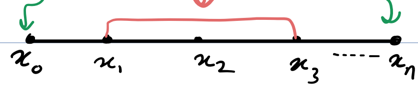
Composite Numerical integration

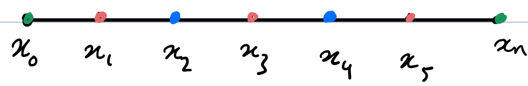
① Composite Trapezoidal rule :

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum f(x_j) + f(x_n)\right] \quad , \quad h = \frac{b-a}{n}$$



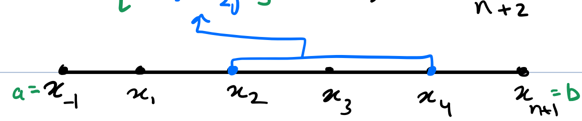
② Composite Simpson's rule : $\boxed{n = \text{must be even}}$

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 2\overset{\text{even point}}{\sum f(x_{2j})} + 4\overset{\text{odd point}}{\sum f(x_{2j-1})} + f(x_n)\right] \quad , \quad h = \frac{b-a}{n}$$



③ Composite Midpoint Rule : "Always start from x_{-1}"

$$\int_{a=x_{-1}}^{b=x_{n+1}} f(x)\,dx = 2h\left[\overset{\text{even point}}{\sum f(x_{2j})}\right] \quad , \quad h = \frac{b-a}{n+2}$$



Q1:

1. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.

a. $\int_1^2 x \ln x \, dx, \quad n = 4$

$h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4} = 0.25$

$x_j = a + jh$

$x_0 = 1$

$x_1 = a + jh = 1.25$

$x_2 = 1 + 2 \times 1.25$

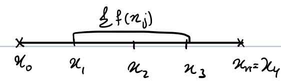
$x_3 = 2$

Stop at $n=4$ → $x_4 = 3$

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3)\right) + f(x_4)\right]$$

$$= \frac{0.25}{2}\left[0 + 4.546842925 + 3.295836866\right]$$

$$= 0.9806514481$$



e. $\int_0^2 e^{2x} \sin 3x \, dx, \quad n = 8$

$h = \frac{b-a}{n} = \frac{2-0}{8} = 0.25$ $\qquad x_j = a + jh$

$x_0 = 0$

$x_1 = 0 + 1 \times 0.25 = 0.25$

$x_2 = 0.5$

$x_3 = 0.75$

$x_4 = 1$

$x_5 = 1.25$

$x_6 = 1.5$

$x_7 = 1.75$

Stop at $n=8$ ← $x_8 = 2$

$$\int_0^2 e^{2x} \sin 3x \, dx = \frac{0.25}{2}\Big[f(x_0) + 2\big[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(7)\big] + f(x_8)\Big]$$

$$= 13.57597939$$

Q3:

3. Use the Composite Simpson's rule to approximate the integrals in Exercise 1.

g. $\int_3^5 \frac{1}{\sqrt{x^2-4}}\,dx, \quad n=8$

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 2\sum_{\text{even}} f(x_{2j}) + 4\sum_{\text{odd}} f(x_{2j-1}) + f(x_n)\right]$$

$h = \frac{5-3}{8} = 0.25$

$$\int_{x_0=3}^{x_8=5} \frac{1}{\sqrt{x^2-4}}\,dx = \frac{0.25}{3}\Big[f(x_0) + 2\underbrace{[f(x_2)+f(x_4)+f(x_6)]}^{0.8848999} + 4\underbrace{[f(x_1)+f(x_3)+f(x_5)+f(x_7)]}_{1.2043744} + f(x_8)\Big]$$

$= 0.6043940$

$x_0 = 3$
$x_1 = 3 + 1\times 0.25 = 3.25$ odd
$x_2 = 3.5$ even
$x_3 = 3.75$ odd
$x_4 = 4$ even
$x_5 = 4.25$ odd
$x_6 = 4.5$ even
$x_7 = 4.75$ odd
$\boxed{x_8 = 5}$ ← Stop at $n=8$ x_n

Q5:

5. Use the Composite Midpoint rule with $n+2$ subintervals to approximate the integrals in Exercise 1.

d. $\int_0^\pi x^2 \cos x\,dx, \quad n=6$

$h = \frac{b-a}{n+2} = \frac{\pi - 0}{6+2} = \frac{\pi}{8}$

$$\int_0^\pi x^2 \cos x\,dx = 2h \sum f(x_{2j})$$ even point

$= 2\,\frac{\pi}{8}[-3.4894321]$

$= -2.7405935$

$x_{-1} = 0$

$x_1 = a + 1\times h = \pi/8$
→ $x_2 = \pi/4$
$x_3 = 3\pi/8$
→ $x_4 = \pi/2$
$x_5 = 5\pi/8$
→ $x_6 = 3\pi/4$
$x_{6+1} = x_7 = 7\pi/4$