Chapter 2

2.1 Biobection | 2.2 | 2.3 | 2.4 | 2.5 | 2.6

The Bisection Method

should always $f(x) = 0$

Q1 : Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x = 0$ on $[0, 1]$.

a b

① Check opposite signs :

$f(a) = f(0) = \sqrt{0} - \cos(0) = -ve$

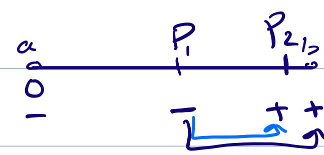
$f(b) = f(1) = \sqrt{1} - \cos(1) = +ve$

② find P_1, P_2, P_3 :

→ $P_n = \frac{a+b}{2}$

→ $f(P_n)$ ⇒ to check sign and find next segences

⇒



$P_1 = \frac{0+1}{2} = 1/2 = \boxed{0,5}$

$f(0.5) = \sqrt{0.5} - \cos(0.5) = -ve$

* Set Calculater in Radain *

$P_2 = \frac{0.5+1}{2} = \boxed{0,75}$

$f(0,75) = +ve$

$P_3 = \frac{0,5+0,75}{2} = \boxed{0.625}$

Q 2 Let $f(x) = 3(x+1)(x-\frac{1}{2})(x-1) = 0$. Use the Bisection method on the following intervals to find p_3.

a. $\overset{a}{[0,}\ \overset{b}{1]}$

$f(0) = +ve$

$f(1) = +ve$

we can't contunue solving the promble in same sign.

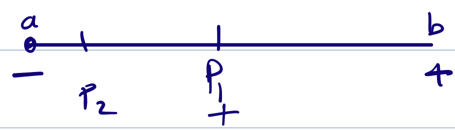
b. $[\overset{a}{-1.25},\ \overset{b}{2.5}]$

$f(a) = -ve$

$f(b) = +ve$

$P_1 = \dfrac{-1.25+2.5}{2} = \boxed{0.625}$

$f(0.625) = +ve$



$P_2 = \dfrac{-1.25+0.625}{2} = \boxed{-0.3125}$

$f(-0.3125) = -ve$

$P_3 = \dfrac{-0.3125+0.625}{2} = \boxed{0.15625}$

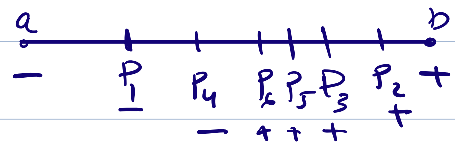
Q3 Use the Bisection method to find solutions accurate to within 10^{-2} (0.01) for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

$$f(x) = x^3 - 7x^2 + 14x - 6$$

a. $\overset{a}{[0}, \overset{b}{1]}$

$f(a) = -ve$

$f(b) = +ve$



$P_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = -ve$ $\qquad |P_n - P| \leq 0.01$

$P_2 = \frac{0.5+1}{2} = 0.75$ $\qquad \rightarrow |P_2 - P_1| = 0,25$ x

$f(0.75) = +ve$

$P_3 = \frac{0.5+0.75}{2} = 0.625$ $\qquad \rightarrow |P_3 - P_2| = 0.125$

$f(0.625) = +ve$

$P_4 = \frac{0.5+0.625}{2} = 0.5625$ $\qquad \rightarrow |P_4 - P_3| = 0.06$

$f(0.5625) = -ve$

$P_5 = \frac{0.5625+0.625}{2} = 0.59375$

$f(0.59375) = +ve$ $\qquad \rightarrow |P_5 - P_4| = 0,03$

$P_6 = \frac{0.5625+0.59375}{2} = 0.578125$

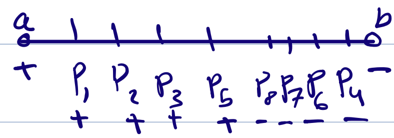
$f(0.578125) = +ve$ $\qquad \rightarrow |P_6 - P_5| = 0.01$

$$P_7 = \frac{0.5625 + 0.578125}{2} = 0.5703125 \quad \rightarrow \quad |P_7 - P_6| = 0.007$$

b. $\overset{a}{[1}, \overset{b}{3.2]}$

$f(1) = +ve$

$f(3.2) = -ve$



$$P_1 = \frac{1+3.2}{2} = 2.1$$

$f(2.1) = +ve$

$$P_2 = \frac{2.1+3.2}{2} = 2.65$$

$f(2.65) = +ve$

$$P_3 = \frac{2.65+3.2}{2} = 2.925$$

$f(2.925) = +ve$

$$P_4 = \frac{2.925+3.2}{2} = 3.0625$$

$f(3.0625) = -ve$

$$P_5 = \frac{2.925+3.0625}{2} = 2.99375 \quad \rightarrow \quad |P_5 - P_4| = > 10^{-2}$$

$f(2.99375) = +ve$

$$P_6 = \frac{2.99375 + 3.0625}{2} = 3.028125 \quad \rightarrow \quad |P_6 - P_5| = > 10^{-2}$$

$f(P_6) = -ve$

$$P_7 = \frac{2.99375 + 3.028125}{2} = 3.0109375 \quad \rightarrow \quad |P_7 - P_6| = > 10^{-2}$$

$f(P_7) = -ve$

$P_8 = \dfrac{2.99375 + 3.0109375}{2} \boxed{= 3.00234375} \rightarrow |P_8 - P_7| = 0{,}008 < 10^{-2}$

$f(P_8) = -ve$

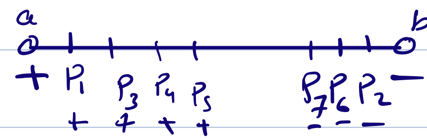
Q4 Use the Bisection method to find solutions accurate to within 10^{-2} (0,01) for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on each interval.

$$f(x) = x^4 - 2x^3 - 4x^2 + 4x + 4$$

b. $[0, 2]$ (a b)

$f(a) = +ve$

$f(2) = -ve$



$P_1 = \dfrac{0+2}{2} = 1$

$f(P_1) = +ve$

$P_2 = \dfrac{1+2}{2} = 1.5$

$f(P_2) = -ve$

$P_3 = \dfrac{1+1.5}{2} = 1.25$

$f(P_3) = +ve$

$P_4 = \dfrac{1.25+1.5}{2} = 1.375$

$f(P_4) = +ve$

$$P_5 = \frac{1.375 + 1.5}{2} = 1.4375$$

$f(P_5) = +ve$

$$P_6 = \frac{1.4375 + 1.5}{2} = 1.46875$$

$f(P_6) = -ve$

$$P_7 = \frac{1.4375 + 1.46875}{2} = 1.453125$$

$f(P_7) = -ve$

$$P_8 = \frac{1.4375 + 1.453125}{2} = \boxed{1.4453125} \longrightarrow |P_8 - P_7| = 0{,}007$$