① Horner's Method :

⇒ if x_0 is Not given do vactor of Constent value

⇒ rewrite $f(x)$ in decending order (higher to lower)

Example 2: Use Horner's method to evaluate $P(x) = 2x^4 - 3x^2 + 3x - 4$ at $x_0 = -2$.

$$P(x) = 2x^4 + 0 - 3x^2 + 3x - 4$$

x^3 is missing

Vactoring -2×2

$$x_0 = -2 \quad \begin{array}{|rrrrr} 2 & 0 & -3 & 3 & -4 \\ \downarrow & -4 & 8 & -10 & 14 \\ \hline 2 & -4 & 5 & -7 & \boxed{10} \end{array} + \;= P(-2) \quad \Rightarrow \quad -2 \quad \begin{array}{|rrrr} 2 & -4 & 5 & -7 \\ \downarrow & -4 & 16 & -42 \\ \hline 2 & -8 & 21 & \boxed{-49} \end{array} + \;= P'(-2)$$

$$x_1 = x_0 - \frac{P(x)}{P'(x)}$$

$$x_1 = -2 - \frac{10}{-49} = -1.796$$

$$-1.796 \quad \begin{array}{|rrrrr} 2 & 0 & -3 & 3 & 4 \\ \downarrow & -3.592 & 6.451 & -6.197 & 5.741 \\ \hline 2 & -3.592 & 3.451 & -3.197 & \boxed{9.741} \end{array} + \;= P(x_1 \quad \Rightarrow \quad -1.796 \quad \begin{array}{|rrrr} 2 & -3.592 & 3.451 & -3.197 \\ \downarrow & -3.592 & 12.902 & -29.369 \\ \hline 2 & -7.184 & 16.353 & \boxed{-32.566} \end{array} + \;= P'(x_1)$$

$$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)} = -1.4418$$

Q 18

1. Find the approximations to within 10^{-4} to all the real zeros of the following polynomials using Newton's method.

a. $f(x) = x^3 - 2x^2 - 5$ $\quad$ $x_0 \Rightarrow$ is not given so use Constent Vector

$= x^3 - 2x^2 + 0 - 5$ $(5x-1)$

x is missing

$$\begin{array}{r|rrrr} -1 & 1 & -2 & 0 & -5 \\ & \downarrow & -1 & 3 & -3 \\ \hline & 1 & -3 & 3 & \boxed{-8} \end{array} \Rightarrow P(x)$$

$$\begin{array}{r|rrr} -1 & 1 & -3 & 3 \\ & \downarrow & -1 & 4 \\ \hline & 1 & -4 & \boxed{7} \end{array} \Rightarrow P'(x)$$

$$x_1 = x_1 - \frac{P(x)}{P'(x)} = -1 - \frac{-8}{7}$$

$x_1 = 0.1428 \approx 0.143$

$$\begin{array}{r|rrrr} 0.143 & 1 & -2 & 0 & -5 \\ & \downarrow & 0.143 & -0.265 & -0.037 \\ \hline & 1 & -1.857 & -0.265 & \boxed{-5.037} \end{array} \Rightarrow P(x)$$

$$\begin{array}{r|rrr} 0.143 & 1 & -1.857 & -0.265 \\ & \downarrow & 0.143 & -0.245 \\ \hline & 1 & -1.714 & \boxed{-0.51} \end{array} \Rightarrow P'(x)$$

$$x_2 = x_1 - \frac{P(x)}{P'(x)}$$

$x_2 = -9.7335$

② Muller's Method :

→ P_0, P_1, P_2 are given to find P_3

→ just memorise "C" formula.

$$a = \frac{(p_1 - p_2)[f(p_0) - f(p_2)] - (p_0 - p_2)[f(p_1) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)}.$$ → given

$$b = \frac{(p_0 - p_2)^2[f(p_1) - f(p_2)] - (p_1 - p_2)^2[f(p_0) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)},$$ → given

$$C = p_3 - p_2 = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}.$$ → memorize

choose sign depend on "b" sign

Q 38 **3.** Repeat Exercise 1 using Müller's method.

a. $f(x) = x^3 - 2x^2 - 5$

by choosing $P_0 = 0$, $P_1 = 1$, $P_2 = 2$ ⇒ Can be any number

$f(P_0) = -5$

$$a = \frac{(p_1 - p_2)[f(p_0) - f(p_2)] - (p_0 - p_2)[f(p_1) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)}.$$

$f(P_1) = -6$

$$b = \frac{(p_0 - p_2)^2[f(p_1) - f(p_2)] - (p_1 - p_2)^2[f(p_0) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)},$$

$f(P_2) = -5 = C$

$2 \pm \sqrt{b^2 - 4ac}$ $2 + \sqrt{(1) - 4(1)(-5)}$

$$a = \frac{(1-2)[-5+5] - (-5+5)[-6+5]}{(0-2)(1-2)(0-1)} = 1$$

$$b = \frac{(0-2)^2[-6+5] - (1-2)^2[-5+5]}{(0-2)(1-2)(0-1)} = 2$$

$$P_3 = P_2 - \frac{-2c}{1 \quad \sqrt{\qquad}} = 2 - \frac{-2(-5)}{\sqrt{\ ^{2} \qquad}} = 0.4808378$$