Composite Integration

Composite trapezoid

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right]$$

Composite Simpson

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\frac{n}{2}-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}-1} f(x_{2j-1}) + f(b)\right]$$

$$h = \frac{b-a}{n}$$

Composite midpoint

$$\int_a^b f(x)\,dx = 2h\sum_{j=0}^{n/2} f(x_{2j})$$

$$h = \frac{b-a}{n+2}$$

EXERCISE SET 4.4

1. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.
   a. $\int_1^2 x \ln x\, dx, \quad n = 4$
   b. $\int_{-2}^2 x^3 e^x\, dx, \quad n = 4$
   c. $\int_0^2 \frac{2}{x^2+4}\, dx, \quad n = 6$
   d. $\int_0^{\pi} x^2 \cos x\, dx, \quad n = 6$
   e. $\int_0^2 e^{2x} \sin 3x\, dx, \quad n = 8$
   f. $\int_1^3 \frac{x}{x^2+4}\, dx, \quad n = 8$
   g. $\int_3^5 \frac{1}{\sqrt{x^2-4}}\, dx, \quad n = 8$
   h. $\int_0^{3\pi/8} \tan x\, dx, \quad n = 8$
2. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.
   a. $\int_{-0.5}^{0.5} \cos^2 x\, dx, \quad n = 4$
   b. $\int_{-0.5}^{0.5} x \ln(x+1)\, dx, \quad n = 6$
   c. $\int_{.75}^{1.75} (\sin^2 x - 2x \sin x + 1)\, dx, \quad n = 8$
   d. $\int_e^{e+2} \frac{1}{x \ln x}\, dx, \quad n = 8$
3. Use the Composite Simpson's rule to approximate the integrals in Exercise 1.
4. Use the Composite Simpson's rule to approximate the integrals in Exercise 2.
5. Use the Composite Midpoint rule with $n+2$ subintervals to approximate the integrals in Exercise 1.
6. Use the Composite Midpoint rule with $n+2$ subintervals to approximate the integrals in Exercise 2.
7. Approximate $\int_0^2 x^2 \ln(x^2+1)\, dx$ using $h = 0.25$. Use
   a. Composite Trapezoidal rule.
   b. Composite Simpson's rule.
   c. Composite Midpoint rule.
8. Approximate $\int_0^2 x^2 e^{-x^2}\, dx$ using $h = 0.25$. Use
   a. Composite Trapezoidal rule.
   b. Composite Simpson's rule.
   c. Composite Midpoint rule.
9. Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\, dx$.
10. The Midpoint rule for approximating $\int_{-1}^1 f(x)\, dx$ gives the value 12, the Composite Midpoint rule with $n = 2$ gives 5, and the Composite Simpson's rule gives 6. Use the fact that $f(-1) = f(1)$ and $f(-0.5) = f(0.5) - 1$ to determine $f(-1)$, $f(-0.5)$, $f(0)$, $f(0.5)$, and $f(1)$.

2)

Composite trapezoid formula

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right]$$

b) $\displaystyle\int_{-0.5}^{0.5} x\ln(x+1)\,dx \qquad h = \frac{0.5+0.5}{6} = \frac{1}{6}$

$x_0 = -0.5 \quad x_1 = -0.5 + \frac{1}{6} = -\frac{1}{3} \quad x_2 = -\frac{1}{3} + \frac{1}{6} = -\frac{1}{6} \quad x_3 = -\frac{1}{6} + \frac{1}{6} = 0 \quad x_4 = 0 + \frac{1}{6} = \frac{1}{6} \quad x_5 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \quad x_6 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} = 0.5$

$$\int_{-0.5}^{0.5} x\ln(x+1)\,dx = \frac{\frac{1}{6}}{2}\left[-0.5\ln(-0.5+1) + 2\left[-\frac{1}{3}\ln\left(-\frac{1}{3}+1\right) - \frac{1}{6}\ln\left(-\frac{1}{6}+1\right) + 0\ln(0+1) + \frac{1}{6}\ln\left(\frac{1}{6}+1\right) + \frac{1}{3}\ln\left(\frac{1}{3}+1\right)\right] + 0.5\ln(1+0.5)\right] = 0.09363$$

c) $\displaystyle\int_{0.75}^{1.75} (\sin^2 x - 2x\sin x + 1)\,dx \qquad h = \frac{1.75-0.75}{8} = \frac{1}{8} = 0.125$

$x_0 = 0.75 \quad x_1 = 0.75 + 0.125 = 0.875 \quad x_2 = 0.875 + 0.125 = 1 \quad x_3 = 1 + 0.125 = 1.125 \quad x_4 = 1.125 + 0.125 = 1.25$

$x_5 = 1.25 + 0.125 = 1.375 \quad x_6 = 1.375 + 0.125 = 1.5 \quad x_7 = 1.5 + 0.125 = 1.625 \quad x_8 = 1.625 + 0.125 = 1.75$

$$\int_{0.75}^{1.75} (\sin^2 x - 2x\sin x + 1)\,dx = \frac{0.125}{2}\Big[(\sin^2(0.75) - 2(0.75)\sin(0.75) + 1) + 2\big[(\sin^2(0.875) - 2(0.875)\sin(0.875) + 1) + (\sin^2(1) - 2(1) + 1) + (\sin^2(1.125) - 2(1.125) + 1) + (\sin^2(1.25) - 2(1.25) + 1)$$
$$+ (\sin^2(1.375) - 2(1.375)\sin(1.375) + 1) + (\sin^2(1.5) - 2(1.5) + 1) + (\sin^2(1.625) - 2(1.625) + 1)\big] + \sin^2(1.75) - 2(1.75) + 1\Big]$$
$$= -0.4893$$

Composite Simpson formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\frac{n}{2}-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b)\right]$$

b) $\displaystyle\int_{-0.5}^{0.5} x\ln(x+1)\,dx \qquad h = \frac{0.5+0.5}{6} = \frac{1}{6}$

$x_0 = -0.5 \quad x_1 = -0.5 + \frac{1}{6} = -\frac{1}{3} \quad x_2 = -\frac{1}{3} + \frac{1}{6} = -\frac{1}{6} \quad x_3 = -\frac{1}{6} + \frac{1}{6} = 0 \quad x_4 = 0 + \frac{1}{6} = \frac{1}{6} \quad x_5 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \quad x_6 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} = 0.5$

$$\int_{-0.5}^{0.5} x\ln(x+1)\,dx = \frac{\frac{1}{6}}{3}\left[-0.5\ln(-0.5+1) + 2\left[-\frac{1}{6}\ln\left(-\frac{1}{6}+1\right) + \frac{1}{6}\ln\left(\frac{1}{6}+1\right)\right] + 4\left[-\frac{1}{3}\ln\left(-\frac{1}{3}+1\right) + 0\ln(0+1) + \frac{1}{3}\ln\left(\frac{1}{3}+1\right)\right] + 0.5\ln(1+0.5)\right] = 0.08809$$

c) $\displaystyle\int_{0.75}^{1.75} (\sin^2 x - 2x\sin x + 1)\,dx \qquad h = \frac{1.75-0.75}{8} = \frac{1}{8} = 0.125$

$x_0 = 0.75 \quad x_1 = 0.75 + 0.125 = 0.875 \quad x_2 = 0.875 + 0.125 = 1 \quad x_3 = 1 + 0.125 = 1.125 \quad x_4 = 1.125 + 0.125 = 1.25$

$x_5 = 1.25 + 0.125 = 1.375 \quad x_6 = 1.375 + 0.125 = 1.5 \quad x_7 = 1.5 + 0.125 = 1.625 \quad x_8 = 1.625 + 0.125 = 1.75$

$$\int_{0.75}^{1.75} (\sin^2 x - 2x\sin x + 1)\,dx = \frac{0.125}{3}\Big[(\sin^2(0.75) - 2(0.75)\sin(0.75) + 1) + 2\big[(\sin^2(1) - 2(1) + 1) + (\sin^2(1.25) - 2(1.25) + 1) + (\sin^2(1.5) - 2(1.5) + 1)\big]$$
$$+ 4\big[(\sin^2(0.875) - 2(0.875)\sin(0.875) + 1) + (\sin^2(1.125) - 2(1.125) + 1) + (\sin^2(1.375) - 2(1.375)\sin(1.375) + 1) + (\sin^2(1.625) - 2(1.625) + 1)\big] + \sin^2(1.75) - 2(1.75) + 1\Big]$$
$$= -4.8901$$

5) a) $\int_1^2 x \ln x \, dx$, $n=4$

Composite midpoint formula

$$\int_a^b f(x)\,dx = 2h \sum_{j=0}^{n/2} f(x_{2j})$$

$$h = \frac{2-1}{4+2} = \frac{1}{6}$$

$x_{-1} = 1$ $\quad x_0 = 1 + \frac{1}{6} = \frac{7}{6}$ $\quad x_1 = \frac{7}{6} + \frac{1}{6} = \frac{4}{3}$ $\quad x_2 = \frac{4}{3} + \frac{1}{6} = \frac{3}{2}$

$x_3 = \frac{3}{2} + \frac{1}{6} = \frac{5}{3}$ $\quad x_4 = \frac{5}{3} + \frac{1}{6} = \frac{11}{6}$ $\quad x_5 = \frac{11}{6} + \frac{1}{6} = 2$

$$\int_1^2 x \ln x \, dx = 2\left(\frac{1}{6}\right)\left[\frac{7}{6}\ln\left(\frac{7}{6}\right) + \frac{3}{2}\ln\left(\frac{3}{2}\right) + \frac{11}{6}\ln\left(\frac{11}{6}\right)\right] = 0.633096$$

9. Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\,dx$.

$x_0 = 0$ $\quad f(0) = 1$

$h = \frac{1-0}{4} = \frac{1}{4}$

$x_1 = 0.25$ $\quad f(0.25) = \alpha$

$x_2 = 0.5$ $\quad f(0.5) = 2.5$

$x_3 = 0.75$ $\quad f(0.75) = \alpha$

$x_4 = 1$ $\quad f(1) = 2$

$$\boxed{\int_0^1 f(x)\,dx = 1.75}$$

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n)\right]$$

$$= \frac{0.25}{2}\left[1 + 2(\alpha + 2.5 + \alpha) + 2\right]$$

$$= \frac{0.25}{2}\left[1 + 2(2\alpha + 2.5) + 2\right]$$

$$= \frac{0.25}{2}\left[1 + 4\alpha + 5 + 2\right]$$

$$= \frac{0.25}{2}\left[8 + 4\alpha\right]$$

$$\frac{1}{8}[8 + 4\alpha]$$

$$= 1 + \frac{1}{2}\alpha$$

$1 + \frac{1}{2}\alpha = 1.75$

$-1 \qquad -1$

$2 \times \frac{1}{2}\alpha = 0.75 \times 2$

$\alpha = 1.5$

$$1 + \frac{1}{2}\alpha = 1.75$$

$-1 \qquad -1$

$$2 \times \frac{1}{2}\alpha = 0.75 \times 2$$

$$\boxed{\alpha = 1.5}$$