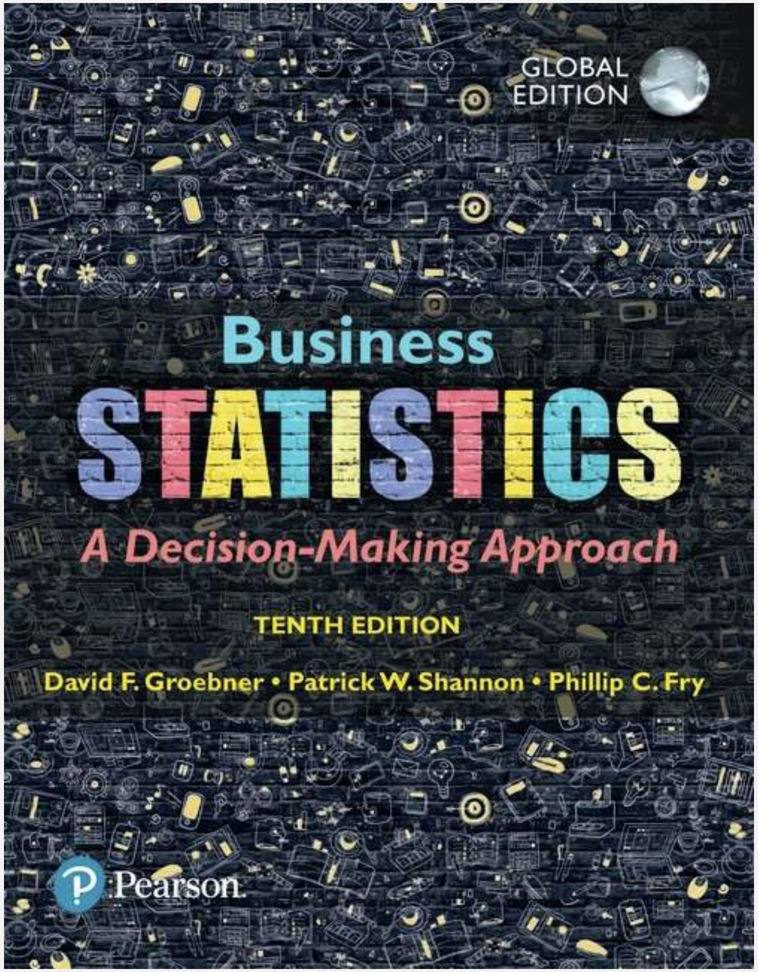
Chapter 7

Introduction to Sampling Distributions



Learning Outcomes

Outcome 1. Understand the concept o

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Outcome 2. Determine the mean and s

distribution of the sample mean $\bar{x}$.

7.1 Sampling Error: What It Is and Why It Happens

- Sampling Error
  - The difference between a measure computed from a sample (a statistic) and the corresponding measure computed from the population (a parameter)

$\bar{x}$ - Sample n

7.1 Sampling Error: What It Is and Why It Happens

Sampling Error

The difference between a measure computed from a sample (a statistic) and the corresponding measure computed from the population (a parameter)

Sampling Error For Means:

$$\bar{x} - \mu$$

Sampling Error For Proportions:

$$\bar{p} - p$$

Sampling Error – Student Test Scores Example

62 students took a business statistics mid-term exam. The scores below reflect the **population** of interest

77	85	55	81	70	82		
79	90	82	72	80	60		
93	90	73	55	84	82		
69	100	51	94	43	79		
64	89	68	58	53	88		N = 62
80	30	60	62	79	91		
49	62	44	62	57	83		
47	78	100	63	74	77		
63	79	100	83	77	29		
85	58	70	33	75	76		
					79		
					78		

Sampling Error – Student Test Scores Example

The population sorted from low score to high score.

29	55	63	76	79	85
30	57	64	77	80	85
33	58	68	77	80	88
43	58	69	77	81	89
44	60	70	78	82	90
47	60	70	78	82	90
49	62	72	79	82	91
51	62	73	79	83	93
53	62	74	79	83	94
55	63	75	79	84	100
					100
					100

Sampling Error – Student Test Scores Example

Compute the Parameters:

Population Mean

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N} = \frac{4,429}{62} = 71.43$$

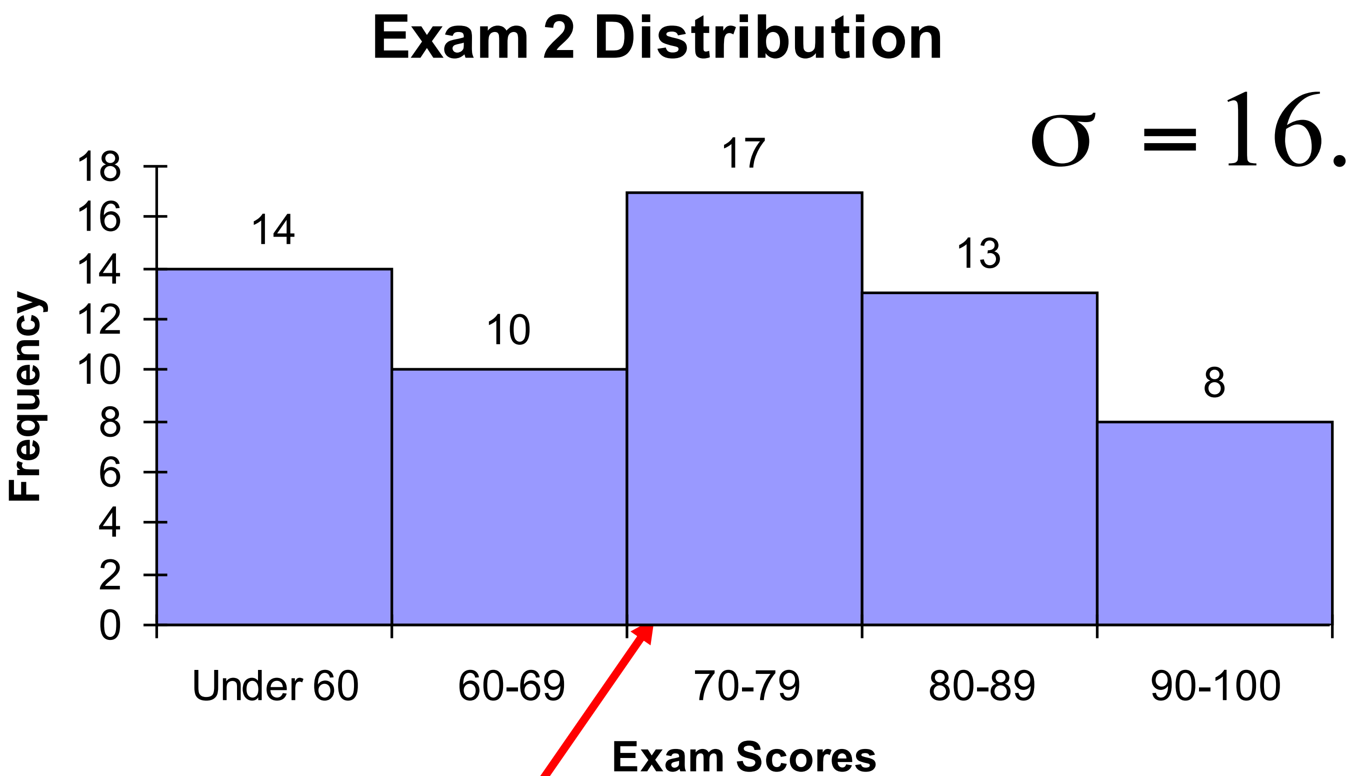
Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}} = \sqrt{\frac{17,498.88}{62}} = 16.80$$

Parameters: $\mu = 71.43$ $\sigma = 16.80$

Sampling Error – Student Test Scores Example

Mid-Term Exam Score Population Distribution



$$\sigma = 16.80$$

$$\mu = 71.43$$

Sampling Error – Student Test Scores Example

To illustrate the concepts of sampling error, select a random sample n = 5 scores from the population

29	55	63	76	79	85		Sample 1
30	57	64	77	80	85		100
33	58	68	77	80	88		44
43	58	69	77	81	89		55
44	60	70	78	82	90		90
47	60	70	78	82	90		75
49	62	72	79	82	91	Mean =	72.80
51	62	73	79	83	93	Sampling Error =	1.37
53	62	74	79	83	94		
55	63	75	79	84	100		
					100		
					100		

$$\mu = 71.43$$

$$\bar{x} - \mu = 1.37$$

Sampling Error – Student Test Scores Example

Sampling error occurs because the sample not a perfect representation of the population

$$\mu = 71.43 \qquad \bar{x} = 72.80$$

$$\bar{x} - \mu = 1.37$$

The sample mean is only 1.37 points higher than the population mean. This sample mean is not a bad representation of the population mean..

Sampling Error – Student Test Scores Example

Take a 2nd sample of n = 5 scores

29	55	63	76	79	85		Sample 1	Sample 2
30	57	64	77	80	85		100	79
33	58	68	77	80	88		44	70
43	58	69	77	81	89		55	33
44	60	70	78	82	90		90	29
47	60	70	78	82	90		75	63
49	62	72	79	82	91	Mean =	72.80	54.80
51	62	73	79	83	93	Sampling Error =	1.37	-16.63
53	62	74	79	83	94			
55	63	75	79	84	100			
					100			
					100			

$$\mu = 71.43$$

$$\bar{x} - \mu = -16.63$$

Sampling Error – Student Test Scores Example

Wow! The sample mean is only 54.80 and is 16.63 points lower than the population mean. This sample would be a misleading value for the center of the population.

$$\mu = 71.43 \qquad \bar{x} = 54.80$$

$$\bar{x} - \mu = -16.63$$

Sampling Error – Student Test Scores Example

Lets take 3 more random sample and calculate the sample mean and sampling error.

	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
	100	79	72	100	100
	44	70	60	83	55
	55	33	29	30	85
	90	29	55	82	58
	75	63	62	62	62
Mean =	**72.80**	**54.80**	**55.60**	**71.40**	**72.00**
Sampling Error =	**1.37**	**-16.63**	**-15.83**	**-0.03**	**0.57**

$$\mu = 71.43$$

Depending on which sample is selected, the sampling error is quite small or very large.

Sampling Error – Student Test Scores Example

Lets take 200 random sample and calculate the sample mean

$\bar{x}$ values (n = 5)

72.80	68.92	64.97	73.32	64.49	66.65	70.74	66.34	85.17	68.97
54.80	69.56	78.61	74.46	73.66	73.27	58.98	61.68	71.51	78.41
55.60	62.94	70.80	75.11	72.01	68.60	75.55	58.93	75.43	72.43
71.40	69.88	64.24	72.34	50.17	81.48	76.52	81.94	75.16	89.90
72.00	63.01	74.03	79.81	67.30	61.35	75.38	75.65	59.37	74.52
63.90	73.42	67.08	64.49	67.05	69.20	88.43	71.26	68.47	66.29
85.58	72.39	88.08	68.58	62.49	61.70	66.49	65.47	68.61	70.67
68.59	64.16	68.82	53.82	69.37	75.15	76.64	67.35	62.72	79.54
78.30	69.15	55.59	75.87	72.17	70.59	73.72	79.62	54.51	64.56
72.83	83.50	82.68	79.92	65.85	72.07	67.47	53.20	65.94	82.27
89.42	62.36	55.00	68.09	69.34	72.12	72.79	63.10	74.89	63.96
70.61	72.82	88.83	72.93	69.08	72.30	60.20	76.95	67.81	80.48
72.70	77.20	72.69	79.71	75.03	71.13	73.66	65.18	66.37	82.26
66.20	68.86	84.10	66.72	76.99	70.36	79.13	80.96	79.81	68.97
77.47	76.24	77.85	77.63	88.56	65.87	72.60	68.49	67.37	60.94
75.94	73.78	69.49	67.51	72.08	73.90	73.29	71.77	55.83	50.07
67.19	63.77	73.90	76.77	78.04	70.60	66.30	80.38	79.94	82.83
51.37	71.08	62.08	66.02	77.70	80.01	56.45	84.86	82.47	67.67
82.54	71.66	76.38	74.27	57.50	69.84	75.70	76.95	68.12	66.49
78.74	87.59	71.50	74.36	76.79	66.26	69.75	69.20	73.37	70.02

Sampling Error – Student Test Scores Example

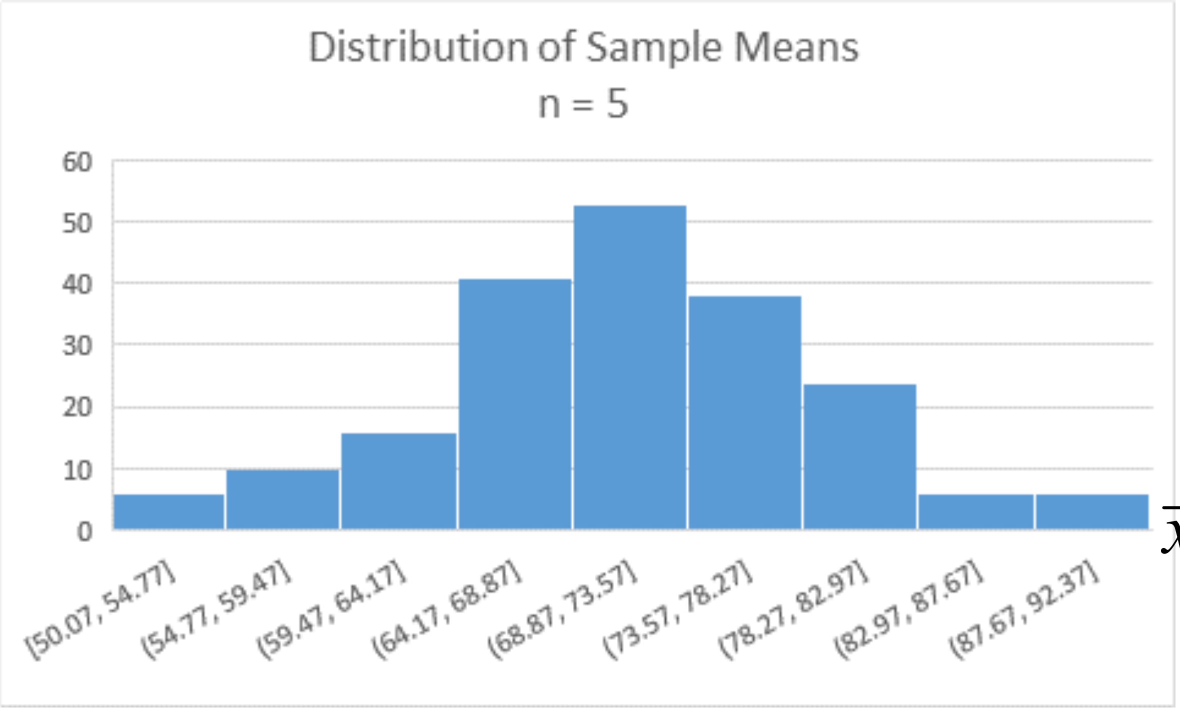
Calculate the sampling error for the 200 different samples.

$$\bar{x} - \mu \text{ (sample error)}$$

1.37	-2.51	-6.46	1.89	-6.94	-4.78	-0.69	-5.09	13.74	-2.46
-16.63	-1.87	7.18	3.03	2.23	1.84	-12.45	-9.75	0.08	6.98
-15.83	-8.49	-0.63	3.68	0.58	-2.83	4.12	-12.50	4.00	1.00
-0.03	-1.55	-7.19	0.91	-21.26	10.05	5.09	10.51	3.73	18.47
0.57	-8.42	2.60	8.38	-4.13	-10.08	3.95	4.22	-12.06	3.09
-7.53	1.99	-4.35	-6.94	-4.38	-2.23	17.00	-0.17	-2.96	-5.14
14.15	0.96	16.65	-2.85	-8.94	-9.73	-4.94	-5.96	-2.82	-0.76
-2.84	-7.27	-2.61	-17.61	-2.06	3.72	5.21	-4.08	-8.71	8.11
6.87	-2.28	-15.84	4.44	0.74	-0.84	2.29	8.19	-16.92	-6.87
1.40	12.07	11.25	8.49	-5.58	0.64	-3.96	-18.23	-5.49	10.84
17.99	-9.07	-16.43	-3.34	-2.09	0.69	1.36	-8.33	3.46	-7.47
-0.82	1.39	17.40	1.50	-2.35	0.87	-11.23	5.52	-3.62	9.05
1.27	5.77	1.26	8.28	3.60	-0.30	2.23	-6.25	-5.06	10.83
-5.23	-2.57	12.67	-4.71	5.56	-1.07	7.70	9.53	8.38	-2.46
6.04	4.81	6.42	6.20	17.13	-5.56	1.17	-2.94	-4.06	-10.49
4.51	2.35	-1.94	-3.92	0.65	2.47	1.86	0.34	-15.60	-21.36
-4.24	-7.66	2.47	5.34	6.61	-0.83	-5.13	8.95	8.51	11.40
-20.06	-0.35	-9.35	-5.41	6.27	8.58	-14.98	13.43	11.04	-3.76
11.11	0.23	4.95	2.84	-13.93	-1.59	4.27	5.52	-3.31	-4.94
7.31	16.16	0.07	2.93	5.36	-5.17	-1.68	-2.23	1.94	-1.41

Sampling Error – Student Test Scores Example

Develop a histogram for the sample means.



$\bar{x}$

$$\mu_{\bar{x}} \approx \frac{\sum_{i=1}^{200} \bar{x}_i}{200} = 71.25$$

$$\sigma_{\bar{x}} \approx \sqrt{\frac{\sum_{i=1}^{200} (\bar{x}_i - 71.25)^2}{200}} = 7.91$$

Parameters

$$\mu = 71.43$$
$$\sigma = 16.80$$

Sampling Error – Student Test Scores Example

Conclusions so far:

- Sampling error can be small or large and positive or negative – the amount of error varies
- The sampling error depends on which sample is selected
- The average of the sample means is approximately equal to the population mean
- The variation in the sample means is less than the variation of the population
- The distribution of sample means is approximately normal

Now let's repeat the exercise with the student test scores but this time, we use a sample size of n = 10

Sampling Error – Student Test Scores Example

First 5 samples of size n = 10

	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
	79	55	49	77	30
	80	75	68	68	55
	77	51	93	49	63
	90	79	60	79	44
	77	80	79	63	75
	82	84	62	30	44
	43	83	30	62	82
	63	44	79	80	79
	79	30	51	75	68
	70	90	93	90	72
Mean =	**74.00**	**67.10**	**66.40**	**67.30**	**61.20**
Sampling Error =	**2.57**	**-4.33**	**-5.03**	**-4.13**	**-10.23**

Sampling Error – Student Test Scores Example

Lets take 200 random sample and calculate the sample mean

$\bar{x}$ values (n = 10)

74.00	65.07	60.02	71.35	68.74	68.23	67.35	65.81	73.21	67.45
67.10	79.13	74.45	67.53	78.08	66.75	66.65	77.55	69.30	64.66
66.40	72.13	77.92	66.26	77.80	69.78	72.19	74.10	76.25	74.14
67.96	73.31	77.33	76.67	65.15	74.25	65.88	68.69	72.97	64.79
67.30	82.09	71.11	79.99	71.86	72.80	80.76	72.01	67.57	64.42
61.20	73.74	77.78	78.83	65.13	64.11	72.50	65.70	65.22	68.31
74.90	69.01	75.82	76.26	74.18	79.52	79.71	67.25	74.02	70.16
68.45	74.51	65.36	71.39	56.95	79.53	69.57	66.11	75.53	74.10
74.64	71.31	71.98	61.41	66.00	58.47	68.67	75.11	66.11	76.35
67.07	70.79	70.82	80.17	68.39	69.17	81.61	69.22	79.93	78.05
72.66	76.45	75.09	74.11	69.22	73.61	71.56	68.09	69.43	74.49
66.07	71.15	69.95	65.66	72.48	68.80	65.91	70.85	75.58	70.15
66.95	79.75	71.65	69.35	75.86	68.33	76.95	62.49	69.67	67.94
75.04	65.42	79.52	72.71	66.71	75.31	68.13	73.06	75.50	76.52
62.82	77.59	72.07	75.42	77.10	69.89	75.64	66.33	65.93	73.95
72.23	68.42	66.71	68.52	78.74	73.19	61.42	75.90	68.40	76.69
71.14	66.54	65.60	76.81	66.99	71.65	70.22	74.42	76.92	76.23
74.99	63.46	70.34	59.69	63.65	71.19	73.71	77.29	71.66	75.76
76.69	77.78	68.45	73.98	71.60	80.24	70.36	71.38	76.08	75.54
61.26	73.23	75.67	77.69	61.41	71.36	68.25	66.59	78.45	69.13

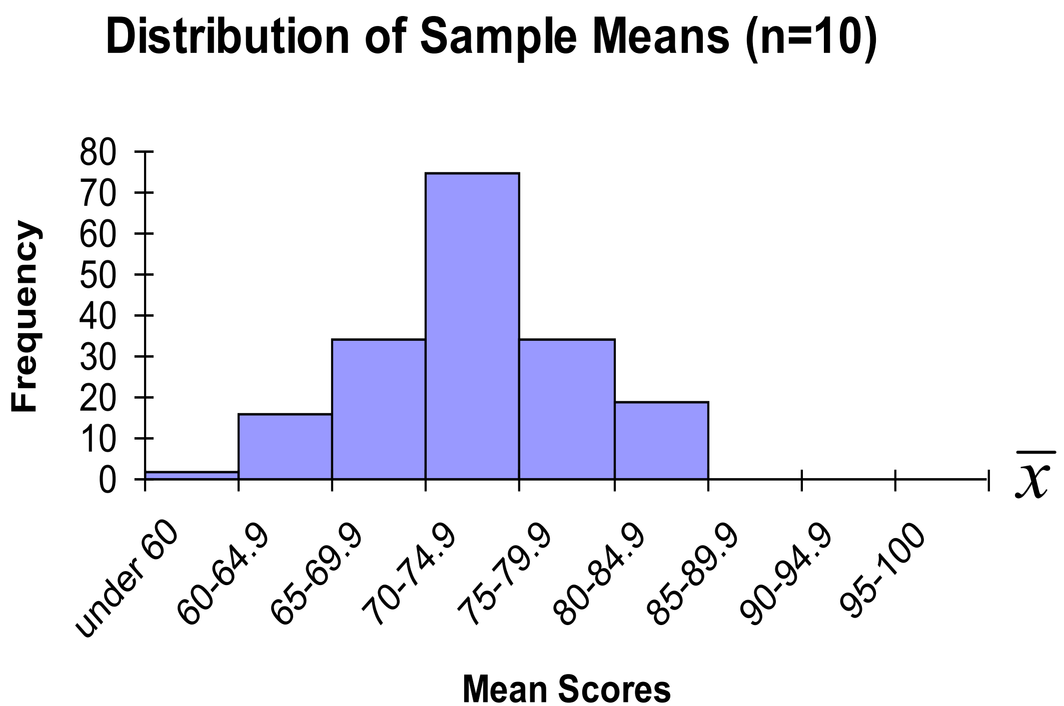
Sampling Error – Student Test Scores Example

Calculate the sampling error for the 200 different samples.

$$\bar{x} - \mu \text{ (sample error)}$$

2.57	-6.36	-11.41	-0.08	-2.69	-3.20	-4.08	-5.62	1.78	-3.98
-4.33	7.70	3.02	-3.90	6.65	-4.68	-4.78	6.12	-2.13	-6.77
-5.03	0.70	6.49	-5.17	6.37	-1.65	0.76	2.67	4.82	2.71
-3.47	1.88	5.90	5.24	-6.28	2.82	-5.55	-2.74	1.54	-6.64
-4.13	10.66	-0.32	8.56	0.43	1.37	9.33	0.58	-3.86	-7.01
-10.23	2.31	6.35	7.40	-6.30	-7.32	1.07	-5.73	-6.21	-3.12
3.47	-2.42	4.39	4.83	2.75	8.09	8.28	-4.18	2.59	-1.27
-2.98	3.08	-6.07	-0.04	-14.48	8.10	-1.86	-5.32	4.10	2.67
3.21	-0.12	0.55	-10.02	-5.43	-12.96	-2.76	3.68	-5.32	4.92
-4.36	-0.64	-0.61	8.74	-3.04	-2.26	10.18	-2.21	8.50	6.62
1.23	5.02	3.66	2.68	-2.21	2.18	0.13	-3.34	-2.00	3.06
-5.36	-0.28	-1.48	-5.77	1.05	-2.63	-5.52	-0.58	4.15	-1.28
-4.48	8.32	0.22	-2.08	4.43	-3.10	5.52	-8.94	-1.76	-3.49
3.61	-6.01	8.09	1.28	-4.72	3.88	-3.30	1.63	4.07	5.09
-8.61	6.16	0.64	3.99	5.67	-1.54	4.21	-5.10	-5.50	2.52
0.80	-3.01	-4.72	-2.91	7.31	1.76	-10.01	4.47	-3.03	5.26
-0.29	-4.89	-5.83	5.38	-4.44	0.22	-1.21	2.99	5.49	4.80
3.56	-7.97	-1.09	-11.74	-7.78	-0.24	2.28	5.86	0.23	4.33
5.26	6.35	-2.98	2.55	0.17	8.81	-1.07	-0.05	4.65	4.11
-10.17	1.80	4.24	6.26	-10.02	-0.07	-3.18	-4.84	7.02	-2.30

Sampling Error – Student Test Scores Example



$$\mu_{\bar{x}} \approx \frac{\sum_{i=1}^{200} \bar{x}_i}{200} = 71.32$$

$$\sigma_{\bar{x}} \approx \sqrt{\frac{\sum_{i=1}^{200} (\bar{x}_i - 71.32)^2}{200}} = 5.12$$

Parameters

$$\mu = 71.43$$
$$\sigma = 16.80$$

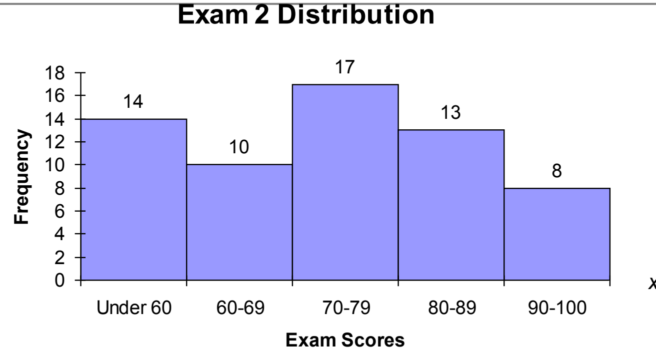
Sampling Error – Student Test Scores Example

Conclusions so far:

- Sampling error can be small or large and positive or negative – the amount of error varies
- The sampling error depends on which sample is selected
- The average of the sample means is approximately equal to the population mean
- The variation in the sample means is less than the variation of the population
- The sampling distribution is approximately normal
- Larger sample sizes result in less variability in the distribution of sample means

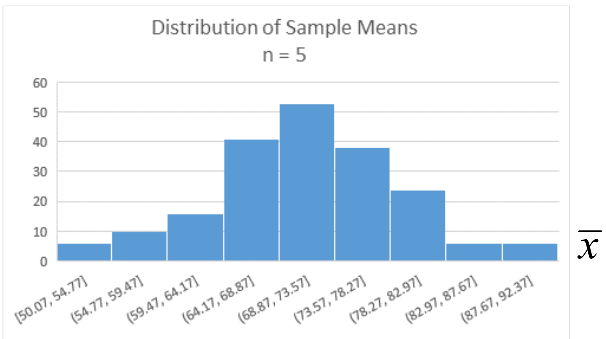
Sampling Distribution – Student Test Scores Example

$$\mu = 71.43$$
$$\sigma = 16.80$$



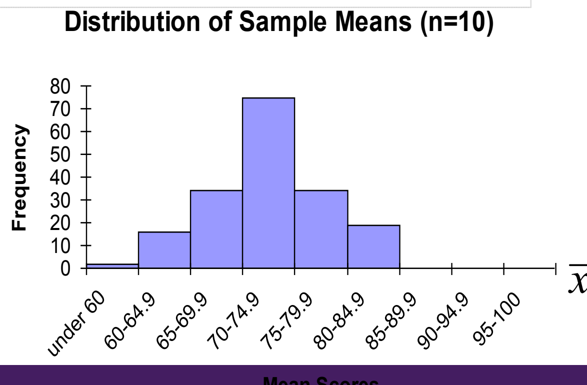
Population Distribution

$$\mu_{\overline{x}} \approx 71.25$$
$$\sigma_{\overline{x}} \approx 7.91$$



Sampling Distribution for 200 samples of n = 5

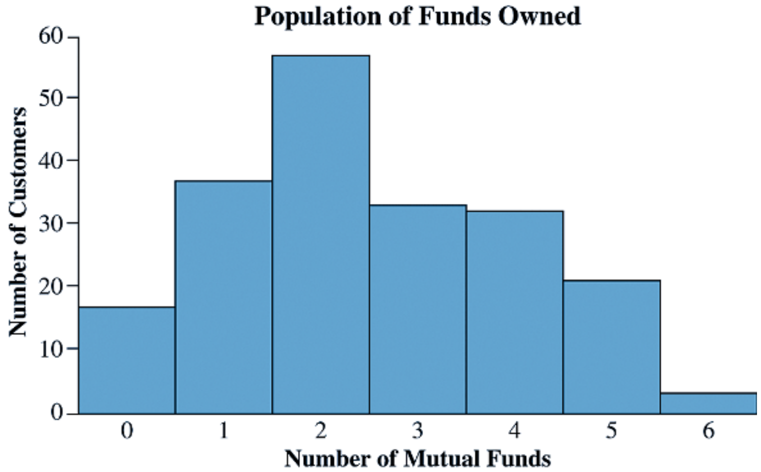
$$\mu_{\overline{x}} \approx 71.32$$
$$\sigma_{\overline{x}} \approx 5.12$$



Sampling Distribution for 200 samples of n = 10

Sampling Distribution – Mutual Fund Portfolio Example

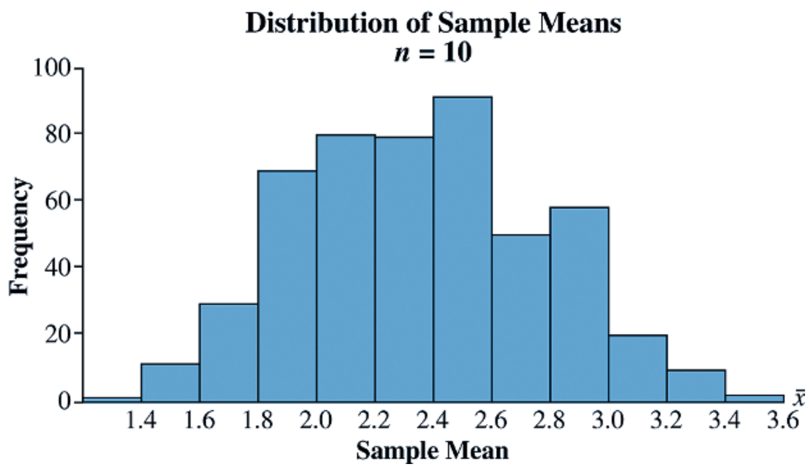
Population



$$\mu = 2.505$$

$$\sigma = 1.503$$

Distribution of sample means for 500 samples of size n = 10



$$\mu_{\overline{x}} \approx 2.41$$

$$\sigma_{\overline{x}} \approx 0.421$$

Sampling Distribution – Theorem 1

Theorem 1

For any population, the average value of all possible sample means computed from all possible random samples of a given size from the population equals the population mean. This is expressed as

$$\mu_{\bar{x}} = \mu$$

The previous examples were simulations based on only a few of the possible samples of a given size that could have been taken from the population. Even then, the results closely match what Theorem 1 says.

Test Score Example Population	n = 5	n= 10
$\mu = 71.43$	$\mu_{\bar{x}} \approx 71.25$	$\mu_{\bar{x}} \approx 71.32$

Sampling Distribution – Theorem 2

Theorem 2

For any population, the standard deviation of the possible sample means computed from all possible random samples of size n is equal to the population standard deviation divided by the square root of the sample size. This is shown as

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

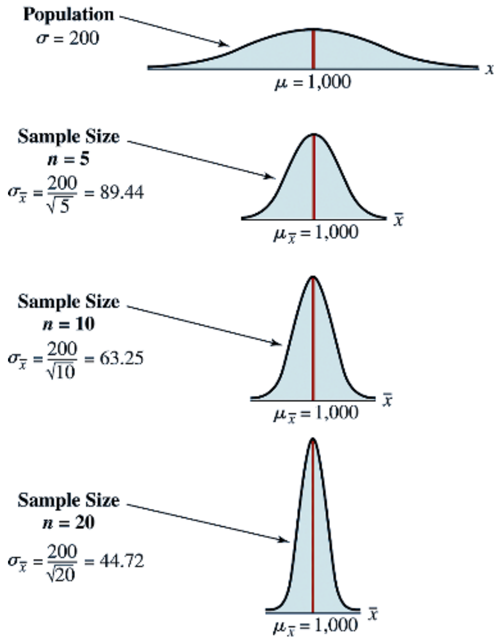
The previous examples were simulations based on only a few of the possible samples of a given size that could have been taken from the population. Even then, the results closely match what Theorem 2 says.

Test Score Example Population	n = 5	n= 10
$\sigma = 16.80$	$\sigma_{\bar{x}} = \frac{16.80}{\sqrt{5}} = 7.51 \approx 7.91$	$\sigma_{\bar{x}} = \frac{16.80}{\sqrt{10}} = 5.31 \approx 5.12$

Sampling Distribution – Theorem 3

Theorem 3

If a population is normally distributed, with mean μ and standard deviation σ, the sampling distribution of the sample mean $\bar{x}$ is also normally distributed with a mean equal to the population mean $(\mu_{\bar{x}} = \mu)$ and a standard deviation equal to the population standard deviation divided by the square root of the sample size $(\sigma_{\bar{x}} = \sigma/\sqrt{n})$.



Sampling Distribution – Theorem 4 – The Central Limit Theorem

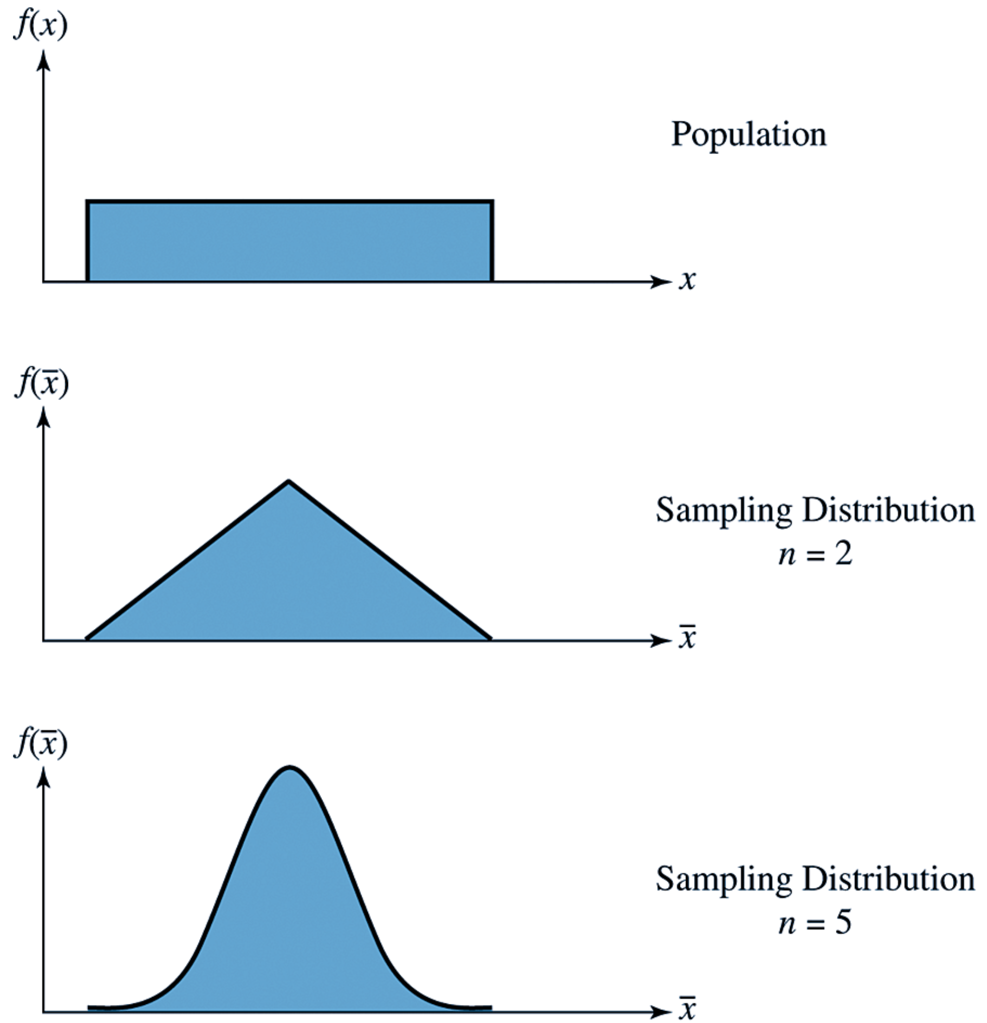
Theorem 4: The Central Limit Theorem

~~For simple random samples of n obser~~vations taken from a population with mean μ and ~~standard deviation σ, regardless of the population's distribution, pro~~vided the sample size is sufficiently large, the distribution of the sample means, $\bar{x}$, is approximately normal with a mean equal to the population mean ($\mu_{\bar{x}} = \mu$) and a standard deviation equal to the population standard deviation divided by the square root of the sample size ($\sigma_{\bar{x}} = \sigma / \sqrt{n}$). The larger the sample size, the better the approximation to the normal distribution.

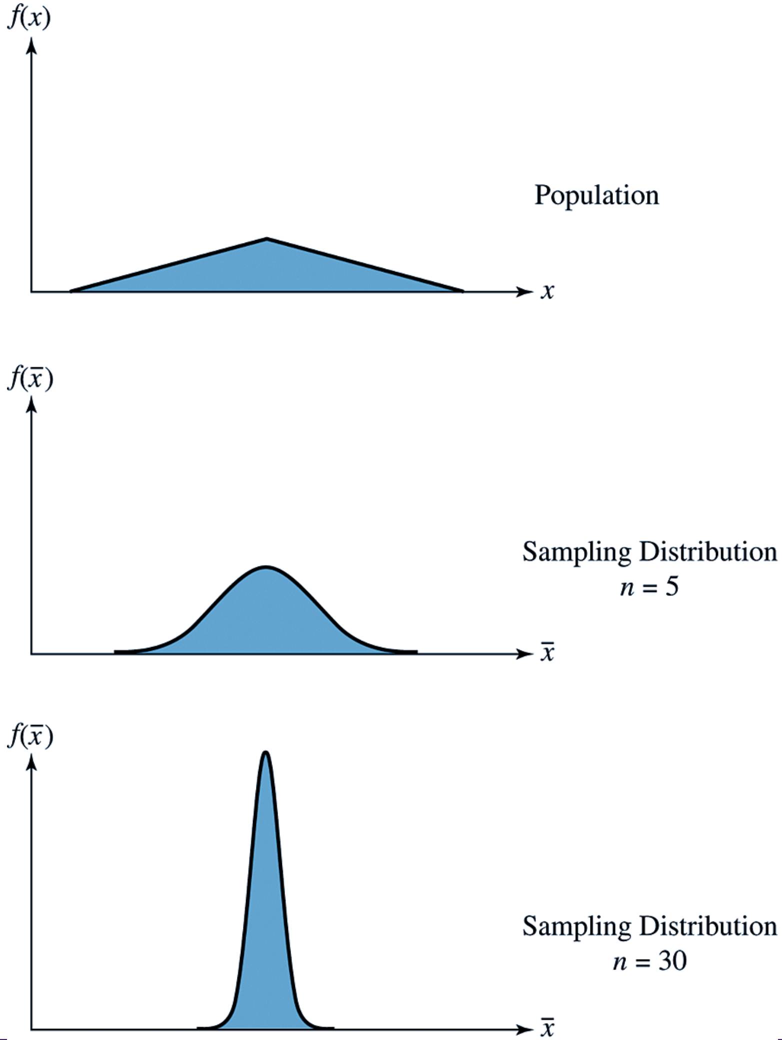
Key Concepts:

- Regardless of population distribution shape
- If sample size is "sufficiently" large
- Sampling distribution will be approximately normal

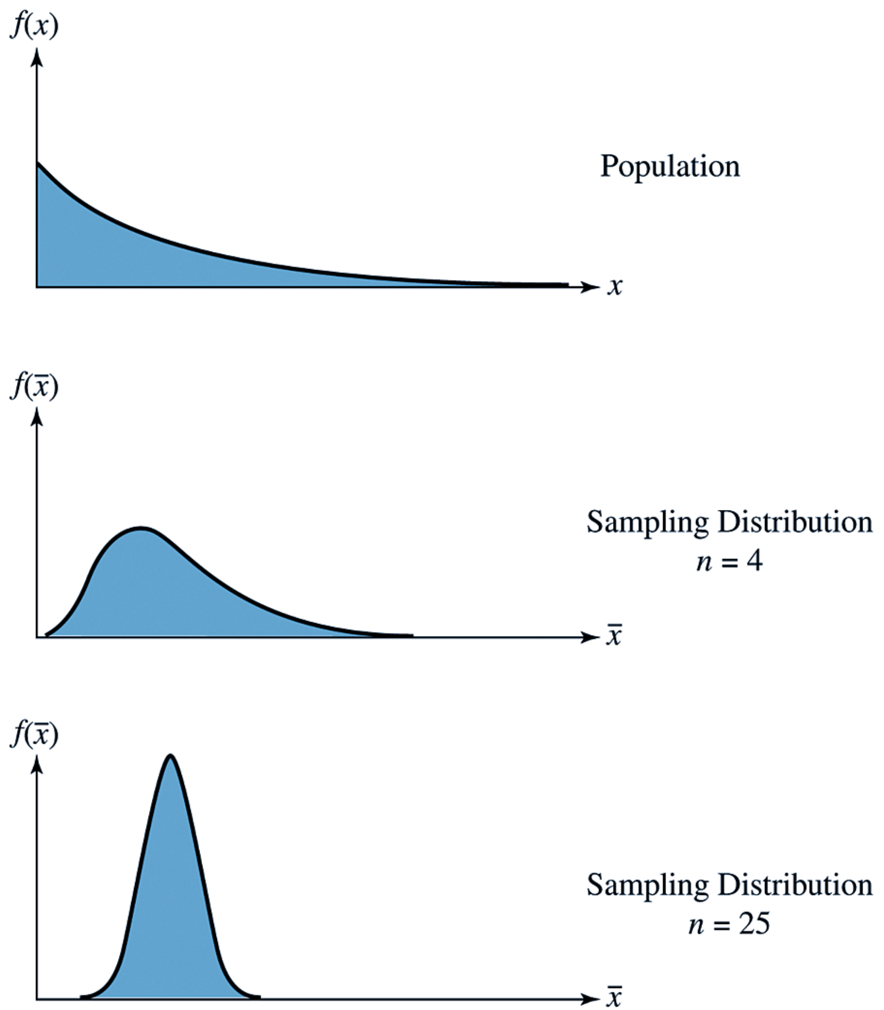
Sampling Distribution – Theorem 3 – The Central Limit Theorem



Sampling Distribution – Theorem 3 – The Central Limit Theorem



Sampling Distribution – Theorem 3 – The Central Limit Theorem



The Central Limit Theorem

- The sample size must be "sufficiently large"
- If the population is quite symmetric, then sample sizes as small as 2 or 3 can provide a normally distributed sampling distribution
- If the population is highly skewed or otherwise irregularly shaped, the required sample size will be larger
- A conservative definition of a sufficiently large sample size is $n \geq 30$

z-Value for Sampli

- The relative distance that a given sample mean is from the center can be determined by <u>standardizing</u> the sampling distribution
- A <u>z-value</u> measures the number of standard deviations a value is from the mean

$\bar{x}$ - Sample mea

μ - Population n

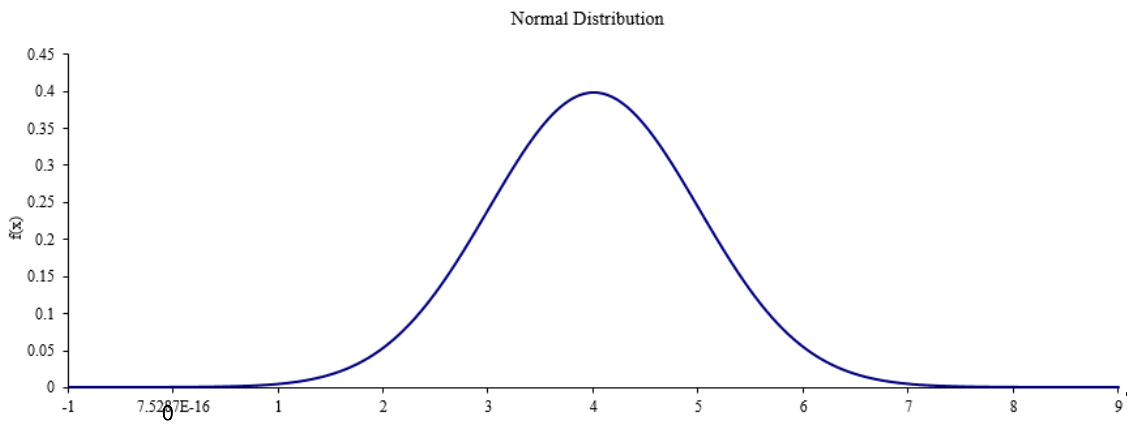
Sampling Distribution – French Fry Example

A food processing company produces French fries for fast food outlets. The distribution of French fry length is thought to be normal with a mean length of 4.0 inches and standard deviation of 1.0 inch. What is the probability of selecting a simple random sample of 100 fries with a sample mean exceeding 4.10 inches?

$$n = 100$$

$$\mu = 4.0$$

$$\sigma = 1.0$$

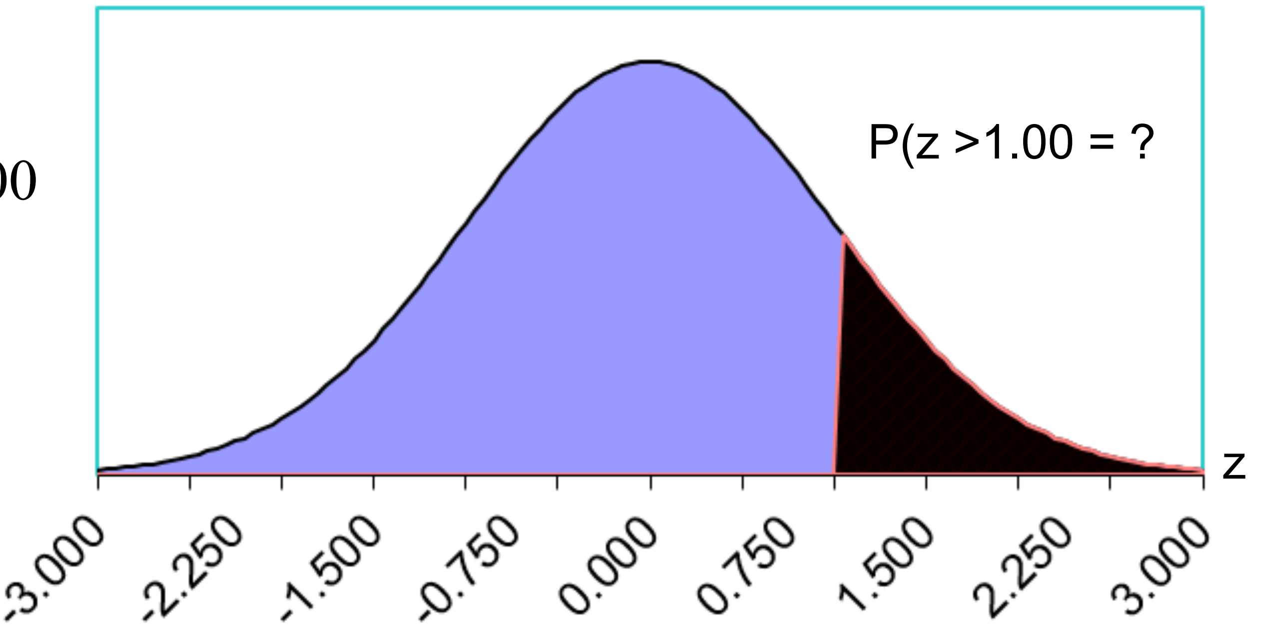


$$\bar{x}$$

$$\bar{x} = 4.1$$

Sampling Distribution – French Fry Example Standardize the Distribution

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.10 - 4}{\frac{1}{\sqrt{100}}} = 1.00$$



From Standard Normal Table:

$$P(0.00 \leq z \leq 1.00) = 0.3413$$

$$P(z > 1.00) = 0.5000 - 0.3413 = 0.1587$$

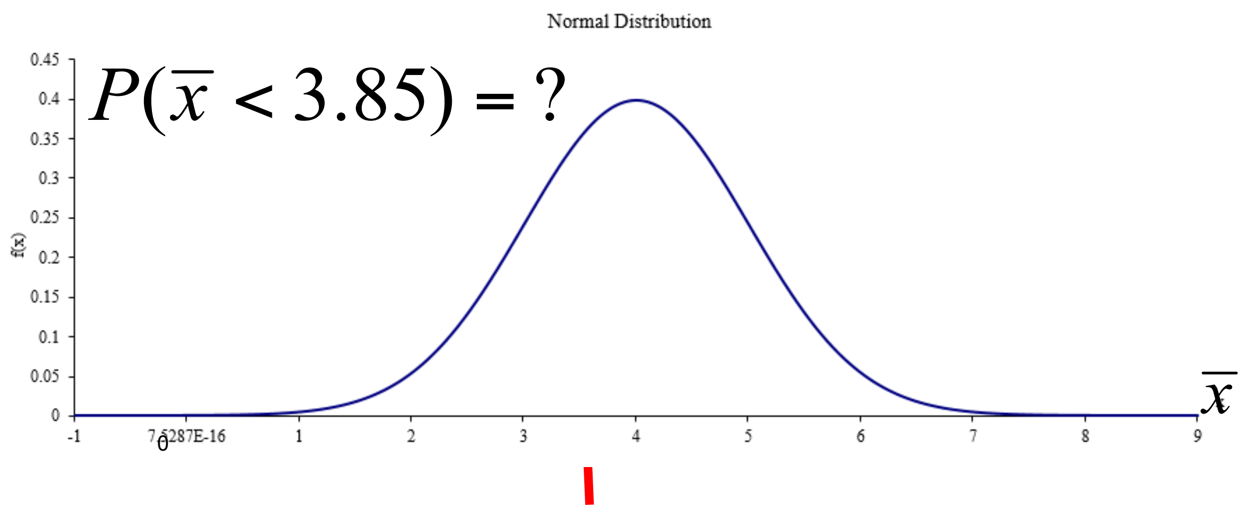
Sampling Distribution – French Fry Example

What is the probability that the mean of a sample of n = 64 fries is less than 3.85 inches

$$n = 100$$

$$\mu = 4.0$$

$$\sigma = 1.0$$

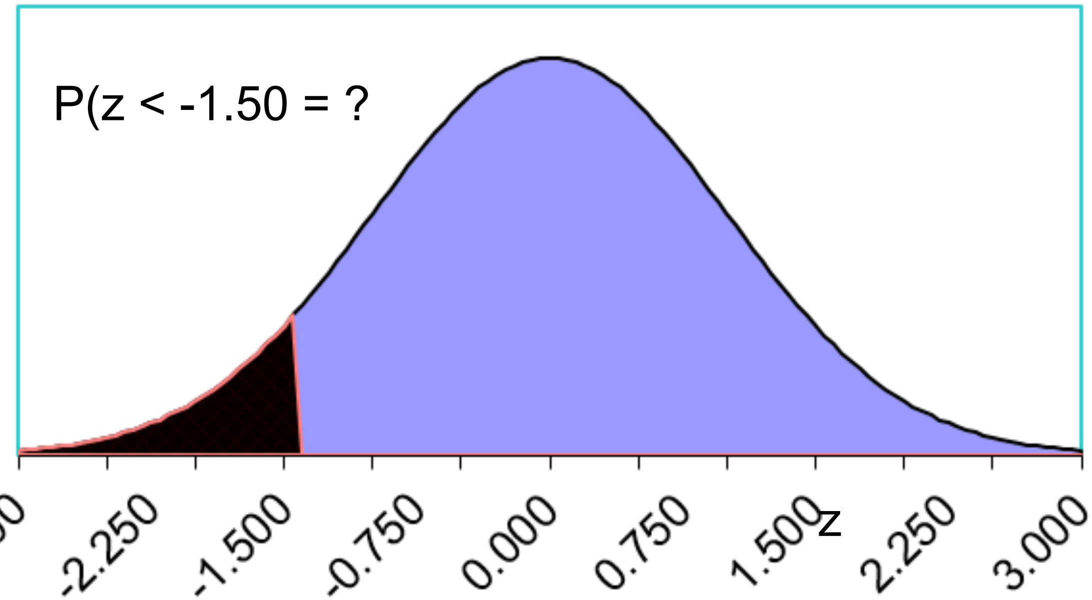


$$\overline{x} = 3.85$$

We need to standardize the sampling distribution.

Sampling Distribution – French Fry Example

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.85 - 4}{\frac{1}{\sqrt{100}}} = 1.50$$



From Standard Normal Table:

$$P(0.00 \geq z \geq -1.50) = 0.4332$$

$$P(z < 1.50) = 0.5000 - 0.4332 = 0.0668$$

7.3 Sampling Distribution of a Proportion

- Population Proportion
  - The fraction of values in a population that have a specific attribute
- Sample Proportion
  - The fraction of items in a sample that have the attribute of interest

p - Population proportion
X - Number of items in the population having the attribute of interest
N - Population size

$\bar{p}$ - Sample prop
x - Number of ite

Sampling Error for a Proportion

Sampling E

p - Populati

- Step 1: Determine the population proportion
- Step 2: Compute the sample proportion
- Step 3: Compute the sampling error

Sampling Distrib

- The best estimate of th
  be $\bar{p}$, the sample propo
- Any inference about ho
  the true population valu
  distribution of this sam

Sampling Distrib

p - Populati

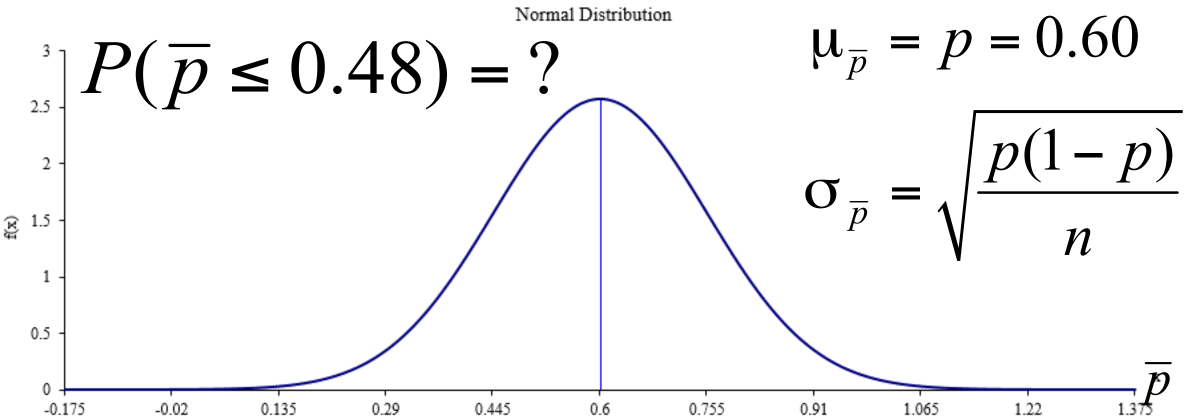
Sampling Distribution for a Proportion – Retail Customers Example

According to past records for a large national retail chain, the proportion of customers who pay for their merchandise with a debit card is 0.60. Suppose a particular store within the chain examines a sample of n = 100 purchases and observes 48 customers used a debit card. What is the probability of a sample proportion of 0.48 or less if the true population proportion is 0.60?

$$P(\overline{p} \leq 0.48) = ?$$

Sampling Distribution for a Proportion – Retail Customers Example

$$P(\bar{p} \le 0.48) = ?$$



$$\bar{p} = 0.48$$

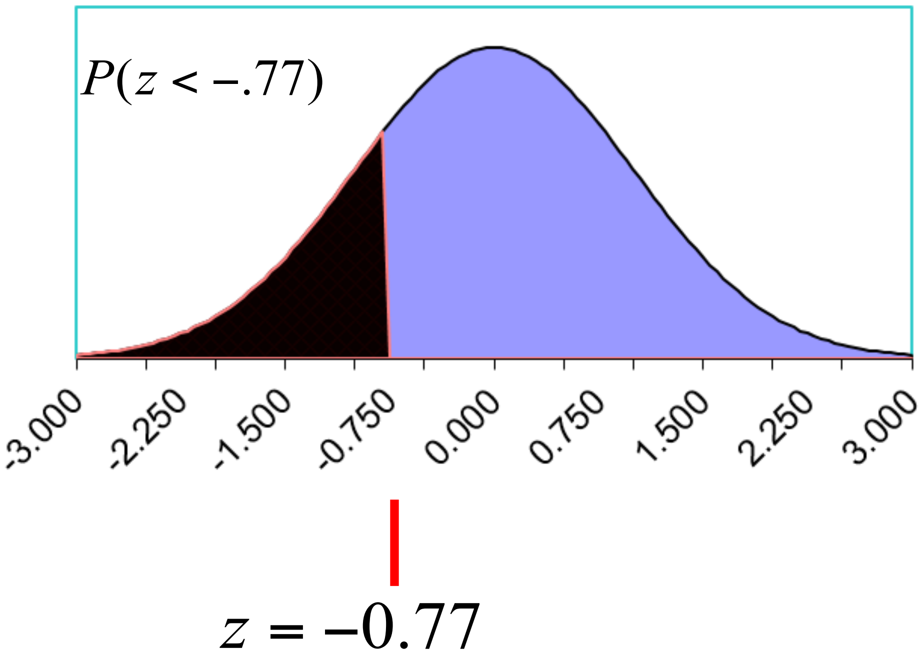
$$\mu_{\bar{p}} = p = 0.60$$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.60)(1-0.60)}{100}} = 0.155$$

Now, standardize the sampling distribution

Sampling Distribution for a Proportion – Retail Customers Example

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.48 - 0.60}{\sqrt{\frac{.60(.40)}{100}}} = -0.77$$



From Standard Normal Table:

$$P(0.00 \geq z \geq -0.77) = 0.2794$$

$$P(z < -0.77) = 0.5000 - 0.2794 = 0.2206$$

Sampling Distribution for a Proportion – Income Tax Example

A big problem for taxpayers has been the recent increase in fraudulent tax returns being filed with the IRS in the name of taxpayers. Suppose the IRS believes that 15 percent of all taxpayers will be affected in 2016. Assuming the true proportion of taxpayers who get "hacked" is 0.15, what is the probability that more than 40 in a random sample of 200 will have had this experience?

$$p = 0.15$$

$$P(\bar{p} > 0.20 = ?$$

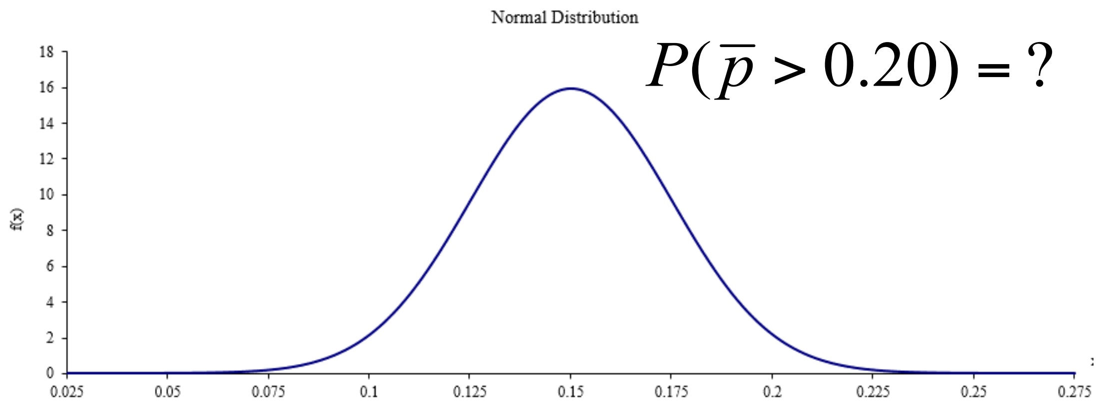
$$\bar{p} = \frac{x}{n} = \frac{40}{200} = 0.20$$

Sampling Distribution for a Proportion – Income Tax Example

$$\mu_{\bar{p}} = p = 0.15$$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} =$$

$$\sigma_{\bar{p}} = \sqrt{\frac{(0.15)(1-0.85)}{200}} = 0.025$$



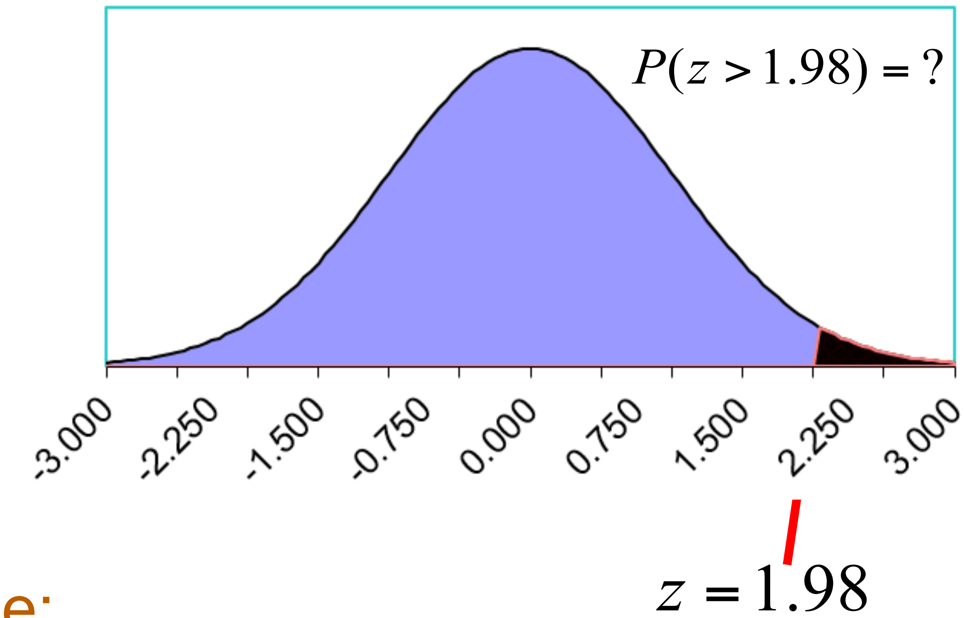
$$P(\bar{p} > 0.20) = ?$$

$$\bar{p} = 0.20$$

Now, standardize the sampling distribution

Sampling Distribution for a Proportion – Income Tax Example

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.20 - 0.15}{\sqrt{\frac{.15(.85)}{200}}} = 1.98$$



From Standard Normal Table:

$$P(0.00 \le z \le 1.98) = 0.4761$$

$$P(z > 1.98) = 0.5000 - 0.4761 = 0.0239$$