Fixed Point Iteration

Q1: Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $\boxed{f(x) = x^4 + 2x^2 - x - 3.}$

a. ✓ $g_1(x) = (3 + x - 2x^2)^{1/4}$

$\Rightarrow$ Show $g(p) = p$, where $f(p) = 0$

$g(x) = (3 + x - 2x^2)^{1/4}$

$(x)^4 = \left((3 + x - 2x^2)^{1/4}\right)^4$

$x^4 = 3 + x - 2x^2$

$x^4 + 2x^2 - x - 3 = 0$

$\Downarrow$

$f(x) = 0$

b. ✓ $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

$= g(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

$\downarrow$

$(x)^2 = \left(\left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}\right)^2$

$2 \times x^2 = \dfrac{x + 3 - x^4}{2} \times 2$

$2x^2 = x + 3 - x^4$

$x^4 + 2x^2 - x - 3 = 0$

$f(x) = 0$

$$g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$$

$$= g(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$$

$$(x)^2 = \left(\left(\frac{x+3}{x^2+2}\right)^{1/2}\right)^2$$

$$\frac{x^2}{1} = \frac{x+3}{x^2+2}$$

$$x^2(x^2+2) = x+3$$

$$x^4 + 2x^2 = x+3$$

$$x^4 + 2x^2 - x - 3 = 0$$

$$f(x) = 0$$

Q2 8 **2. a.** Perform four iterations, if possible, on each of the functions g defined in Exercise 1. Let $p_0 = 1$ and $p_{n+1} = g(p_n)$, for $n = 0, 1, 2, 3$.

from Q 1 → **a.** $g_1(x) = (3 + x - 2x^2)^{1/4}$

$P_0 = 1$

$P_1 = g(P_0) = (3+(1)-2(1)^2)^{1/4} = 1.18920711$

$P_2 = g(P_1) = (3+(1.18920711)-2(1.18920711)^2)^{1/4} = 1.0800577$

$P_3 = g(P_2) = 1.1496714$

$P_4 = g(P_3) = 1.10782055$

Q7: Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. Use $p_0 = 1$.

find $g(x)$ and $g'(x)$:

$$x^4 - 3x^2 - 3 = 0$$

$$(x^4)^{1/4} = (3x^2 + 3)^{1/4}$$

$$x = (3x^2+3)^{1/4} \quad \rightsquigarrow \quad g(x) = x$$

$$\downarrow$$

$$g(x) = (3x^2+3)^{1/4}$$

$$g'(x) = \frac{1}{4}(3x^2+3)^{-\frac{3}{4}} \cdot (6x)$$

use Chain Rule $\Rightarrow$

$$\boxed{\frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)}$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$g'(x) = \frac{6x}{4(3x^2+3)^{3/4}}$$

find $g'(x)$ w/ Interval [1, 2]: Sholud be less 1

$$g'(1) = \frac{6(1)}{4(3(1)^2+3)^{3/4}} = 0.3912711 < 1$$

$$g'(2) = \frac{6(2)}{4(3(2)^2+3)^{3/4}} = 0.3935979 < 1$$

find (p) Iteration:

$p_0 = 1$

$p_1 = g(p_0) = (3(1)^2+3)^{1/4} = 1.5650845$

$p_2 = g(p_1) = 1.7935728 \quad > 10^{-2}$

$p_3 = g(p_2) = 1.8859437 \quad > 10^{-2}$

$p_4 = g(p_3) = 1.9228478 \quad > 10^{-2}$

$p_5 = g(p_4) = 1.9375075 \quad |p_5 - p_4| = 0.01$, No it's equal 10^{-2}

$p_6 = g(p_5) = 1.9433169 \quad |p_6 - p_5| = 0.005 < 10^{-2}$, Q 8 8

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on [1, 2]. Use $p_0 = 1$.

① find $g(x)$ and $g'(x)$:

$x^3 - x - 1 = 0$

$(x^3)^{1/3} = (x+1)^{1/3}$

$x = (x+1)^{1/3}$

$\downarrow$

$g(x) = (x+1)^{1/3}$

$g'(x) = \frac{1}{3}(x+1)^{-2/3} \cdot (1)$

$\Rightarrow g'(1) = \dfrac{1}{3(1+1)^{2/3}} = 0,20 > 1$

$\Rightarrow g'(2) = \dfrac{1}{3(2+1)^{2/3}} = 0,16 > 1$

$P_0 = 1$

$P_1 = g(P_0) = (x+1)^{1/3} = 1.2599210$

$P_2 = g(P_1) = 1.3122938$

$P_3 = g(P_2) = \underline{1.32}23538$ $\quad |P_3 - P_2| = 0,01 \nless 10^{-2}$

$P_4 = g(P_3) = \underline{1.32}46874$ $\quad |P_4 - P_3| = 0,002 < 10^{-2}$

2digit is Simailer