All polynomials are functions but not all functions are polynomials.

If a function is a polynomial, it is always continuous.

ex: $f(x) = -x^3 + 3x^2 - 2x + 5$

Continuity conditions:

1) Limit must exist

2) Value must exist

3) Limit value = Exact value

Rolle's theorem:

1) f(x) ∈ continuous [a, b] closed interval

2) f'(x) exists in (a, b) open interval

3) f(a) = f(b)

Then f'(c) = 0, c ∈ [a, b]

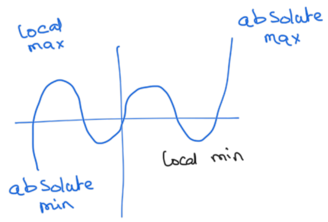
there could be more that 1 c

Extreme value theorem:

Where f'(x) = 0, it is called a critical point which can be max or min

local max / local min can be in multiple points

absolute max / absolute min are just one point, either the highest or lowest point



Steps to find critical point:

1) Find the derivative

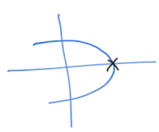
2) make the derivative equal to 0

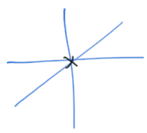
3) a) if the value we found is within the interval, accept it
   b) if the value isn't within the interval, reject it

4)calculate f value that the critical points add end points or the interval

Intermediate value theorem:

If the function is continuous and is going from posited to negative one the y-axis, there will be at least one point where the x-axis is intersected or 0 is at x





Taylors theorem:

Converts any function to a polynomial approximately

The degree will be given to know when you should stop the formula

You can use polynomials as functions but the degree of x won't change

$$P_n(x) \approx f(x)$$

$$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)(x-x_0)^3}{3!} + \dots$$

Ex 1 : $f(x) = 2 - e^x + 2x$ $[0,1]$

① $f'(x) = -e^x + 2$

② $-e^x + 2 = 0$

$\cancel{\ln e^x = \ln 2}$

$x = \boxed{\ln 2} = 0.69$ ←

$f(0) = 2 - e^0 + 2(0) = 1$ ← absolute min

$f(1) = 2 - e^1 + 2(1) = 1.28$

$f(\ln 2) = 2 - e^{\ln 2} + 2(\ln 2) = 1.38$ ← absolute max

$[1,2]$

$f(1) = 2 - e^1 + 2(1) = 1.28$ ← absolute min

$f(2) = 2 - e^2 + 2(2) = 1.38$ ← absolute max

9. Let $f(x) = x^3$.

a. Find the second Taylor polynomial $P_2(x)$ about $x_0 = 0$.

9 ⓐ $f(x) = x^3$ $\quad x_0 = 0$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f(0) = 0^3 = 0$

$f'(0) = 3(0)^2 = 0$

$f''(0) = 6(0) = 0$

$$P_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!}$$

$$= 0$$

Note: You cannot change x^3 to a second degree

ⓒ $x_0 = 1$

$f(1) = 1^3 = 1$

$f'(1) = 3(1)^2 = 3$

$f''(1) = 6(1) = 6$

$$P_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!}$$

$$= 1 + 3(x-1) + \frac{\cancel{6}^{3}(x-1)^2}{\cancel{2!}}$$

$$= 1 + 3x - 3 + 3(x^2 - 2x + 1)$$

$$= 1 + 3x \cancel{-3} + 3x^2 - 6x \cancel{+3}$$

$$= 3x^2 - 3x + 1$$

7. Show that $f'(x)$ is 0 at least once in the given intervals. (Rolle's theorem)

b. $f(x) = (x-1)\tan x + x \sin \pi x, \quad [0, 1]$

7 b $f(x) = (x-1)\tan x + x\sin \pi x \quad [0,1]$

Always right this no matter what the question is about $\left\{ \begin{array}{l} \text{Continuous in } [0,1] \\ \text{derivative exists in } (0,1) \end{array} \right.$

$f(0) = (0-1)\tan 0 + 0\sin(\pi \cdot 0) = 0$

$f(1) = (1-1)\tan 1 + 1\sin(\pi \cdot 1) = 0$

$f(0) = f(1)$

By Rolle's theorem there exists a point $c \in [0,1]$ such that $f'(c) = 0$

1. Show that the following equations have at least one solution in the given intervals.

a. $x\cos x - 2x^2 + 3x - 1 = 0, \quad [0.2, 0.3]$ and $[1.2, 1.3]$

a $= f(0.2) = -0.28$

By IVT $\quad f(0.3) = 0.0066$

$f(a)$ and $f(b)$ are opposite in sign therefore there exists a $c \in [0.2, 0.3]$ such that $f(c) = 0$

//// $[1.2, 1.3]$