3. Repeat Exercise 1 using ~~Algorithm 6.2~~. Partial pivoting

c. $2x_1 - 3x_2 + 2x_3 = 5,$

$-4x_1 + 2x_2 - 6x_3 = 14,$

$2x_1 + 2x_2 + 4x_3 = 8.$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 2 & -3 & 2 & 5 \\ 2 & 2 & 4 & 8 \end{array}\right] \xrightarrow{\frac{1}{2}R_1+R_2,\ \frac{1}{2}R_1+R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & -2 & -1 & 12 \\ 0 & 3 & 1 & 15 \end{array}\right]$$

$\max\{|2|, |-4|, |2|\} = 4$

$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}, \quad \frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$

$\frac{1}{2}R_1 + R_2 \qquad \frac{1}{2}R_1 + R_3$

$\max\{|-2|, |3|\}$

$$\xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & -2 & -1 & 12 \end{array}\right]$$

$\frac{a_{32}}{a_{22}} = \frac{-2}{3} \qquad \frac{2}{3}R_2 + R_3$

$$\xrightarrow{\frac{2}{3}R_2+R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & 0 & -\frac{1}{3} & 22 \end{array}\right]$$

$-3 \cancel{x} \quad -\frac{1}{3}x_3 = 22 \cancel{x} -3$

$x_3 = -66$

$3x_2 + x_3 = 15$

$3x_2 - 66 = 15$
$\quad +66 \quad +66$

$\frac{3x_2}{3} = \frac{81}{3}$

$x_2 = 27$

$-4x_1 + 2x_2 - 6x_3 = 14$

$-4x_1 + 2(27) - 6(-66) = 14$

$-4x_1 + 450 = 14$
$\quad -450 \quad -450$

$\frac{-4x_1}{-4} = \frac{-436}{-4}$

$x_1 = 109$

5. Repeat Exercise 1 using ~~Algorithm 6.3~~. Scaled partial pivoting

c. $2x_1 - 3x_2 + 2x_3 = 5,$

$-4x_1 + 2x_2 - 6x_3 = 14,$

$2x_1 + 2x_2 + 4x_3 = 8.$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_1+R_2} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & -4 & -2 & 24 \\ 0 & 5 & 2 & 3 \end{array}\right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & -4 & -2 & 24 \end{array}\right]$$

$S_1 = \max\{|2|, |-3|, |2|\} = 3$

$S_2 = \max\{|-4|, |2|, |-6|\} = 6$

$S_3 = \max\{|2|, |4|, |2|\} = 4$

$\frac{|a_{11}|}{S_1} = \frac{2}{3}, \quad \frac{|a_{21}|}{S_2} = \frac{4}{6} = \frac{2}{3}, \quad \frac{|a_{31}|}{S_3} = \frac{2}{4} = \frac{1}{2}$

$\frac{a_{21}}{a_{11}} = \frac{-4}{2} = -2, \quad \frac{a_{31}}{a_{11}} = \frac{2}{2} = 1$

$\frac{|a_{22}|}{S_2} = \frac{|-4|}{6} = \frac{2}{3} \qquad \frac{|a_{32}|}{S_3} = \frac{5}{4}$

$\frac{a_{32}}{a_{22}} = \frac{-4}{5}$

$$\xrightarrow{\frac{4}{5}R_2+R_3} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -\frac{2}{5} & \frac{132}{5} \end{array}\right]$$

$-\frac{5}{2} \times -\frac{2}{5} x_3 = \frac{132}{5} \times -\frac{5}{2}$

$x_3 = -66$

$5x_2 + 2x_3 = 3$

$5x_2 + 2(-66) = 3$

$5x_2 - 132 = 3$
$\quad +132 \quad +132$

$\frac{5x_2}{5} = \frac{135}{5}$

$x_2 = 27$

$2x_1 - 3x_2 + 2x_3 = 5$

$2x_1 - 3(27) + 2(-66) = 5$

$2x_1 - 213 = 5$
$\quad +213 \quad +213$

$2x_1 = 218$

$x_1 = 109$

Ex 6.2 Solve by partial pivoting.

(3c)

$$2x_1 - 3x_2 + 2x_3 = 5$$
$$-4x_1 + 2x_2 - 6x_3 = 14$$
$$2x_1 + 2x_2 + 4x_3 = 8$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right]$$

$$\max\{|2|, |-4|, |2|\} = 4$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 2 & -3 & 2 & 5 \\ 2 & 2 & 4 & 8 \end{array}\right]$$

$$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}, \quad \frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$$

$$\xrightarrow[\frac{1}{2}R_1 + R_3]{\frac{1}{2}R_1 + R_2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & -2 & -1 & 12 \\ 0 & 3 & 1 & 15 \end{array}\right]$$

$$\max\{|-2|, |3|\} = 3$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{array}{c} \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & \boxed{-2} & \boxed{-1} & 12 \end{array}\right]$$

$$\frac{a_{32}}{a_{22}} = \frac{-2}{3}$$

$$R_3 = R_3 + \frac{2}{3}R_2$$
$$= -2 + \frac{2}{3}(3)$$
$$-1 + \frac{2}{3}(1)$$

$$\xrightarrow{\frac{2}{3}R_2 + R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & 0 & -\frac{1}{3} & 22 \end{array}\right]$$

$$x_3 \quad -\frac{1}{3}x_3 = 22 \times 3$$

$$\boxed{x_3 = -66}$$

$$3x_2 + x_3 = 15$$
$$3x_2 - 66 = 15$$
$$\frac{3x_2}{3} = \frac{15 + 66}{3}$$
$$\boxed{x_2 = 27}$$

Ex 6.2 Solve by partial pivoting.

(3)

$$-4x_1 + 2x_2 - 6x_3 = 14$$
$$-4x_1 = 14 - 2x_2 + 6x_3$$
$$\frac{\cancel{-4}x_1}{\cancel{-4}} = \frac{14 - 2(27) + 6(-66)}{-4}$$

$$\boxed{x_1 = \cancel{59.5}\ 109}$$

$$(x_1, x_2, x_3) = (59.5, 27, -66)$$

with 2 row interchange $R_1 \leftrightarrow R_2$ then $R_2 \leftrightarrow R_3$.

$$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}, \quad \frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$$

$$\xrightarrow[\frac{1}{2}R_1 + R_3]{\frac{1}{2}R_1 + R_2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & -2 & -1 & 12 \\ 0 & 3 & 1 & 15 \end{array}\right]$$

$$\max\{|-2|, |3|\} = 3$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & -2 & -1 & 12 \end{array}\right]$$

$$\frac{a_{32}}{a_{22}} = \frac{-2}{3}$$

$$\xrightarrow{\frac{2}{3}R_2 + R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & 0 & -\frac{1}{3} & 22 \end{array}\right]$$

$$-\frac{1}{3}x_3 = 22$$

$$\boxed{x_3 = -66}$$

$$3x_2 + x_3 = 15$$
$$3x_2 - 66 = 15$$
$$\frac{3x_2}{3} = \frac{15 + 66}{3}$$
$$\boxed{x_2 = 27}$$

5c) Solve the system of linear eqns using scaled partial pivoting.

$$2x_1 - 3x_2 + 2x_3 = 5$$
$$-4x_1 + 2x_2 - 6x_3 = 14$$
$$2x_1 + 2x_2 + 4x_3 = 8$$

$$\begin{array}{c c c} \left|\frac{2}{3}\right| & & \frac{2}{3} \\ \left|\frac{-4}{-6}\right| & \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right] & \frac{2}{3} \\ \frac{2}{4} & & \frac{1}{2} \end{array}$$

$$s_1 = \max\{|2|, |-3|, |2|\} = 3$$
$$s_2 = \max\{|-4|, |2|, |-6|\} = 6$$
$$s_3 = \max\{|2|, |2|, |4|\} = 4$$

$$\frac{|a_{11}|}{s_1} = \frac{2}{3}, \quad \frac{|a_{21}|}{s_2} = \frac{4}{6} = \frac{2}{3}, \quad \frac{|a_{31}|}{s_3} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{a_{21}}{a_{11}} = \frac{-4}{2} = -2, \quad \frac{2}{2} = 1 = \frac{a_{31}}{a_{11}}$$

$$\xrightarrow[-R_1 + R_3]{2R_1 + R_2} \begin{array}{c} \\ 0.66\ \frac{4}{6} \\ 1.25\ \frac{5}{4} \end{array} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & -4 & -2 & 24 \\ 0 & 5 & 2 & 3 \end{array}\right]$$

$$\frac{|a_{22}|}{s_2} = \frac{|-4|}{6} = \frac{2}{3}, \quad \frac{|a_{32}|}{s_3} = \frac{5}{4}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & -4 & -2 & 24 \end{array}\right]$$

5c) Solve the system of linear eqns using scaled partial pivoting.

$$\frac{a_{32}}{a_{22}} = \frac{-4}{5}$$

$$\xrightarrow{\frac{4}{5}R_2 + R_3} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -\frac{2}{5} & \frac{132}{5} \end{array}\right]$$

$$-\frac{2}{5}x_3 = \frac{132}{5}$$

$$\boxed{x_3 = -66}$$

$$5x_2 + 2x_3 = 3$$

$$x_2 = 27$$

$$2x_1 - 3x_2 + 2x_3 = 5$$

$$x_1 = 109$$

$(x_1, x_2, x_3) = (109, 27, -66)$ with one row interchange $R_2 \leftrightarrow R_3$

$\frac{a_{11}}{s_1}, \frac{a_{21}}{s_2}, \frac{a_{31}}{s_3}$

whichever is bigger is on top

$\frac{a_{22}}{s_2}, \frac{a_{32}}{s_3}$

Whichever is bigger is on top

Partial pivoting

$\max \{ |a_{11}| \ |a_{21}| \ |a_{31}| \}$

Scaled partial pivoting

a_{11}