Newton's Method and Eextention

Secent method

false Postion method

① newton's method :

⇒ formula : $\boxed{P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})} \quad , n \geq 1}$

⇒ need to Solve : P_0 will given in Q

Q1 : **1.** Let $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_2.

$$P_n = P_{n-1} \quad \frac{f(P_{n-1})}{f'(P_{n-1})}$$

→ find $f'(x)$:

$$f(x) = x^2 - 6$$

$$f'(x) = 2x$$

→ find P_2 :

$$P_0 = 1$$

$$P_1 = 1 - \frac{(1)^2 - 6}{2(1)} = 3.5$$

$$P_2 = 3.5 - \frac{(3.5)^2 - 6}{2(3.5)} = 2.6071428$$

Q2 : **2.** Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find p_2. Could $p_0 = 0$ be used?

$$P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})}$$

→ find $f'(x)$:

$$f(x) = -x^3 - \cos x$$

$$f'(x) = -3x^2 - (-\sin x) = -3x^2 + \sin x$$

note ⇒ $\frac{d}{dx} \cos x = -\sin x$

→ find P_2 :

$$P_0 = -1$$

$$P_1 = P_0 - \frac{-(P_0)^3 - \cos(P_0)}{-3(P_0)^2 + \sin(P_0)} = -0.88033289$$

$$P_2 = P_1 - \frac{-(P_1)^3 - \cos(P_1)}{-3(P_1)^2 + \sin(P_1)} = -0.8656841$$

→ If we can use $P_0 = 0$?

NO, we can't due to $f'(P_0) \neq 0$

Q5: **5.** Use Newton's method to find solutions accurate to within $\boxed{10^{-4}}$ for the following problems.

a. $x^3 - 2x^2 - 5 = 0, \quad [1, 4]$

$$P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})}$$

→ find $f'(x)$:

$f(x) = x^3 - 2x^2 - 5$

$f'(x) = 3x^2 - 4x$

→ find $p > 10^{-4}$:

for P_0 you can take any number between [1, 4]

or you can take mean value :

$= P_0 = \frac{1+4}{2} = 2{,}5 \approx 2$

$\Rightarrow \quad P_0 = 2$

$P_1 = 2 - \frac{(2)^3 - 2(2)^2 - 5}{3(2)^2 - 4(2)} = 3.25$

$P_2 = P_1 - \frac{(P_1)^3 - 2(P_1)^2 - 5}{3(P_1)^2 - 4(P_1)} = 2.8110367$

$P_3 = P_2 - \frac{(P_2)^3 - 2(P_2)^2 - 5}{3(P_2)^2 - 4(P_2)} = 2.6979894$

$P_4 = P_3 - \frac{(P_3)^3 - 2(P_3)^2 - 5}{3(P_3)^2 - 4(P_3)} = \underline{2.6906771}$

$$P_5 = P_4 - \frac{(P_4)^3 - 2(P_3)^2 - 5}{3(P_3)^2 - 4(P_3)} = \boxed{2.6906474} \rightarrow |P_5 - P_4| = 0.00002 < 10^{-4}$$

Q5/d : **d.** $x - 0.8 - 0.2 \sin x = 0, \quad [0, \pi/2]$

① find $f'(x)$:

$\rightarrow f(x) = x - 0.8 - 0.2 \sin x$

$f'(x) = 1 - 0.2 \cos x$

note : $\frac{d}{dx} \sin x = \cos x$

② find $P > 10^{-4}$

$$P_0 = \frac{0 + \frac{\pi}{2}}{2} = \frac{\pi}{4}$$

$$P_1 = \frac{\pi}{4} - \frac{(\pi/4) - 0.8 - 0.2 \sin(\pi/4)}{1 - 0.2 \cos(\pi/4)} = 0.9671208$$

$$P_2 = P_1 - \frac{(P_1) - 0.8 - 0.2 \sin(P_1)}{1 - 0.2 \cos(P_1)} = 0.9643346$$

$$P_3 = P_2 - \frac{(P_2) - 0.8 - 0.2 \sin(P_2)}{1 - 0.2 \cos(P_2)} = \boxed{0.9643338} \longrightarrow |P_3 - P_2| = 0.0000008 < 10^{-4}$$

(2) Secent method :

Formula: $\boxed{P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}}$

→ Always start w/ P_2

→ Should give P_0 and P_1 to find P_2

Q3: **3.** Let $f(x) = x^2 - 6$. With $p_0 = 3$ and $p_1 = 2$, find p_3.

a. Use the Secant method.

$$P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}$$

→ find P_3 :

was given $P_0 = 3$, $P_1 = 2$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 2.4$$

⇒ $f(P_0) = (3)^2 - 6 = 3$

⇒ $f(P_1) = (2)^2 - 6 = -2$

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_1)}{f(P_2) - f(P_1)} = \boxed{2.45454545}$$

⇒ $f(P_2) = (2.4)^2 - 6 = -0.24$

Q4 : **4.** Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find p_3.

a. Use the Secant method.

$\Rightarrow$ find P_3 : $P_0 = -1$, $P_1 = 0$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = -0.6850734$$

$\Rightarrow f(P_0) = -(-1)^3 - \cos(-1) = 0.4596976$

$\Rightarrow f(P_1) = -(0)^2 - \cos(0) = -1$

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_1)}{f(P_2) - f(P_1)} = \boxed{-1.2519648}$$

$\Rightarrow f(P_2) = -(P_2)^3 - \cos(P_3) = -0.4528014$

Q5 : **5.** Use Newton's method to find solutions accurate to within 10^{-4} for the following problems.

c. $x - \cos x = 0, \quad [0, \pi/2]$

we can assume $P_0 = 0$ $P_1 = \pi/2$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 0.6110154$$

$\Rightarrow f(P_0) = 0 - \cos(0) = -1$

$\Rightarrow f(P_1) = \pi/2 - \cos(\pi/2) = \pi/2$

$$P_3 = P_2 - \frac{f(P_2)(P_2-P_1)}{f(P_2)-f(P_1)} = 0.7232695$$

$$\Rightarrow f(P_2) = (P_2) - \cos(P_2) = -0.2080505$$

$$P_4 = P_3 - \frac{f(P_3)(P_3-P_2)}{f(P_3)-f(P_2)} = 0.7395670$$

$$\Rightarrow f(P_3) = (P_3) - \cos(P_3) = -0.0263763$$

$$P_5 = P_4 - \frac{f(P_4)(P_4-P_3)}{f(P_4)-f(P_3)} = \boxed{0.7395203} \rightarrow$$

4 digit are similar

$$|P_5 - P_4| = 0.00004 < 10^{-4}$$

$$\Rightarrow f(P_4) = (P_4) - \cos(P_4) = 0.0008065$$

Remark: How to Know you Not going wrong in Calculation by the similarity in outcomes.

d. $x - 0.8 - 0.2\sin x = 0, \quad [0, \pi/2]$

$P_0 = 0$ and $P_1 = \pi/2$

$$\Rightarrow P_2 = P_1 - \frac{f(P_1)(P_1-P_0)}{f(P_1)-f(P_0)} = 0.9167206$$

$$\Rightarrow f(P_0) = 0 - 0.8 - 0.2\sin(0) = -0.8$$

$$\Rightarrow f(P_1) = \pi/2 - 0.8 - 0.2\sin(\pi/2) = 0.570796$$

$$P_3 = P_2 - \frac{f(P_2)(P_2-P_1)}{f(P_2)-f(P_1)} = 0.9615658$$

$f(P_2) - f(P_1)$

$\Rightarrow f(P_2) = (P_2) - 0.8 - 0.2\sin(P_2) = -0.0420161$

$$P_4 = P_3 - \frac{f(P_3)(P_3 - P_2)}{f(P_3) - f(P_2)} = \underline{0.964}3449$$

$$f(P_3) = (P_3) - 0.8 - 0.2\sin(P_3) = -0.0024519$$

$$P_5 = P_4 - \frac{f(P_4)(P_4 - P_3)}{f(P_4) - f(P_3)} = \underline{0.9643}338$$

$\rightarrow$ 4 digit are Similer

$|P_5 - P_4| = 0,00001 < 10^{-4}$

$\Rightarrow f(P_4) = 0.0000976$

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③ False Position Method :

$$P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}$$

$\Rightarrow$ in false position metheed we will use Bisection method and Secent method. How?

① Check $f(P_0)$ and $f(P_1)$ is must in Oppsite sign's

② find P_2 by using formula that used in Secent method

③ find $f(P_2)$ depends on sign replace Interval

Q3 :

3. Let $f(x) = x^2 - 6$. With $p_0 = 3$ and $p_1 = 2$, find p_3.

b. Use the method of False Position.

① Check $f(P_0)$ and $f(P_1)$ is must in Oppsite sign's :

$\Rightarrow f(P_0) = (3)^2 - 6 = 3$, +ve

$\Rightarrow f(P_1) = (2)^2 - 6 = -2$, -ve

② find P_2 by using formula that used in Secent method

$$\Rightarrow P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)}$$

$$= 2 - \frac{(-2)(2-3)}{(-2)-(3)} = 2.4$$

③ find $f(P_2)$ depends on sign replace Interval

$f(P_2) = (2.4)^2 - 6 = -0.24 \Rightarrow [3, 2.4]$

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_1)}{f(P_2) - f(P_1)} = 2.4 - \frac{(-0.24)(2.4-3)}{-0.24 - 3} = \boxed{2.4444444}$$

Q98 **9.** Repeat Exercise 5 using the method of False Position.

5. Use Newton's method to find solutions accurate to within $\boxed{10^{-4}}$ for the following problems.

d. $x - 0.8 - 0.2 \sin x = 0, \quad [\overset{P_0}{0}, \overset{P_1}{\pi/2}]$

$f(P_1) = (0) - 0.8 - 0.2\sin(0) = -0.8$, $-ve$

$f(P_1) = (\pi/2) - 0.8 - 0.2\sin(\pi/2) = 0.5707963$, $+ve$

$P_2 = P_1 - \dfrac{f(P_1)\,(P_1 - P_0)}{f(P_1) - f(P_0)} = 0.9167204$

$\Rightarrow f(P_2) = (P_2) - 0.8 - 0.2\sin(P_2) = -0.0420016$, $-ve$ $\Rightarrow [0, -0.9615512]$

$P_3 = P_2 - \dfrac{f(P_2)\,(P_2 - P_1)}{f(P_2) - f(P_1)} = 0.9615512$

$\Rightarrow f(P_3) = (P_3) - 0.8 - 0.2\sin(P_3) = -0.0024648$, $-ve$ $\Rightarrow [0, -0.0024648]$

$P_4 = P_3 - \dfrac{f(P_3)\,(P_3 - P_2)}{f(P_3) - f(P_2)} = 0.\underline{964}3460$

$\Rightarrow f(P_4) = P_4 - 0.8 - 0.2\sin(P_4) = 0.0000107$, $+ve$ $\Rightarrow [0.0000107, -0.0024648]$

$P_5 = P_4 - \dfrac{f(P_4)\,(P_4 - P_3)}{f(P_4) - f(P_3)} = 0.\underline{964}3339$ → 4 digit are Similer

$|P_5 - P_4| = 0.00001 < 10^{-4}$