1) $\lim\limits_{k \to \infty} 1 = 1$ 2) $\lim\limits_{k \to \infty} 2 + \frac{1}{k} = 2 + \cancelto{0}{\frac{1}{\infty}} = 2$ 3) $\lim\limits_{k \to \infty} \frac{3}{k^2} = \frac{3}{\infty} = 0$ 4) $\lim\limits_{k \to \infty} e^{-k} \sin k = \lim\limits_{k \to \infty} \frac{\sin k}{e^k} = \frac{\sin \infty}{\infty} = \frac{c}{\infty} = 0$

$e^{\infty} = \infty$

Example 3 Show that

$$\mathbf{x}^{(k)} = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, x_4^{(k)})^t = \boxed{\left(1, 2 + \frac{1}{k}, \frac{3}{k^2}, e^{-k} \sin k\right)^t}.$$

converges to $\mathbf{x} = \boxed{(1, 2, 0, 0)}^t$ with respect to the l_∞ norm.

Solution Because

$$\lim_{k\to\infty} 1 = 1, \quad \lim_{k\to\infty} (2 + 1/k) = 2, \quad \lim_{k\to\infty} 3/k^2 = 0 \quad \text{and} \quad \lim_{k\to\infty} e^{-k} \sin k = 0,$$

Theorem 7.6 implies that the sequence $\{\mathbf{x}^{(k)}\}$ converges to $(1, 2, 0, 0)^t$ with respect to the l_∞ norm. ■

Norm of matrix in $L_\infty \to \|A\|_\infty$
① add each row
② take max

Example 5 Determine $\|A\|_\infty$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}.$$

$\sum a_{1j} = |1| + |2| + |-1| = 4$

$\sum a_{2j} = |0| + |3| + |-1| = 4$ $\quad \|A\|_\infty = \max\{4,4,7\} = 7$

$\sum a_{3j} = |5| + |-1| + |1| = 7$

Norm of vector = length

$\|x\| = 0$

$x = (x_1, x_2) \to \|x\| = \sqrt{(x_1)^2 + (x_2)^2}$

$x = (x_1, x_2, x_3, x_4) \to \|x\| = \sqrt{(x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2}$

$\|\alpha x\| = |\alpha| \, \|x\|$

α: scalar constant

$\|x+y\| \le \|x\| + \|y\|$

$\|x\|_\infty = \max |x_i|$ ← infinite space

$x = (2,-3,5,-9) \to \|x\|_\infty = \max\{|2|, |-3|, |5|, |-9|\} = 9$

$\|x\|_2$ ← finite space

Distance in L_2

$\|x-y\|_2 = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 \ldots\ldots (x_n-y_n)^2}$

Distance in L_∞

$\|x-y\|_\infty = \max\{|x_i - y_i|\}$

Solution We have

$$\sum_{j=1}^{3} |a_{1j}| = |1| + |2| + |-1| = 4, \quad \sum_{j=1}^{3} |a_{2j}| = |0| + |3| + |-1| = 4,$$

and

$$\sum_{j=1}^{3} |a_{3j}| = |5| + |-1| + |1| = 7.$$

So, Theorem 7.11 implies that $\|A\|_\infty = \max\{4, 4, 7\} = 7$. ■

In the next section, we will discover an alternative method for finding the l_2 norm of a matrix.

EXERCISE SET 7.1

1. Find l_∞ and l_2 norms of the vectors.
   a. $\mathbf{x} = (3, -4, 0, \frac{3}{2})^t$
   b. $\mathbf{x} = (2, 1, -3, 4)^t$
   c. $\mathbf{x} = (\sin k, \cos k, 2^k)^t$ for a fixed positive integer k $\quad$ K is positive
   d. $\mathbf{x} = (4/(k+1), 2/k^2, k^2 e^{-k})^t$ for a fixed positive integer k

1) a) $\|x\|_\infty = \max\{|3|, |-4|, |0|, |\frac{3}{2}|\} = 4$

$\|x\|_2 = \sqrt{3^2 + (-4)^2 + 0^2 + (\frac{3}{2})^2} = 5.22$

1) c) $\|x\|_\infty = \max\{|\sin k|, |\cos k|, |2^k|\} = 2^k$

$\|x\|_2 = \sqrt{(\sin k)^2 + (\cos k)^2 + (2^k)^2} = \sqrt{1 + 2^{2k}}$

2. Find l_∞ and l_2 norms of the vectors.
   a. $\mathbf{x} = (2, -2, 1)^t$
   b. $\mathbf{x} = (-4/5, -2/5, 1/5, 2/5)^t$
   c. $\mathbf{x} = ((2+k)/k, 1/\sqrt{k}, -3)^t$ for a fixed positive integer k
   d. $\mathbf{x} = ((3k+1)/(2k), 2, 0, 1/k)^t$ for a fixed positive integer k
3. Prove that the following sequences are convergent and find their limits.
   a. $\mathbf{x}^{(k)} = (1/k, e^{1-k}, -2/k^2)^t$
   b. $\mathbf{x}^{(k)} = \left(e^{-k} \cos k, k \sin(1/k), 3 + k^{-2}\right)^t$
   c. $\mathbf{x}^{(k)} = (ke^{-k^2}, (\cos k)/k, \sqrt{k^2+k} - k)^t$
   d. $\mathbf{x}^{(k)} = (e^{1/k}, (k^2+1)/(1-k^2), (1/k^2)(1+3+5+\cdots+(2k-1)))^t$

3) b) $\lim_{k\to\infty} e^{-k} \cos k = \lim_{k\to\infty} \frac{\cos k}{e^k} = \frac{c}{\infty} = 0$

$\lim_{k\to\infty} k \sin(\frac{1}{k}) = 0$

$\frac{1}{\infty} = 0 \Rightarrow \sin 0 = 0 \Rightarrow k(0) = 0$

$\lim_{k\to\infty} 3 + k^{-2} = \lim_{k\to\infty} 3 + \frac{1}{k^2} = 3 + \cancel{\frac{1}{\infty^2}} = 3$

x^k converges to $(0, 0, 3)$

4. Prove that the following sequences are convergent and find their limits.
   a. $\mathbf{x}^{(k)} = (2 + 1/k, -2 + 1/k, 1 + 1/k^2)^t$
   b. $\mathbf{x}^{(k)} = ((2+k)/k, k/(2+k), (2k+1)/k)^t$
   c. $\mathbf{x}^{(k)} = ((3k+1)/k^2, (1/k)\ln k, k^2 e^{-k}, 2k/(1+2k))^t$
   d. $\mathbf{x}^{(k)} = \left(\frac{\cos k}{k}, \frac{\sin k}{k}, \frac{1-k}{k^2+1}, \frac{3k-2}{4k+1}\right)^t$

5. Find the l_∞ norm of the matrices.

a. $\begin{bmatrix} 10 & 15 \\ 0 & 1 \end{bmatrix}$

$a_{1j} = |10| + |15| = 25$

$a_{2j} = |0| + |1| = 1$

$\|A\|_\infty = \max\{25, 1\} = 25$

b. $\begin{bmatrix} 10 & 0 \\ 15 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix}$

6. Find the l_∞ norm of the matrices.

a. $\begin{bmatrix} 10 & -1 \\ -1 & 11 \end{bmatrix}$

b. $\begin{bmatrix} 11 & -11 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 1 \\ 1 & 0 & 0 \\ \pi & -1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 1/3 & -1/3 & 1/3 \\ -1/4 & 1/2 & 1/4 \\ 2 & -2 & -1 \end{bmatrix}$

7. The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

a.
$$\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63},$$
$$\frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168},$$
$$\mathbf{x} = \left(\frac{1}{7}, -\frac{1}{6}\right)^t,$$
$$\tilde{\mathbf{x}} = (0.142, -0.166)^t.$$

b.
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t,$$
$$\tilde{\mathbf{x}} = (-0.33, -7.9, 5.8)^t.$$

c.
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t,$$
$$\tilde{\mathbf{x}} = (-0.2, -7.5, 5.4)^t.$$

$b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

7) c) $x = (0, -7, 5)^t$

$\tilde{x} = (-0.2, -7.5, 5.4)^t$

$\|x - \tilde{x}\|_\infty = \max\{|0-(-0.2)|, |-7-(-7.5)|, |5-5.4|\}$

$\max\{0.2, 0.5, 0.4\} = 0.5$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.3 \\ 1.8 \end{bmatrix} = \tilde{b}$

$\|A\tilde{x} - b\|_\infty = \max\{|1-1|, |-1.3-(-1)|, |1.8-2|\}$

$\max\{0, 0.3, 0.2\} = 0.3$

d.
$$0.04x_1 + 0.01x_2 - 0.01x_3 = 0.06,$$
$$0.2x_1 + 0.5x_2 - 0.2x_3 = 0.3,$$
$$x_1 + 2x_2 + 4x_3 = 11,$$
$$\mathbf{x} = (1.827586, 0.6551724, 1.965517)^t,$$
$$\tilde{\mathbf{x}} = (1.8, 0.64, 1.9)^t.$$

8. The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

a.
$$3.9x_1 + 1.5x_2 = 5.4,$$
$$6.8x_1 - 2.9x_2 = 3.9,$$
$$\mathbf{x} = (1, 1)^t,$$
$$\tilde{\mathbf{x}} = (0.98, 1.02)^t.$$

b.
$$x_1 + 2x_2 = 3,$$
$$1.001x_1 - x_2 = 0.001,$$
$$\mathbf{x} = (1, 1)^t,$$
$$\tilde{\mathbf{x}} = (1.02, 0.98)^t.$$

c.
$$x_1 + x_2 + x_3 = 2\pi,$$
$$-x_1 + x_2 - x_3 = 0,$$
$$x_1 + x_3 = \pi,$$
$$\mathbf{x} = (0, \pi, \pi)^t,$$
$$\tilde{\mathbf{x}} = (0.1, 3.18, 3.10)^t.$$

d.
$$0.04x_1 + 0.01x_2 - 0.01x_3 = 0.0478,$$
$$0.4x_1 + 0.1x_2 - 0.2x_3 = 0.413,$$
$$x_1 + 2x_2 + 3x_3 = 0.14,$$
$$\mathbf{x} = (1.81, -1.81, 0.65)^t,$$
$$\tilde{\mathbf{x}} = (2, -2, 1)^t.$$

THEORETICAL EXERCISES

9. **a.** Verify that the function $\|\cdot\|_1$, defined on $\mathbb{R}^n$ by

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|,$$

is a norm on $\mathbb{R}^n$.

b. Find $\|\mathbf{x}\|_1$ for the vectors given in Exercise 1.

c. Prove that for all $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|_1 \geq \|\mathbf{x}\|_2$.

Norm of vector = length

$\|x\| = 0$

$x = (x_1, x_2) \longrightarrow \|x\| = \sqrt{x_1^2 + x_2^2}$

$x = (x_1, x_2, x_3, x_4) \rightarrow \|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2 + x_4^2}$

$\|\alpha x\| = |\alpha|\,\|x\|$

scalar constant

$\|x + y\| \leq \|x\| + \|y\|$

$\|x\|_\infty = \max |x_i|$ infinite space

$x = (2, -3, 5, -9) \rightarrow \|x\|_\infty = \max\{|2|, |-3|, |5|, |-9|\} = 9$

$\|x\|_2 \rightarrow$ finite space

Distance in l_2

$\|x - y\|_2 = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 \ldots (x_n - y_n)^2}$

Distance in l_∞

$\|x - y\|_\infty = \max\{|x_i - y_i|\}$

Go to page 442

Norm of matrix in $L_\infty \Rightarrow \|A\|_\infty$

① add each row

② Take max

Example 5 Determine $\|A\|_\infty$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}.$$

$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}$

1st row any

$\sum_j a_{1j} = |1| + |2| + |-1| = 4$

$\sum_j a_{2j} = |0| + |3| + |-1| = 4$

$\sum_j a_{3j} = |5| + |-1| + |1| = 7$

$\|A\|_\infty = \max\{4, 4, 7\} = 7$

Solution We have

$$\sum_{j=1}^{3} |a_{1j}| = |1| + |2| + |-1| = 4, \quad \sum_{j=1}^{3} |a_{2j}| = |0| + |3| + |-1| = 4,$$

and

$$\sum_{j=1}^{3} |a_{3j}| = |5| + |-1| + |1| = 7.$$

So, Theorem 7.11 implies that $\|A\|_\infty = \max\{4, 4, 7\} = 7$. ∎

In the next section, we will discover an alternative method for finding the l_2 norm of a matrix.

EXERCISE SET 7.1

1a

$\|x\|_\infty = \max\{|3|, |4|, |0|, |\tfrac{3}{2}|\} = 4$

$\|x\|_2 = \sqrt{3^2 + (-4)^2 + 0^2 + (\tfrac{3}{2})^2} = 5.22$

1c

$\|x\|_\infty = \max\{\underbrace{|\sin k|, |\cos k|}_{\max: 1}, |2^k|\} = 2^k$

k can be 1 and above So $k = 1$ means it's 2

$\|x\|_2 = \sqrt{(\sin k)^2 + (\cos k)^2 + (2^k)^2}$

$\sqrt{1 + 2^{2k}}$

$\sin^2 x + \cos^2 x = 1$

3b

$\lim_{k \to \infty} e^{-k} \cos k = \lim_{k \to \infty} \frac{\cos k}{e^k} = 0$

$\frac{\cos \infty}{e^\infty} = \frac{c}{\infty} = 0$

$\lim_{k \to \infty} k\left(\sin\left(\tfrac{1}{k}\right)\right) = 0$

$\frac{1}{\infty} = 0$

$\sin 0 = 0$

$k(0) = 0$

can be decimal

$\frac{1}{\infty} = 0$

$\lim_{k \to \infty} 3 + k^{-2} = 3 + \frac{1}{k^2} = 3$

x^k converges to $x = (0, 0, 3)$

I_∞ (max)

$\|x\|_2 = \sqrt{3^2 + (-4)^2 + 0^2 + (\tfrac{3}{2})^2} = 5.22$

1. Find l_∞ and l_2 norms of the vectors.

a. $\mathbf{x} = (3, -4, 0, \tfrac{3}{2})^t$ $\quad \|x\|_{max} = 4$

b. $\mathbf{x} = (2, 1, -3, 4)^t$

means k is positive

c. $\mathbf{x} = (\sin k, \cos k, 2^k)^t$ for a fixed positive integer k

d. $\mathbf{x} = (4/(k+1), 2/k^2, k^2 e^{-k})^t$ for a fixed positive integer k

$\frac{4}{k+1}$ $\quad \frac{4}{2}$ $\quad \frac{2}{k^2}$ $\quad \frac{k^2}{e^k}$

2. Find l_∞ and l_2 norms of the vectors.

a. $\mathbf{x} = (2, -2, 1)^t$

b. $\mathbf{x} = (-4/5, -2/5, 1/5, 2/5)^t$

c. $\mathbf{x} = ((2+k)/k, 1/\sqrt{k}, -3)^t$ for a fixed positive integer k

d. $\mathbf{x} = ((3k+1)/(2k), 2, 0, 1/k)^t$ for a fixed positive integer k

3. Prove that the following sequences are convergent and find their limits.

lim

a. $\mathbf{x}^{(k)} = (1/k, e^{1-k}, -2/k^2)^t$

b. $\mathbf{x}^{(k)} = (e^{-k}\cos k, k\sin(1/k), 3 + k^{-2})^t$ $\quad \lim_{k \to \infty} e^{-k}\cos k = \lim_{k \to \infty} \frac{\cos k}{e^k}$

~~**c.** $\mathbf{x}^{(k)} = (ke^{-k^2}, (\cos k)/k, \sqrt{k^2 + k} - k)^t$~~ complicated, no need to do

~~**d.** $\mathbf{x}^{(k)} = (e^{1/k}, (k^2 + 1)/(1 - k^2), (1/k^2)(1 + 3 + 5 + \cdots + (2k - 1)))^t$~~

4. Prove that the following sequences are convergent and find their limits.

a. $\mathbf{x}^{(k)} = (2 + 1/k, -2 + 1/k, 1 + 1/k^2)^t$

b. $\mathbf{x}^{(k)} = ((2+k)/k, k/(2+k), (2k+1)/k)^t$

c. $\mathbf{x}^{(k)} = ((3k+1)/k^2, (1/k)\ln k, k^2 e^{-k}, 2k/(1+2k))^t$

d. $\mathbf{x}^{(k)} = \left(\dfrac{\cos k}{k}, \dfrac{\sin k}{k}, \dfrac{1-k}{k^2+1}, \dfrac{3k-2}{4k+1}\right)^t$

5. Find the l_∞ norm of the matrices.

a. $\begin{bmatrix} 10 & 15 \\ 0 & 1 \end{bmatrix}$ $\quad a_{1j} = |10| + |15| = \boxed{25}$, $\; a_{2j} = |0| + |1| = \boxed{1}$, $\; \|A\|_\infty = \max\{25, 1\} = \boxed{25}$

b. $\begin{bmatrix} 10 & 0 \\ 15 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

Solved one it above the set

d. $\begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix}$

6. Find the l_∞ norm of the matrices.

a. $\begin{bmatrix} 10 & -1 \\ -1 & 11 \end{bmatrix}$

b. $\begin{bmatrix} 11 & -11 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 1 \\ 1 & 0 & 0 \\ \pi & -1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 1/3 & -1/3 & 1/3 \\ -1/4 & 1/2 & 1/4 \\ 2 & -2 & -1 \end{bmatrix}$

$\boxed{7c}$ $x = (0, -7, 5)^t$

$\tilde{x} = (-0.2, -7.5, 5.4)^t$

$\|x - \tilde{x}\|_\infty = \max\{|0+0.2|, |-7+7.5|, |5-5.4|\}$

$\max\{0.2, 0.5, 0.4\} = 0.5$

$\|A\tilde{x} - b\|_\infty$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.3 \\ 1.8 \end{bmatrix}$ $\tilde{b}$

$b - \tilde{b}$

$b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\|A\tilde{x} - b\|_\infty = \max\{|1-1|, |-1.3+1|, |1.8-2|\}$

$\max\{|0|, |-0.3|, |-0.2|\} = \boxed{0.3}$

7. The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

a.
$$\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63},$$
$$\frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168},$$
$$\mathbf{x} = \left(\frac{1}{7}, -\frac{1}{6}\right)^t,$$
$$\tilde{\mathbf{x}} = (0.142, -0.166)^t.$$

b.
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t,$$
$$\tilde{\mathbf{x}} = (-0.33, -7.9, 5.8)^t.$$

c. A b
$$x_1 + 2x_2 + 3x_3 = 1,$$
$$2x_1 + 3x_2 + 4x_3 = -1,$$
$$3x_1 + 4x_2 + 6x_3 = 2,$$
$$\mathbf{x} = (0, -7, 5)^t, \text{ exact}$$
$$\tilde{\mathbf{x}} = (-0.2, -7.5, 5.4)^t. \text{ approx}$$

$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -7 \\ 5 \end{bmatrix}$ $\begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$

d.
$$0.04x_1 + 0.01x_2 - 0.01x_3 = 0.06,$$
$$0.2x_1 + 0.5x_2 - 0.2x_3 = 0.3,$$
$$x_1 + 2x_2 + 4x_3 = 11,$$
$$\mathbf{x} = (1.827586, 0.6551724, 1.965517)^t,$$
$$\tilde{\mathbf{x}} = (1.8, 0.64, 1.9)^t.$$

8. The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

$\lim_{k\to\infty} 2+\frac{1}{k} = 2+\frac{1}{\infty}^{0} = 2$

transpose

$e^{\infty}=\infty$

Example 3 Show that

$$\mathbf{x}^{(k)} = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, x_4^{(k)})^t = \underbrace{\left(1, 2+\frac{1}{k}, \frac{3}{k^2}, e^{-k}\sin k\right)^{\boxed{t}}}.$$

$\lim_{k\to\infty} e^{-k}\sin k = \frac{\sin k}{e^k}$

$= \frac{\sin\infty}{e^\infty}$

$= \frac{\sin\infty}{\infty}^{0}$

$\lim_{k\to\infty} \frac{3}{k^2} = \frac{3}{\infty^2}^{0} = 0$

lim

converges to $\mathbf{x} = (1, 2, 0, 0)^t$ with respect to the l_∞ norm.

$\lim_{k\to\infty} 1 = 1$ ✓

$\lim_{k\to\infty} 2+\frac{1}{k} = 2$ ✓

$\lim_{k\to\infty} \frac{3}{k^2} = 0$ ✓

(0,1)
max: 1

$\lim_{k\to\infty} e^{-k}\sin k = \lim_{k\to\infty} \frac{\sin k}{e^k} = 0$ ✓

max go to infinity

Solution Because

$$\lim_{k\to\infty} 1 = 1, \quad \lim_{k\to\infty} (2+1/k) = 2, \quad \lim_{k\to\infty} 3/k^2 = 0 \quad \text{and} \quad \lim_{k\to\infty} e^{-k}\sin k = 0,$$

Theorem 7.6 implies that the sequence $\{\mathbf{x}^{(k)}\}$ converges to $(1, 2, 0, 0)^t$ with respect to the l_∞ norm. ■