Divied Difference

General formula:

$x_0 \rightarrow \underline{f(x_0)}$

$x_1 \rightarrow f(x_1)$

$x_2 \rightarrow f(x_2)$

$x_3 \rightarrow f(x_3)$

$$\underline{f(x_0-x_1)} = \frac{f(x_1)-f(x_0)}{x_1-x}$$

$$f(x_1-x_2) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$f(x_2-x_3) = \frac{f(x_3)-f(x_2)}{x_3-x_2}$$

$$\underline{f(x_0,x_1,x_2)} = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$

$$f(x_1,x_2,x_3) = \frac{f(x_2,x_3)-f(x_2-x_1)}{x_3-x_1}$$

$$\underline{f(x_0,x_1,x_2,x_3)} = \frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$$

$\Rightarrow$ Polynomeail: "underlined"

$$P_n(x)=\overset{\text{dego}}{f(x_0)}+\overset{\text{deg1}}{f(x_0,x_1)(x-x_0)}+\overset{\text{deg2}}{f(x_0,x_1,x_2)(x-x_0)(x-x_1)}+\overset{\text{deg3}}{f(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2)}+\cdots$$

Q2: **2.** Use Eq. (3.10) or Algorithm 3.2 to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

$x \Rightarrow 0.43$

$x_0 = 0$

$x_1 = 0.25$ $\Rightarrow$

$x_2 = 0.5$

$x_3 = 0.75$

$$= \frac{f(x_1)-f(x_0)}{x_1-x}$$

$$\boxed{f(x_0,x_1) = 2{,}59488}$$

$$f(x_1,x_2) = 4.27824$$

$$f(x_2,x_3) = 7.05364$$

$$= \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$

$$\boxed{f(x_0,x_1,x_2) = 3.66672}$$

$$f(x_1,x_2,x_3) = 5.5508$$

$$= \frac{f(x_0,x_1,x_2)-f(x_1,x_2,x_3)}{x_3-x_0}$$

$$\boxed{f(x_0,x_1,x_2,x_3) = 2.5121066}$$

$$P_1(0.43) = f(x_0) + f(x_0,x_1)(x-x_0) = 2.1157984$$

$$P_2(0.43) = P_1(x) + f(x_0,x_1,x_2)(x-x_0)(x-x_1) = 2.3996025$$

$$P_3(0.43) = P_2(x) + f(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2) = 2.3859919$$

Netown's forward method :

deg 1 | deg 2 | deg 3

$x_0 \to f(x_0)$

$f(x_0 - x_1) = \dfrac{f(x_1) - f(x_0)}{h}$

$x_1 \to f(x_1)$

$f(x_0, x_1, x_2) = \dfrac{f(x_0, x_1) - f(x_1, x_2)}{2h}$

$f(x_1 - x_2) = \dfrac{f(x_2) - f(x_1)}{h}$

$f(x_0, x_1, x_2, x_3) = \dfrac{f(x_0, x_1, x_2) - f(x_1, x_2, x_3)}{3h}$

$x_2 \to f(x_2)$

$f(x_1, x_1, x_2) = \dfrac{f(x_1, x_2) - f(x_2, x_3)}{2h}$

$f(x_2 - x_3) = \dfrac{f(x_3) - f(x_2)}{h}$

$x_3 \to f(x_3)$

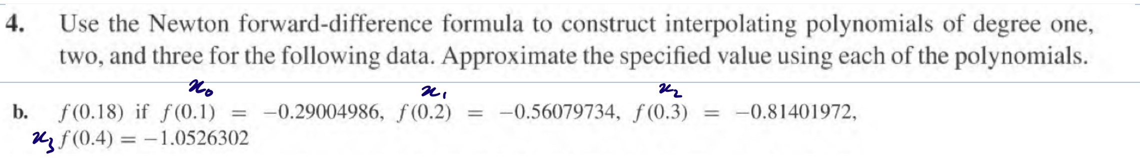
$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \dfrac{x - x_0}{h}$, In forward method "S" is +ve

$\Rightarrow$ polynomial :

$P_n(x) = f(x_0) + f(x_0, x_1) S{\cdot}h + f(x_0, x_1, x_2) S(S-1) h^2 + f(x_0, x_1, x_2, x_3) S(S-2) h^3$ & ----.

Q4 :



4. Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

b. $f(0.18)$ if $\overset{x_0}{f(0.1)} = -0.29004986$, $\overset{x_1}{f(0.2)} = -0.56079734$, $\overset{x_2}{f(0.3)} = -0.81401972$, x_3 $f(0.4) = -1.0526302$

$x \Rightarrow 0.18$

$x_0 = 0.1$

$f(x_0, x_1) = -0.3384343$

$x_1 = 0.2$

$f(x_0, x_1, x_2) = 0.0136915$

$f(x_1, x_2) = -0.3165279$

$f(x_0, x_1, x_2, x_3) = -0.0009483$

$x_2 = 0.3$

$f(x_1, x_2, x_3) = 0.0114155$

$f(x_2, x_3) = -0.2982631$

$x_3 = 0.4$

$h = 0.1$

$S = \dfrac{0.18 - 0.1}{0.1} = 0.8$

$p(0.43) = f(x_0) + f(x_0, x_1) s \cdot h + f(x_0, x_1, x_2) s(s-1) h^2 + f(x_0, x_1, x_2, x_3) s(s-1) h^3$

$= -0.3171464$

Q9:

9. **a.** Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

	x_0	x_1	x_2	x_3	x_4
x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

$x \Rightarrow 0.05$

$x_0 = 0$

$x_1 = 0.2$

$x_2 = 0.4$

$x_3 = 0.6$

$x_4 = 0.8$

$f(x_0, x_1) = 1.107$

$f(x_1, x_2) = 1.3521$

$f(x_2, x_3) = 1.6515$

$f(x_3, x_4) = 2.0171$

$f(x_0, x_1, x_2) = 0.61275$

$f(x_1, x_2, x_3) = 0.7485$

$f(x_2, x_3, x_4) = 0.914$

$f(x_0, x_1, x_2, x_3) = 0.22625$

$f(x_1, x_2, x_3, x_4) = 0.2758333333$

$f(x_0, x_1, x_2, x_3, x_4) = 0.04010416625$

$h = 0.2$

$$S = \frac{0.5 - 0}{0.2} = 2.5$$

$P(0.05) = f(x_0) + Sh(f(x_0, x_1)) + S(S-1)h^2(f(x_0, x_1, x_2)) + S(S-1)h^3(f(x_0, x_1, x_2, x_3) + S(S-1)h^4($

$f(x_0, x_1, x_2, x_3, x_4) =$

$= 1.6524406$

Centered differene (Stirling formula):

$x_{-2} \to f(x_{-2})$				
	$f(x_{-2}, x_{-1}) = \frac{f(x_{-1}) - f(x_{-2})}{x_{-1} - x_{-2}}$			
$x_{-1} \to f(x_{-1})$		$f(x_{-2}, x_{-1}, x_0)$		
	$\boxed{f(x_{-1}, x_0) = \frac{f(x_0) - f(x_{-1})}{x_0 - x_1}}$		$\boxed{f(x_{-2}, x_{-1}, x_0, x_1)}$	
$\boxed{x_0 \to f(x_0)}$		$\boxed{f(x_{-1}, x_0, x_1)}$		$\boxed{f(x_{-2}, x_{-1}, x_0, x_1, x_2)}$
	$\boxed{f(x_0 - x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x}}$		$\boxed{f(x_{-1}, x_0, x_1, x_2)}$	
$x_1 \to f(x_1)$		$f(x_0, x_1, x_2$		
	$f(x_1 - x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$			
$x_2 \to f(x_2)$				

$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \frac{x - x_0}{h}$, In Centeral method "S" can be +ve and −ve

$\Rightarrow$ polynomial:

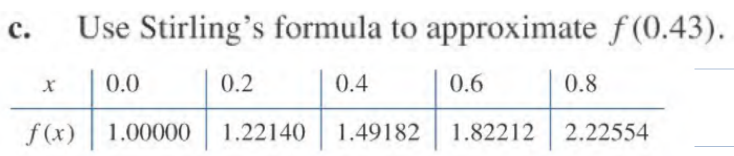
$$P_n(x) = f(x_0) + \frac{s \cdot h}{2}\left[f(x_{-1}, x_0) + f(x_0, x_1)\right] + s^2 h^2 \left[f(x_{-1}, x_0, x_1)\right] + \frac{s(s^2-1)}{2} \cdot h^3 \left[f x_{-2}, x_{-1}, x_0, x_1) + f(x_{-1}, x_0, x_1, x_2)\right]$$

$$+ s^2(s^2-1)h^4 \left[f(x_{-2}, x_{-1}, x_0, x_1, x_2)\right] + \ldots$$

above $x_0 \Rightarrow x_{-1}, x_{-2} \ldots$

belwo $x_0 \Rightarrow x_1, x_2 \ldots$

Q9:



c. Use Stirling's formula to approximate $f(0.43)$.

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

$h = 0.2$

$S = \frac{x - x_0}{h} = \frac{0.43 - 0.4}{0.2} = .15$

	d1	de2	deg3	
$x_{-2} = 0$				
	$f(x_{-2}, x_{-1}) = 1.107$			
$x_{-1} = 0.2$		$f(x_{-2}, x_{-1}, x_0) = 0.61275$		
	$\boxed{f(x_{-1}, x_0) = 1.3521}$		$\boxed{f(x_{-2}, x_{-1}, x_0, x_1) = 0.22625}$	
$\boxed{x_0 = 0.4}$		$\boxed{f(x_{-1}, x_0, x_1) = 0.7485}$		$f(x_{-2}, x_{-1}, x_0, x_1, x_2) =$ $\boxed{0.0619791 25}$
	$\boxed{f(x_0, x_1) = 1.6515}$		$\boxed{f(x_{-1}, x_0, x_1, x_2) = 0.2758333}$	
$x_1 = 0.6$		$f(x_0, x_1, x_2) = 0.914$		
	$f(x_1, x_2) = 2.0171$			
$x_2 = 0.8$				

deg1 deg2 deg3

$$P(0.43) = f(x_0) + \frac{sh}{2}\left(f(x_{-1},x_0) + f(x_0,x_1)\right) + s^2h^2 f(x_{-1},x_0,x_1) + \frac{s(s^2-1)h^3}{2}\left(f(x_{-1},x_0,x_1)\right) +$$

$$s^2(s^2-1)h^4\left(f(x_{-2},x_{-1},x_0,x_1,x_2)\right) = 1.53722913 \rightarrow \text{not sure}$$

Newton's Backword method :

deg 1 | deg 2 | deg 3

$x_0 \rightarrow f(x_0)$

$f(x_0 - x_1) = \frac{f(x_1) - f(x_0)}{h}$

$x_1 \rightarrow f(x_1)$

$f(x_0, x_1, x_2) = \frac{f(x_0, x_1) - f(x_1, x_2)}{2h}$

$f(x_1 - x_2) = \frac{f(x_2) - f(x_1)}{h}$

$f(x_0, x_1, x_2, x_3) = \frac{f(x_0, x_1, x_2) - f(x_1, x_2, x_3)}{3h}$

$x_2 \rightarrow f(x_2)$

$f(x_1, x_1, x_2) = \frac{f(x_1, x_2) - f(x_2, x_3)}{2h}$

$f(x_2 - x_3) = \frac{f(x_3) - f(x_2)}{h}$

$x_3 \rightarrow f(x_3)$

⇒ polynomial :

$$P_n(x) = f(x_0) + \frac{s \cdot h}{2}\left[f(x_{-1}, x_0) + f(x_0, x_1)\right] + s^2 h^2 \left[f(x_{-1}, x_0, x_1)\right] + \frac{s(s^2-1)}{2} \cdot h^3 \left[f x_{-2}, x_{-1}, x_0, x_1) + f(x_{-1}, x_0, x_1, x_2)\right]$$

$$+ s^2(s^2-1)h^4 \left[f(x_{-2}, x_{-1}, x_0, x_1, x_2)\right] + \ldots\ldots$$

above $x_0 \Rightarrow x_{-1}, x_{-2} \ldots\ldots$

belwo $x_0 \Rightarrow x_1, x_2 \ldots\ldots$

$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \frac{x - x_0}{h}$, In Backword method "s" is -ve

9. **a.** $f(0.05) \approx 1.05126$ **b.** $f(0.65) \approx 1.91555$ **c.** $f(0.43) \approx 1.53725$