Neville's

degree 1 | degree 2 | degree 3

$x_0 \to P_0$

$x_1 \to P_1$

$x_2 \to P_2$

$x_3 \to P_3$

$$P_{01} = \frac{(x-x_0)P_1 - (x-x_1)P_0}{x_1 - x_0}$$

$$P_{12} = \frac{(x-x_1)P_2 - (x-x_2)P_1}{x_2 - x_1}$$

$$P_{23} = \frac{(x-x_2)P_3 - (x-x_3)}{x_3 - x_2}$$

$$P_{012} = \frac{(x-x_0)P_{12} - (x-x_2)P_{01}}{x_2 - x_0}$$

$$P_{123} = \frac{(x-x_1)P_{23} - (x-x_3)P_{12}}{x_3 - x_1}$$

$$P_{0123} = \frac{(x-x_0)P_{123} - (x-x_3)P_{012}}{x_3 - x_0}$$

Q1 8

1. Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:

a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$

$x \Rightarrow 8.4$

deg. 1 | deg. 2 | deg. 3

$x_0 = 8.1 \to P_0 = 16.94410$

$x_1 = 8.3 \to P_1 = 17.56492$

$x_2 = 8.6 \to P_2 = 18.50515$

$x_3 = 8.7 \to P_3 = 18.82091$

$P_{01} = 17.87533$

$\boxed{P_{12} = 17.87833}$

$P_{23} = 17.87363$

$$= \frac{(x-x_0)P_{12} - (x-x_2)P_{01}}{x_2 - x_0}$$

$\boxed{P_{012} = 17.87713}$

$P_{123} = 17.877155$

$$\frac{(x-x_0)P_{123} - (x-x_3)P_{012}}{x_3 - x_0}$$

$\boxed{P_{0123} = 17.87714}$

for degree 1 ⇒ we need to choose the P that $x = 8.4$ close/between it's x's

for degree 2,3 ⇒ we Choose what is similare to deg 1

$$P_{01} = \frac{(8.4-8.1)(17.56492) - (8.4-8.3)(16.94410)}{8.3 - 8.1} = 17.87533$$

and P_{12}, P_{23} is the same

Q18

d. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

$x \Rightarrow 0.9$

$x_0 = 0.6 \rightarrow P_0 = -0.17694460$			
	$P_{01} = 0.3951460$		
$x_1 = 0.7 \rightarrow P_1 = 0.01375227$		$P_{012} = 0.4527055$	
	$P_{12} = 0.4335149$		$\boxed{P_{0123} = 0.4451620}$
$x_2 = 0.8 \rightarrow P_2 = 0.22363362$		$\boxed{P_{123} = 0.4426476}$	
	$\boxed{P_{23} = 0.4472140}$		
$x_3 = 1 \rightarrow P_3 = 0.65809197$			

Q38

Q58 find a missing parts in Neville's method:

5. Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{01} = 2.6$		
$x_2 = 0.5$	P_2 ?	$P_{1,2}$?	$P_{0,1,2} =$?	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

Determine $P_2 = f(0.5)$.

$$P_{0123} = \frac{(x - x_0)P_{123} - (x - x_3)P_{012}}{x_3 - x_0}$$

$$3.016 = \frac{(0.4-0)\,2.96 - (0.4-0.75)\,P_{012}}{0.75-0}$$

$$3.016 = \frac{1.184 - (-0.35)\,P_{012}}{0.75}$$

$$0.75 \times 3.016 = 1.184 - (-0.35)\,P_{012}$$

$$\frac{2.262 - 1.184}{0.35} = \frac{0.35}{0.35}P_{012}$$

$$\frac{3.016(0.75)}{1.534} = \frac{(1.534)}{1.534}P_{012}$$

$$\boxed{P_{012} = 1.4745762}$$

$$P_{012} = \frac{1.078}{0.35}$$

$$\boxed{P_{012} = 3.08}$$

$$P_{012} = \frac{(x-x_0)P_{12} - (x-x_2)P_{01}}{x_2 - x_0}$$

$$3.08 = \frac{(0.4-0)P_{12} - (0.4-0.5)\,2.6}{0.5-0}$$

$$0.5 \times 3.08 = \frac{(0.4)P_{12} + 0.26}{0.5} \times 0.5$$

$$1.54 = (0.4)P_{12} + 0.26$$

$$\frac{1.28}{0.4} = \frac{0.4}{0.4} P_{12}$$

$$\boxed{P_{12} = 3.2}$$

$$P_{12} = \frac{(x-x_1)P_2 - (x-x_2)P_1}{x_2 - x_1}$$

$$3.2 = \frac{(0.4-0.25)P_2 - (0.4-0.5)\,2}{0.5-0.25}$$

$$0.25 \times 3.2 = \frac{(0.15)P_2 + 0.2}{0.25} \times 0.25$$

$$0.8 = (0.15)P_2 + 0.2$$

$$\frac{0.6}{0.15} = \frac{0.15}{0.15} P_2$$

$$\boxed{P_2 = 4}$$

Q7