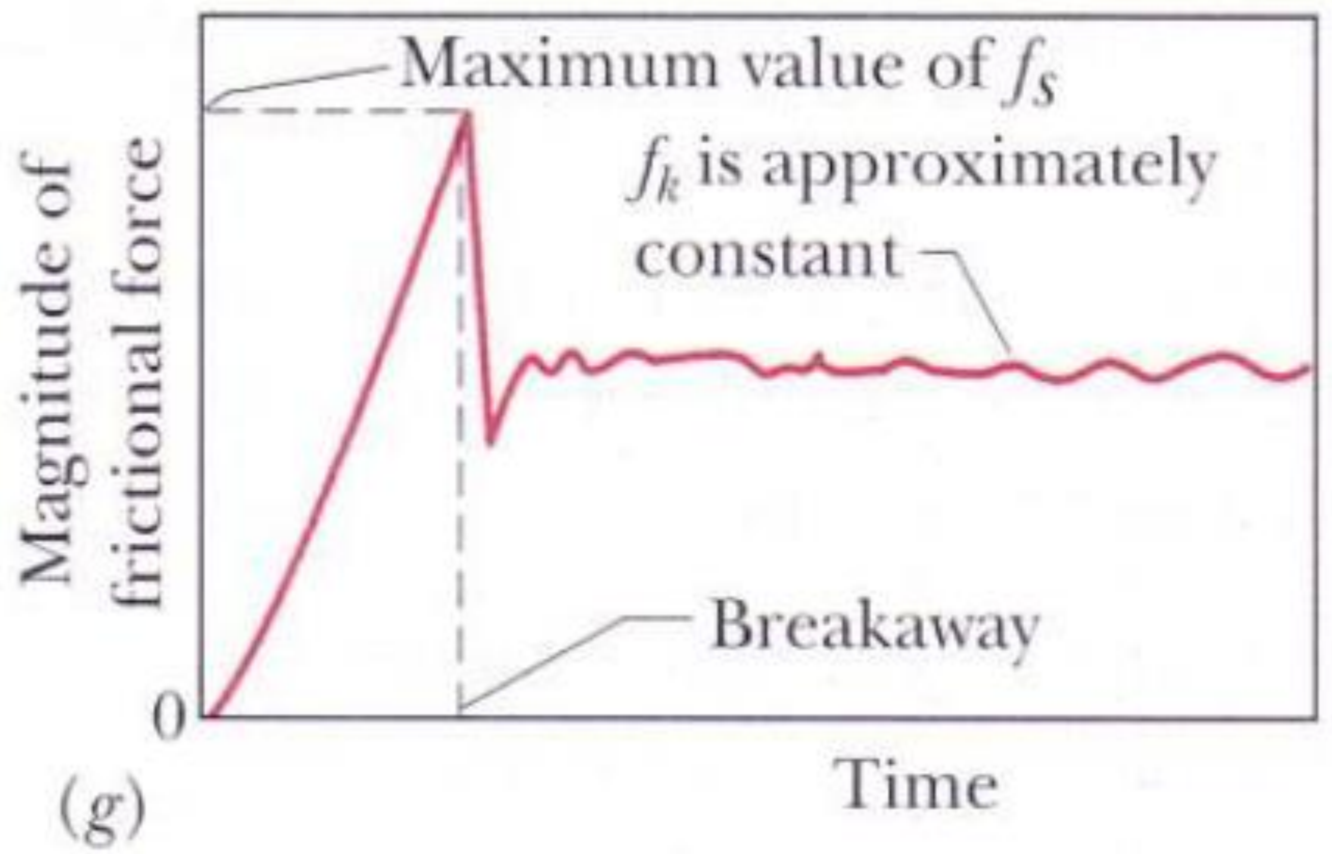
$$-f_k = ma_x \leftrightarrow f_k = -ma_x$$

$$\sum F_y = 0$$

$$N - W = 0 \rightarrow N = W = mg$$

$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787)}{9.81} = 0.0802$$

Frictional Forces



COEFFICIENT OF FRICTION

$$f_{s,max} = \mu_s N$$

The general case for equilibrium condition

$$f_s \leq \mu_s N$$

Once the body starts to slip

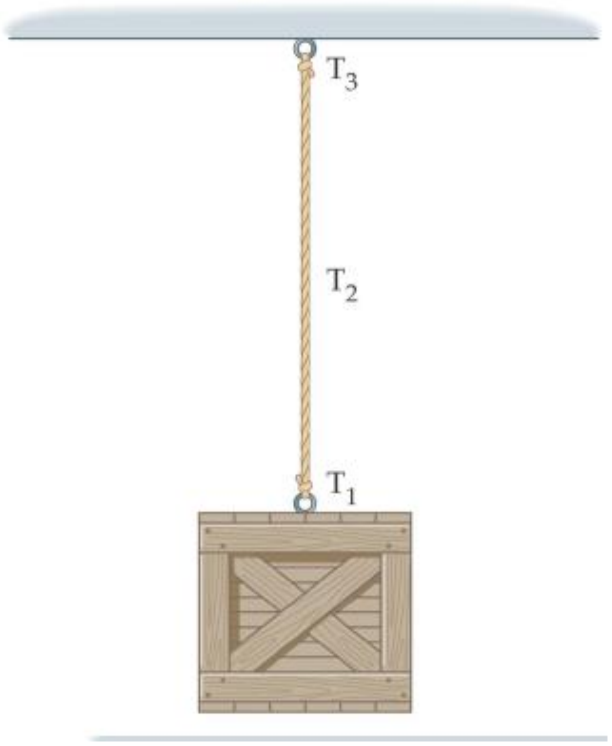
$$f_k = \mu_k N$$

μ_k is the coefficient of dynamic or kinetic friction

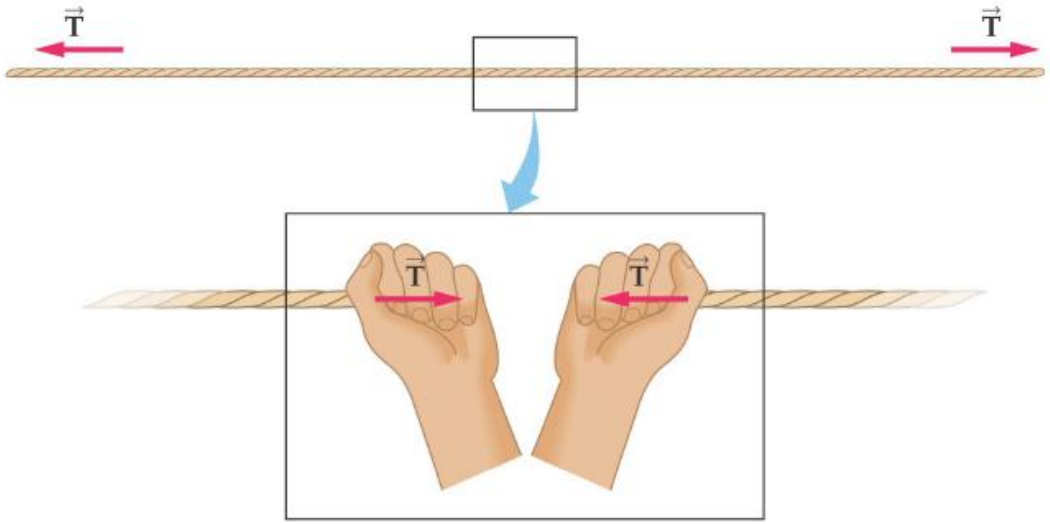
$$\mu_k < \mu_s$$

Strings and Springs

The tension in a real rope will vary along its length, due to the weight of the rope.



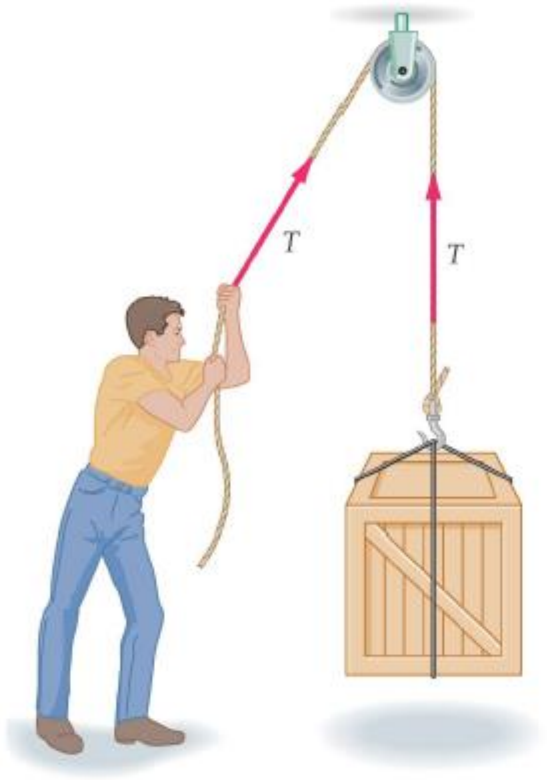
When you pull on a string or rope, it becomes taut. We say that there is tension in the string.



Strings and Springs

- An ideal pulley is one that simply changes the direction of the tension without changing its magnitude.

Assume: All ropes, wires, strings are massless and hence, that the tension is the same throughout their length.



Strings and Springs

Hooke's Law for springs states that the force increases with the amount the spring is stretched or compressed.

The spring Force

$$F = -kx$$

If F is the magnitude of the spring

$$F = kx$$

The constant of proportionality is called the spring constant (k)

k is measured in (N/m) in SI.

Problems

- **20.** Pulling up on a rope, you lift a 4.35 kg bucket of water from a well with an acceleration of 1.78 m/s^2. What is the tension in the rope?

T=50.4N

- **21.** When a 9.09 kg mass is placed on top of a vertical spring, the spring compresses 4.18 cm. Find the force constant of the spring.

k=2.13kN/m

- **24**. A backpack full of books weighing 52.0 N rests on a table in a physics laboratory classroom. A spring with a force constant of 150 N/m is attached to the backpack and pulled horizontally.

(a) If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table? Fs=3.0N

(b) Does your answer to part (a) Change if the mass of the backpack is doubled? Explain.

Assignment

Problem 20: Pulling up on a rope, you lift a 4.35kg bucket of water from a well with an acceleration of 1.78m\s^2. What is the Tension in the rope?

Given: $m = 4.35kg, \quad a = 1.78\, m\backslash s^2,$

$$T = ?????$$

Solution:

$$\Sigma F = ma,$$

$$T - W = ma$$

$$T = W + ma$$

$$T = mg + ma = m(g + a)$$

$$T = 4.35 \times (9.81 + 1.78) = 50.4N$$

Assignment

Problem 21: When a 9.09kg mass is placed on top of a vertical spring, the spring compresses 4.18cm. Fined the force constant of the spring?

Given: $m = 9.09kg, \quad x = 4.18\ cm = 0.0418m,$

$$F = ?????$$

Solution:

$$F = kx$$

$$k = \frac{F}{x} = \frac{W}{x}$$

$$k = \frac{mg}{x} = \frac{9.09 \times 9.81}{0.0418} = 2133.3\ N/m$$

$$k = 2.13\ kN/m$$

Assignment

Problem 24: A backpack full of books weighing 52.0N rests on a table in a physics laboratory classroom. A spring with a force constant 150N/m is attached to the backpack and pulled horizontally.

a) If the spring is pulled until it stretches 2.00cm and the pack remains at rest, What is the force of friction exerted on the backpack by the table?

b) Does your answer to Part a Changes if the mass of the backpack is doubled? Explain.

Given: $W = 52N,$ $x = 2cm = 0.02m,$ $k = 150\ N/m,$ $f_k = ?????$

Solution:

a) the pack remains constant means the friction is static friction.

$$\Sigma F = 0,$$

$$F - f_s = 0$$

$$F = f_s = kx$$

$$f_s = 150 \times 0.02 = 3N$$

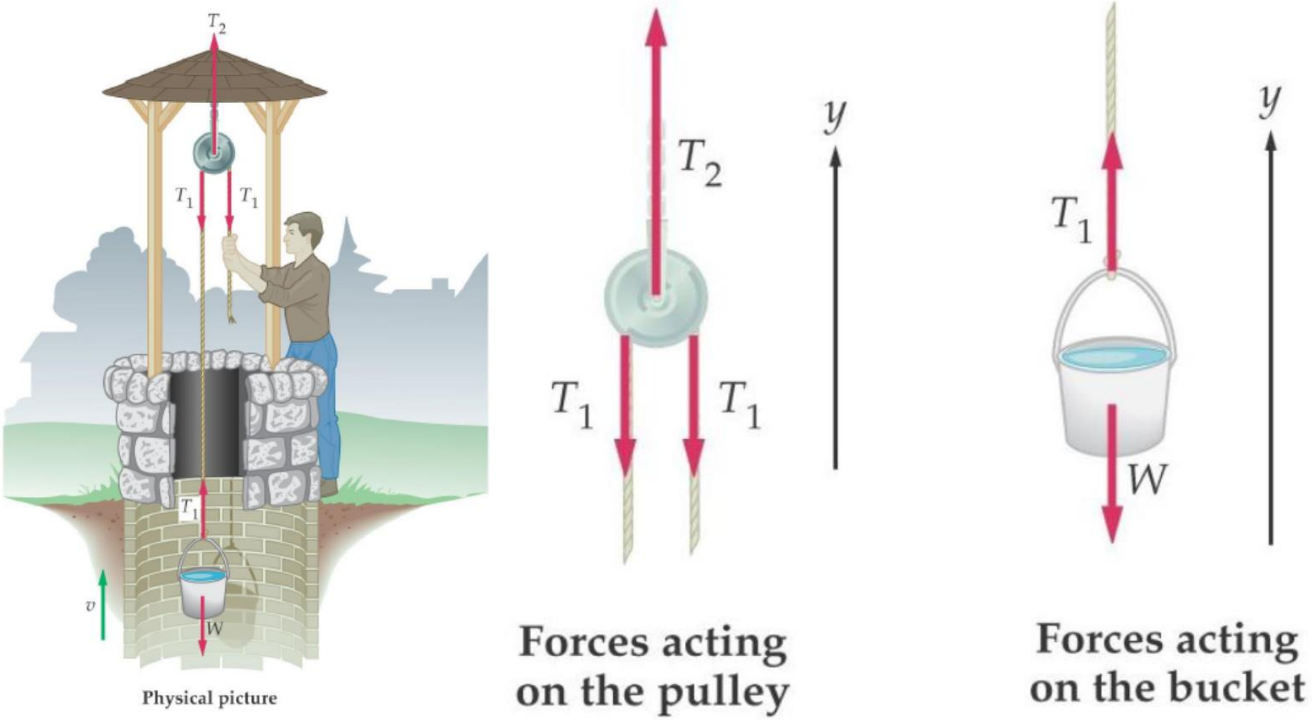
b) No

Translational Equilibrium

When the object in Translational Equilibrium, we mean that the net force acting on it is zero.

$\sum F_x = 0$ $\sum F_y = 0$

This allows the calculations of unknown forces



Problem

Problem 34: pulling a string on a bow back with a force of 28.7lb, an archer prepare to shoot an arrow. If the archer pulls in the center of the string, and the angle between the two halves is 138°, what is the tension in the string?

Given: $F = 28.7\ Ib,$ $\theta = 138^{o}\ ,$ $T = ???,$

Solution:

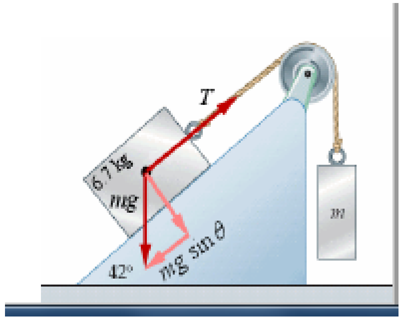
θ= 69.0O ,T=40.0 Ib

Problem

Problem 37: Two blocks are connected by a string as shown. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. find the mass of the hanging block that will cause the system to be in equilibrium? (the pulley is assumed to be ideal)

Given: $m_1 = 6.7kg,$ $\theta = 42^o$, $m_2 =???,$

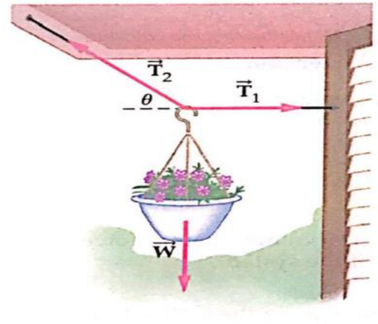
Solution:



m=4.5kg

Example 5

To hang a 6.20kg pot of flowers, a gardener uses two wires- one attached horizontally to a wall , the other sloping upward at an angle of $\theta = 40.0^o$ and attached to the ceiling, Find the tension in each wire?



Physical picture

Given: $m = 6.20kg,$ $\theta = 40.0^o$, $T = ???,$

Solution:

$\sum F_y = 0$

$T_2 \sin\theta - W = 0$

$T_2 \sin\theta = W = mg = 6.2 \times 9.81 = 60.822$

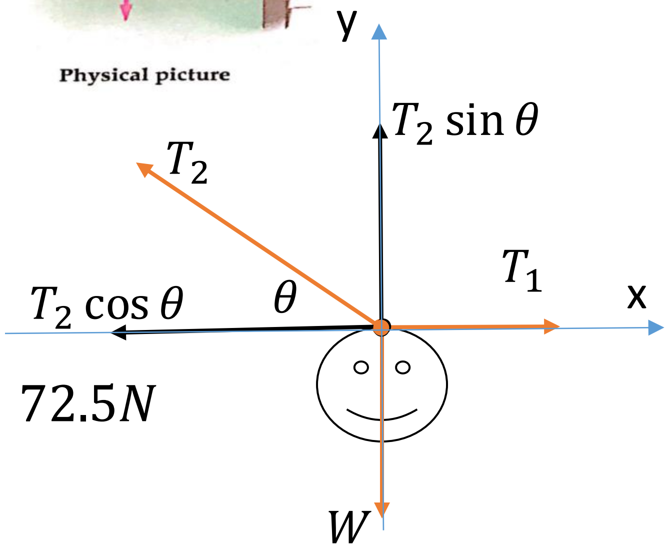
$$T_2 = \frac{60.822}{\sin 40} = 94.6\ N$$

$\sum F_x = 0$

$T_1 - T_2 \cos\theta = 0$

$T_1 = T_2 \cos\theta$

$$T_1 = 94.6 \times \cos 40 = 72.5N$$

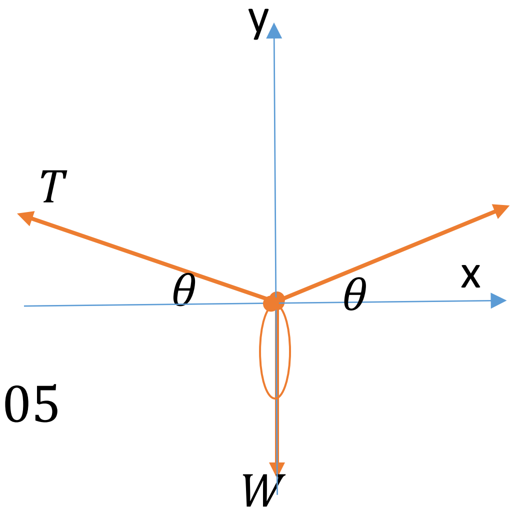


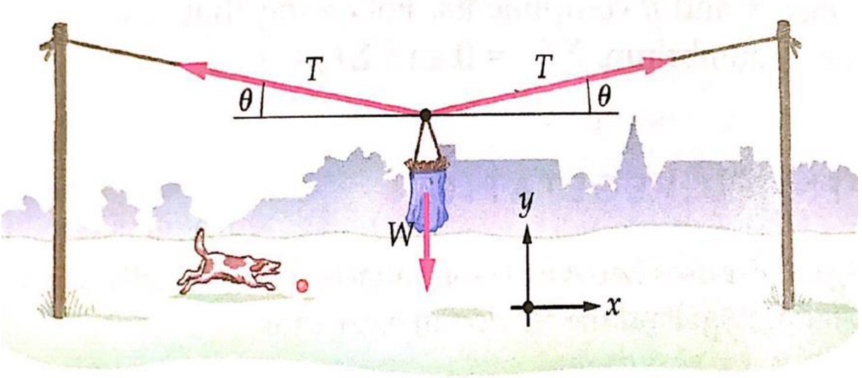
Example 3

A 1.84kg bag of clothespins hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50^o$. Find the tension T in the clothes line?

Given: $m = 1.84kg,$ $\theta = 3.50^o$, $T = ???,$

Solution:





$$\sum F_y = 0$$

$$T\sin\theta + T\sin\theta - W = 0$$

$$2T\sin\theta = W = mg = 1.84 \times 9.81 = 18.05$$

$$T_2 = \frac{18.05}{\sin 3.5} = 148\ N$$

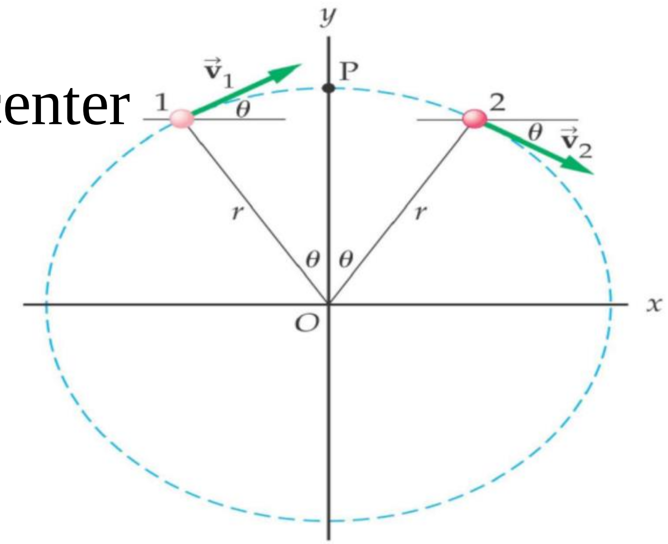
Circular Motion

An object moving in a circle must have a force acting on it; otherwise it would move in a straight line.

Other wards: To make an object move in a circle with constant speed; a force must act on it and the direction of the force is **towards the center** of the circle.

That force follows that it must be accelerating toward the center

Of the circle, called **centripetal acceleration.**



Circular Motion

When an object moves in a circle of radius r with constant speed v, its centripetal acceleration is a_{cp};

$$a_{cp} = \frac{v^2}{r}$$

A force must be applied to an object to give it circular motion. For an object of mass m, the net force (centripetal force) acting on it must have a magnitude given by:

$$f_{cp} = ma_{cp} = m\frac{v^2}{r}$$

For example, f_{cp}, might be tension in a string, friction, force of gravity.

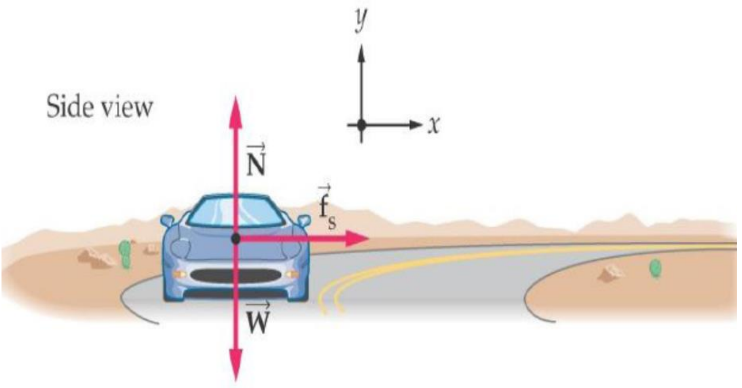
Circular Motion

Example 8: A 1200kg car rounds a corner of radius 45m. If the coefficient of static friction between the tires and the road is 0.82, what is the greatest speed the car can have in the corner without skidding?

Given: $m = 1200kg$, $r = 45\,m$, $\mu_s = 0.82$, $v = ???$,

Solution:

$$f_{cp} = m\frac{v^2}{r}, \qquad f_{cp} = \mu_s . N = \mu_s mg$$



$$\mu_s mg = m\frac{v^2}{r}$$

$$v = \sqrt{\mu_s gr} = \sqrt{0.82 \times 9.81 \times 45} = 19\,m/s$$

Circular Motion

Example 9: If a roadway is banked at the proper angle, a car can round a corner without any assistance from friction between the tires and the road. Find the appropriate banking angle for a 900kg car traveling at 20.5m/s in a turn of radius 85m?

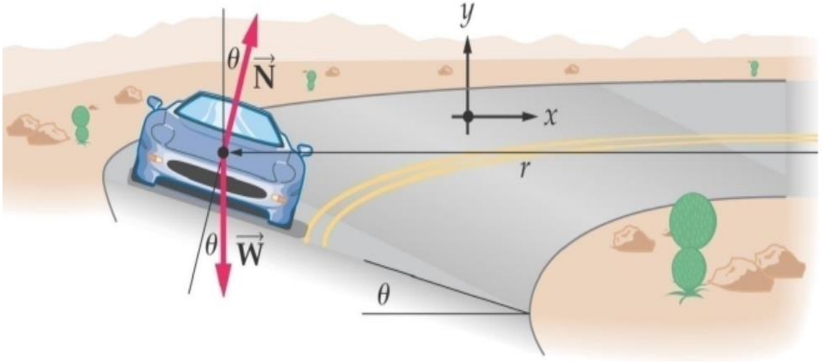
Given: $m = 900kg, \quad r = 85\ m, \quad v = 20.5m/s, \quad \theta = ???$

Solution: $\sum F_y = 0 \quad \rightarrow \quad N\cos\theta - W = 0\ , \quad N = \frac{mg}{\cos\theta}$

$$\sum F_x = ma_x \quad \rightarrow \quad N\sin\theta = m\frac{v^2}{r},$$

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r} \rightarrow \quad \tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = \tan^{-1}\left(\frac{20.5^2}{9.81 \times 85}\right) = 26.7\ ^o$$



Circular Motion

Problem 1: A 2kg ball on a string is rotated about a circle of radius 10m. The maximum tension allowed in the string is 50N. What is the maximum speed of the ball?

Given: $m = 2kg, \quad r = 10\, m, \quad T = 50N, \quad v = ???,$

Solution:

$$f_{cp} = m\frac{v^2}{r}$$

$$v = \sqrt{f_{cp}.r/_{m}} = \sqrt{T \times r/_{m}} = \sqrt{(50 \times 10)/2} = 15.8\, m/s$$

Circular Motion

Problem 2: A 55.0kg ice skater is moving at 4.00m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800m around the pole.

a) Determine the force exerted by the horizontal rope on her arms?

b) Compare this force with her weight?

Given: $m = 55kg,\quad r = 0.8\,m,\quad v = 4m/s\,,\quad f_{cp} = ???\,,$

Solution:

a)
$$f_{cp} = m\frac{v^2}{r}$$
$$f_{cp} = 55 \times \frac{4^2}{0.8} = 1100N$$

b)
$$W = mg = 55 \times 9.81 = 539N$$

Circular Motion

Question: A 4kg ball on a string is rotated about a circle of radius 10m, the maximum tension allowed in the string is 52N. What is the maximum speed of the ball?

Given: $m = 4kg,\quad r = 10\,m,\quad T = 52N,\quad v = ??,$

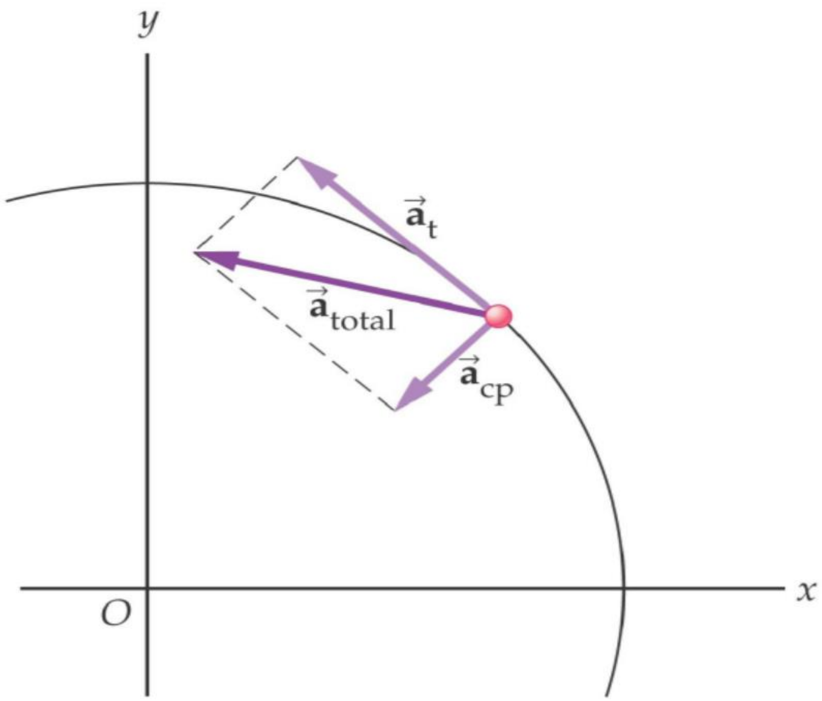
Solution:

$$f_{cp} = m\frac{v^2}{r}$$

$$v = \sqrt{f_{cp}.r/_m} = \sqrt{T \times r/_m} = \sqrt{(52 \times 10)/4} = 11.4\,m/s$$

Circular Motion

An object may be changing its speed as it moves in a circle; in that case, there is a tangential acceleration as well



Problems

3:-A baseball player slides into third base with an initial speed of 4.0m/s. If the coefficient of kinetic friction between the player and the ground is 0.46,how far does the player slide before coming to rest. (Ans 1.8 m)

4:- A child goes down a playground slide with an acceleration of $1.26 m/s^2$.Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33^0 below the horizontal.(Ans 0.496)

5:-Hoping into your Porsche, you floor it and accelerate at $12 m/s^2$ without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible. (Ans 1.2)

Chapter Summary

- Friction is due to microscopic roughness.
- Kinetic friction: $f_k = \mu_k N$
- Static friction: $f_s = \mu_s N$
- An object is in translation equilibrium if the net force acting on it is zero.

Equivalently, an object in equilibrium if it has zero acceleration.

Chapter Summary

- Tension: The force transmitted through a string, and have same value throughout an ideal string.

- The force exerted by a spring is proportional to the amount of stretch or compression, and in the opposite direction.

$$F = -kx$$

- The force required to move an object of mass m in a circle of radius r is:

$$f_{cp} = m\frac{v^2}{r}$$