

The background of the slide is an abstract, marbled pattern in shades of blue, green, and white. The pattern consists of organic, flowing shapes and veins, creating a textured, watercolor-like effect. The colors are most concentrated on the left side, where the blue and green are darker, and become lighter and more white towards the right.

3.2

Data Approximation and Neville's Method

Neville's Method

Disadvantages: No polynomial, we'll find the value only (unlike Lagrange)

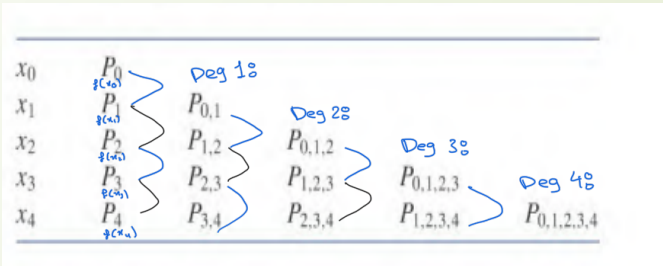
Theorem 3.5 Let f be defined at $x_0, x_1, \dots, x_k$ and let x_j and x_i be two distinct numbers in this set. Then

$$P(x) = \frac{(x - x_j)P_{0,1,\dots,j-1,j+1,\dots,k}(x) - (x - x_i)P_{0,1,\dots,i-1,i+1,\dots,k}(x)}{(x_i - x_j)}$$

is the k th Lagrange polynomial that interpolates f at the $k + 1$ points $x_0, x_1, \dots, x_k$.

$$P_{0,1} = \frac{1}{x_1 - x_0}[(x - x_0)P_1 + (x - x_1)P_0], \quad P_{1,2} = \frac{1}{x_2 - x_1}[(x - x_1)P_2 + (x - x_2)P_1],$$

$$P_{0,1,2} = \frac{1}{x_2 - x_0} [(x - x_0)P_{1,2} + (x - x_2)P_{0,1}],$$



in this set. Then

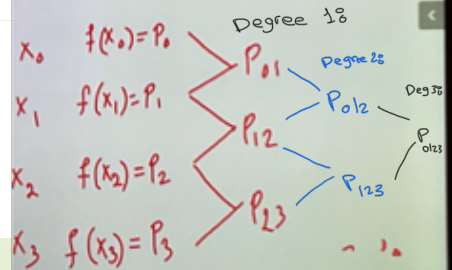
$\mu_4(x)$

$$y = x_1 + x_2 + \dots + x_n$$

$$(-x_2)P_1],$$

$$p_{01}(x) = \frac{(x-x_0)q_1 - (x-x_1)q_0}{x_1 - x_0}$$

$$p_{12}(x) = \frac{(x-x_1)p_2 - (x-x_2)p_1}{x_2-x_1}$$



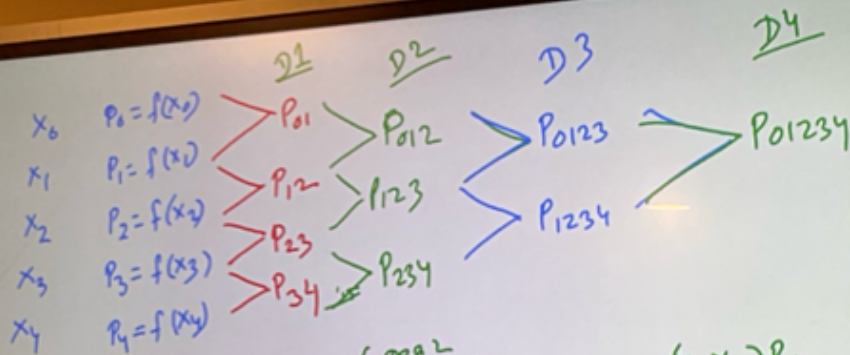
$$P_{012}(x) = \frac{(x-x_0)P_{12} - (x-x_2)P_{01}}{x_2 - x_0}$$

drop the middle value

$$P_{123}(x) = \frac{(x-x_1)P_{23} - (x-x_3)P_{12}}{x_3 - x_1}$$

$$P_{0123}(x) = \frac{(x - x_0)P_{123} - (x - x_3)P_{012}}{x_3 - x_0}$$

(max to be in an exam)



deg 1

$$P_{01}(x) = \frac{(x-x_1)P_0 - (x-x_0)P_1}{x_1-x_0}$$

$$P_{12}(x) = \frac{(x-x_1)P_2 - (x-x_2)P_1}{x_2-x_1}$$

$$P_{23}(x) = \frac{(x-x_2)P_3 - (x-x_3)P_2}{x_3-x_2}$$

$$P_{34}(x) = \frac{(x-x_3)P_4 - (x-x_4)P_3}{x_4-x_3}$$

deg 2

$$P_{012}(x) = \frac{(x-x_2)P_{01} - (x-x_1)P_2}{x_2-x_0}$$

$$P_{123}(x) = \frac{(x-x_1)P_{23} - (x-x_3)P_2}{x_3-x_1}$$

$$P_{234}(x) = \frac{(x-x_2)P_{34} - (x-x_4)P_3}{x_4-x_2}$$

Degree 3:

$$P_{0123}(x) = \frac{(x-x_0)P_{123} - (x-x_3)P_{012}}{x_3-x_0}$$

$$P_{1234}(x) = \frac{(x-x_0)P_{234} - (x-x_4)P_{123}}{x_4-x_0}$$

Degree 4:

$$P_{01234}(x) = \frac{(x-x_0)P_{1234} - (x-x_4)P_{0123}}{x_4-x_0}$$

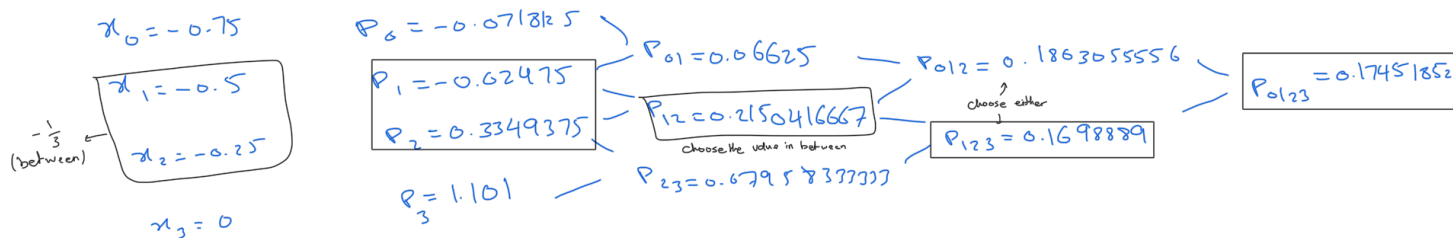
x_0	$P_0 = Q_{0,0}$				
x_1	$P_1 = Q_{1,0}$	$P_{0,1} = Q_{1,1}$			
x_2	$P_2 = Q_{2,0}$	$P_{1,2} = Q_{2,1}$	$P_{0,1,2} = Q_{2,2}$		
x_3	$P_3 = Q_{3,0}$	$P_{2,3} = Q_{3,1}$	$P_{1,2,3} = Q_{3,2}$	$P_{0,1,2,3} = Q_{3,3}$	
x_4	$P_4 = Q_{4,0}$	$P_{3,4} = Q_{4,1}$	$P_{2,3,4} = Q_{4,2}$	$P_{1,2,3,4} = Q_{4,3}$	$P_{0,1,2,3,4} = Q_{4,4}$

EXERCISE SET 3.2

1. Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:
 - a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$
 - b. $f\left(-\frac{1}{3}\right)$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$
 - c. $f(0.25)$ if $f(0.1) = 0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$
 - d. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

1. Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:

b. $f(-\frac{1}{3})$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$



$$P_{01}(-\frac{1}{3}) = \frac{(-\frac{1}{3} + 0.75)(-0.02475) - (-\frac{1}{3} + 0.5)(-0.0718125)}{-0.5 - (-0.75)} = 0.06625$$

$$P_{12}(-\frac{1}{3}) = \frac{(-\frac{1}{3} + 0.5)(0.3349375) - (-\frac{1}{3} + 0.25)(-0.02475)}{-0.25 + 0.5} = 0.215041667$$

$$P_{23}(-\frac{1}{3}) = \frac{(-\frac{1}{3} + 0.25)(1.101) - (-\frac{1}{3} - 0)(0.3349375)}{0 + 0.25} = 0.0795833333$$

$$\frac{\text{Deg } 28}{P_{012}} = \frac{(-\frac{1}{3} + 0.75)(0.2150416667) - (-\frac{1}{3} + 0.25)(0.606625)}{-0.25 + 0.75} = 0.180305556$$

$$P_{123} = \frac{(-\frac{1}{3} + 0.5)(0.8795833333) - (-\frac{1}{3} - 0)(0.2150416667)}{0 + 0.5} = 0.1698889$$

$$\frac{\text{Deg } 30}{P_{0123}} = \frac{(-\frac{1}{3} + 0.75)(0.1698889) - (-\frac{1}{3} - 0)(0.180305556)}{0 + 0.75} = 0.17451852$$

3. Use Neville's method to approximate $\sqrt{3}$ with the following functions and values.
 - a. $f(x) = 3^x$ and the values $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$.
 - b. $f(x) = \sqrt{x}$ and the values $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, and $x_4 = 5$.
 - c. Compare the accuracy of the approximation in parts (a) and (b).
4. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.
5. Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$				
$x_1 = 0.25$	$P_1 = 2$	$P_{01} = 2.6$			
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$		
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$	

Determine $P_2 = f(0.5)$.

6. Neville's method is used to approximate $f(0.5)$, giving the following table.

$x_0 = 0$	$P_0 = 0$			
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$		
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$	

Determine $P_2 = f(0.7)$.

7. Suppose $x_j = j$, for $j = 0, 1, 2, 3$, and it is known that

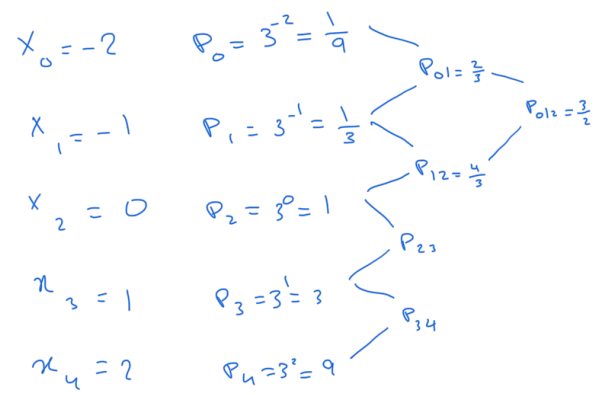
$$P_{0,1}(x) = 2x + 1, \quad P_{0,2}(x) = x + 1, \quad \text{and} \quad P_{1,2,3}(2.5) = 3.$$

Find $P_{0,1,2,3}(2.5)$.

$$\sqrt{3} = 3^x \Rightarrow 3^{\frac{1}{2}} = 3^x \Rightarrow x = \frac{1}{2}$$

3. Use Neville's method to approximate $\sqrt{3}$ with the following functions and values.

a. $f(x) = 3^x$ and the values $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$.



$(x - x_0) P$

$$P_{01} = \frac{(\frac{1}{2} + 2) \frac{1}{3} - (\frac{1}{2} + 1) \frac{1}{9}}{-1 + 2} = \frac{2}{3}$$

$$P_{012} = \frac{(\frac{1}{2} + 2) \frac{4}{3} - (\frac{1}{2} - 0) \frac{2}{3}}{0 + 2} = \frac{3}{2}$$

$$P_{12} = \frac{(\frac{1}{2} + 1)(1) - (\frac{1}{2} - 0) \frac{1}{3}}{0 + 1} = \frac{4}{3}$$

b. $f(x) = \sqrt{x}$ and the values $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, and $x_4 = 5$.

$$f(x) = \sqrt{x} = \sqrt{3}$$

$$x = 3$$

$$\begin{array}{ll} x_0 = 0 & p_0 = \sqrt{0} = 0 \\ x_1 = 1 & p_1 = \sqrt{1} = 1 \end{array} \quad \begin{array}{l} > \\ > \end{array} \quad \begin{array}{l} p_{01} = 3 \\ p_{01} = 3 \end{array}$$
$$\begin{array}{ll} x_2 = 2 & p_2 = \sqrt{2} \\ x_3 = 4 & p_3 = \sqrt{4} = 2 \\ x_4 = 5 & p_4 = \sqrt{5} \end{array}$$

$$p_{01} = \frac{(3-0)1 - (3-1)0}{1-0} = 3$$

5. Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{01} = 2.6$		
$x_2 = 0.5$	$P_2 = 4$	$P_{1,2} = 3.2$	$P_{0,1,2} = 3.08$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

$f(x_i)$

Start backwards to find P_2

Determine $P_2 = f(0.5)$.

$$P_{0123} = \frac{(x - x_0) P_{123} - (x - x_3) P_{012}}{x_3 - x_0}$$

$$3.016 = \frac{(0.4 - 0) 2.96 - (0.4 - 0.75) P_{012}}{0.75 - 0}$$

$P_{012} = 3.08$ using calculator

$$P_{012} = \frac{(x - x_0) P_{12} - (x - x_2) P_{01}}{x_2 - x_0}$$

$$3.08 = \frac{(0.4 - 0) P_{12} - (0.4 - 0.5)(2.6)}{0.5 - 0} \quad P_{12} = 3.2$$

$$P_{12} = \frac{(x - x_1) P_2 - (x - x_2) P_1}{x_2 - x_1}$$

$$3.2 = \frac{(0.4 - 0.25) P_2 - (0.4 - 0.5)(2)}{0.5 - 0.25} \quad P_2 = 4$$

7. Suppose $x_j = j$, for $j = 0, 1, 2, 3$, and it is known that

$$P_{0,1}(x) = 2x + 1, \quad P_{0,2}(x) = x + 1, \quad \text{and} \quad P_{1,2,3}(2.5) = 3.$$

Find $P_{0,1,2,3}(2.5)$

$x = 2.5$

$$P_{0,1}(2.5) = 2(2.5) + 1 = 6$$

$$P_{0,2}(2.5) = 2.5 + 1 = 3.5$$

$$P_{1,2,3}(2.5) = 3$$

$$P_{0,1,2}(2.5) = \frac{(x-x_0)P_{1,2} - (x-x_2)P_{0,1}}{x_1-x_0} = \frac{(2.5-0)3.5 - (2.5-2)(6)}{2-0} = \frac{8.75 - 3}{2} = 2.875$$

$$P_{0,1,2,3}(2.5) = \frac{(x-x_0)P_{1,2,3} - (x-x_3)P_{0,1,2}}{x_2-x_0} = \frac{(2.5-0)3 - (2.5-3)(2.875)}{3-0} = \frac{7.5 - (-2.875)}{3} = \frac{10.375}{3} = 3.458\bar{3}$$

7. $x_j = j, j = 0, 1, 2, 3$

$$P_{0,1}(x) = 2x + 1, \quad P_{0,2}(x) = x + 1, \quad P_{1,2,3}(2.5) = 3, \quad P_{0,1,2,3}(2.5) = ?$$

$$P_{0,1}(2.5) = 2(2.5) + 1 = 6$$

$$P_{0,2}(2.5) = 2.5 + 1 = 3.5$$

$$P_{1,2,3}(2.5) = 3$$

$$P_{0,1,2}(2.5) = \frac{(x-x_0)P_{1,2} - (x-x_2)P_{0,1}}{x_1-x_0} = \frac{(2.5-0)3.5 - (2.5-2)(6)}{2-0} = 2.875$$

$$P_{0,1,2,3}(2.5) = \frac{(x-x_0)P_{1,2,3} - (x-x_3)P_{0,1,2}}{x_2-x_0} = \frac{(2.5-0)3 - (2.5-3)(2.875)}{3-0} = 3.458\bar{3}$$