Numerical differentiation

Approximation formula:

⇒ **Forward - End point:** $h>0$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$, where $h = x_1 - x_0 =$

⇒ Error Bound $= \frac{h}{2} f''(\xi)$

⇒ **Backword - Endpoint:** $h<0$

$$f'(x_0) = \frac{f(x_0+(-h)) - f(x_0)}{-h}$$, h should be $-ve$

⇒ Error Bound $= \frac{h}{2} f''(\xi)$

⇒ **Three point formula:** *Can use forword n/ Backword*

Three Point - Endpoint ⇒ $f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]$

Error.B $= \frac{h^2}{3} f'''(\xi)$

Three point - Midpoint ⇒ $f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$

, Error. B $= \frac{h^2}{6} f'''(\xi_1)$ using x_0-h and x_0+h

⇒ **Five point formula:** *Can use forword n/ Backword*

Five point - Endpoint ⇒ $f'(x_0) = \frac{1}{12h}\left[-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)\right]$

Error.B $= \frac{h^4}{5} f^5(\xi)$

Five point - Mid point $f'(x_0) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right]$

Error.B $= \frac{h^4}{30} f^5(\xi)$

Q1:

1. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

h=0.1

a.

x	$f(x)$	$f'(x)$
0.5	0.4794	0.852 → f
0.6	0.5646	0.852 → B
0.7	0.6442	0.796 → B

forward: h=0.1.

$$f(0.5) = \frac{f(0.6) - f(0.5)}{0.1} = \boxed{0.852}$$

$$f(0.6) = \frac{f(0.7) - f(0.6)}{0.1} = \boxed{0.796}$$

$$f(0.7) = \frac{f(0.8) - f(0.7)}{0.1}$$ = Can't be solved bc. 0.4 not in table.

Backword: h = -0.1

$$f(0.5) = \frac{f(0.5+(-0.1)) - f(0.5)}{-0.1} = \frac{f(0.4) - f(0.5)}{-0.1}$$ = Can't be solved bc. 0.4 not in table.

$$f(0.6) = \frac{f(0.5) - f(0.6)}{-0.1} = \boxed{0.852}$$

$$f(0.7) = \frac{f(0.6) - f(0.7)}{-0.1} = \boxed{0.796}$$

Q3:

3. The data in Exercise 1 were taken from the following functions. Compute the actual errors in Exercise 1, and find error bounds using the error formulas.

a. $f(x) = \sin x$

① find $f'(x)$ and $f''(x)$:

$f'(x) = \cos x$

$f''(x) = -\sin x$ ('ll use for E.Bound)

from Q1

② Actual. Error: $|\text{Actual.E} - \text{Apper.E}|$

① $\Rightarrow f'(0.5) = \cos(0.5) = 0.8775$

$f'(0.6) = 0.8253$

$f'(0.7) = 0.7648$

② A.E for $(0.5) = |0.852 - 0.8775| = \underline{0.0255}$

$(0.6) = |0.852 - 0.8253| = \underline{0.0267}$

$(0.7) = |0.796 - 0.7648| = \underline{0.0312}$

③ find E. Bound & $\left|\frac{h}{2} f''(\xi)\right|$

for 0.5 $[0.5, 0.6] = \left|\frac{0.1}{2}(-\sin(\xi))\right| = |0.05(-\sin(\xi))| = |-0.05 \times 0.5646| = \underline{0.02823}$

$0.5 \leq \sin(\xi) \leq 0.6$ ↩ Substitut 0.6 and 0.5 in Sin

$0.4794 \quad \boxed{0.5646}$

for 0.6 $[0.6, 0.7] = |-0.05 \times 0.6442| = \underline{0.03221}$

$0.6 \leq \sin(\xi) \leq 0.7$

$0.5646 \quad \boxed{0.6442}$

for 0.7 $[0.6, 0.7] = \underline{0.03221}$

Q58

5. Use the most accurate <u>three-point formula</u> to determine each missing entry in the following tables.

d.		x	$f(x)$	$f'(x)$
forword	x_0	2.0	3.6887983	
mid-Point	x_1	2.1	3.6905701	
	x_2	2.2	3.6688192	
Backwords	x_3	2.3	3.6245909	

$$f'(x_0)=\frac{1}{2h}\left[-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right]$$

$$\text{Error. B}=\frac{h^2}{3}f'''(\xi_0)$$

Three point - Midpoint $\Rightarrow f'(x_0)=\frac{1}{2h}\left[f(x_0+h)-f(x_0-h)\right]$

, Error. B $=\frac{h^2}{6}f'''(\xi_1)$ using x_0-h and x_0+h

$h=0.1$

Three-point-Endpoint :

Endpoint (forward) : $h>0$

$$f'(x_0)=\frac{1}{2h}\left[-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right]=0.1353$$

$$f'(x_1)=-0.1051$$

Endpoint (Backword) : $h<0 \Rightarrow -0.1$

$$f'(2.2)=\frac{1}{2(-0.1)}\left[-3\overset{2.2}{f(x_2)}+4\overset{2.1}{f(x_2+h)}-\overset{2.0}{f(x_2+2h)}\right]$$

$$=-0.3351$$

$$f'(2.3)=-0.5546$$

Three-Point-Midpoint : mus use mid point only "x_1, x_2"

$$f'(x_1)=\frac{1}{2h}\left[\overset{2.2}{f(x_1+h)}-\overset{2.1}{f(x_1-h)}\right]=-0.09989$$

$$f'(x_2)=-0.329896$$