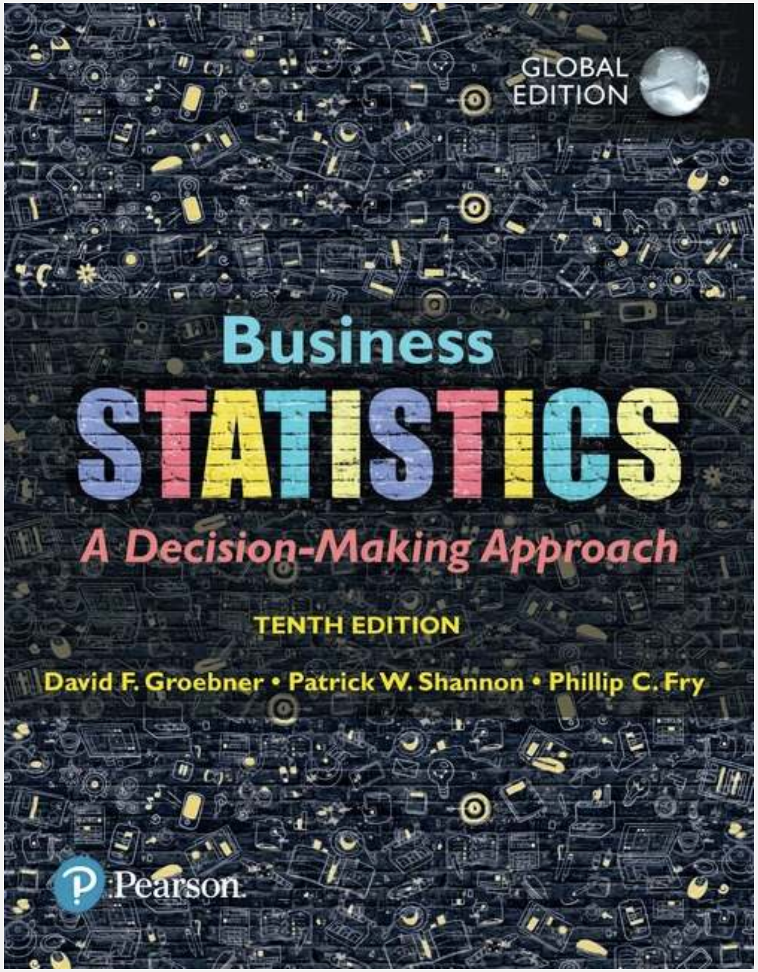
Chapter 6

Introduction to Continuous Probability Distributions



Learning Outcomes

Outcome 1. Convert a normal distribution to a standard normal distribution.

Outcome 2. Determine probabilities using the standard normal distribution.

Outcome 3. Calculate values of the random variable associated with specified probabilities from a normal distribution.

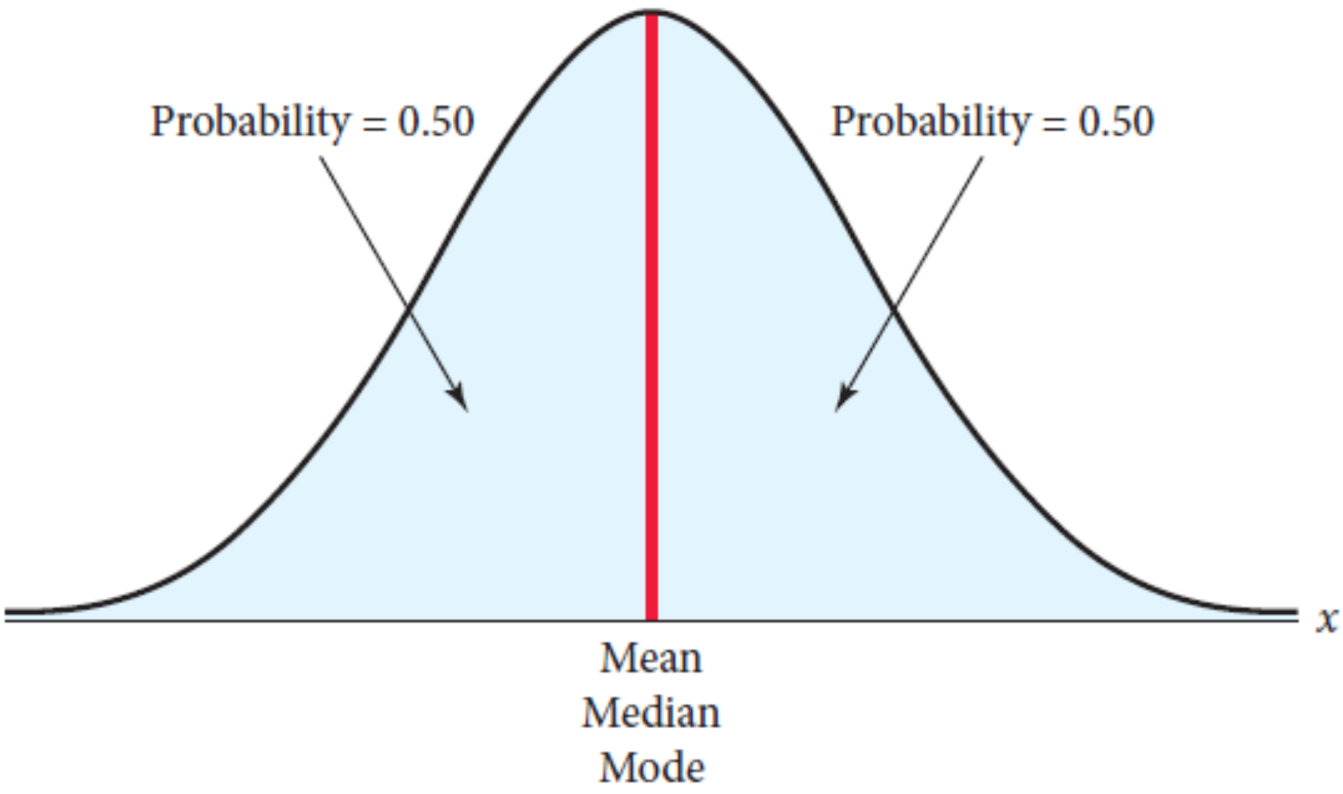
Outcome 4. Calculate probabilities associated with a uniformly distributed random variable.

Outcome 5. Determine probabilities using an exponential probability distribution.

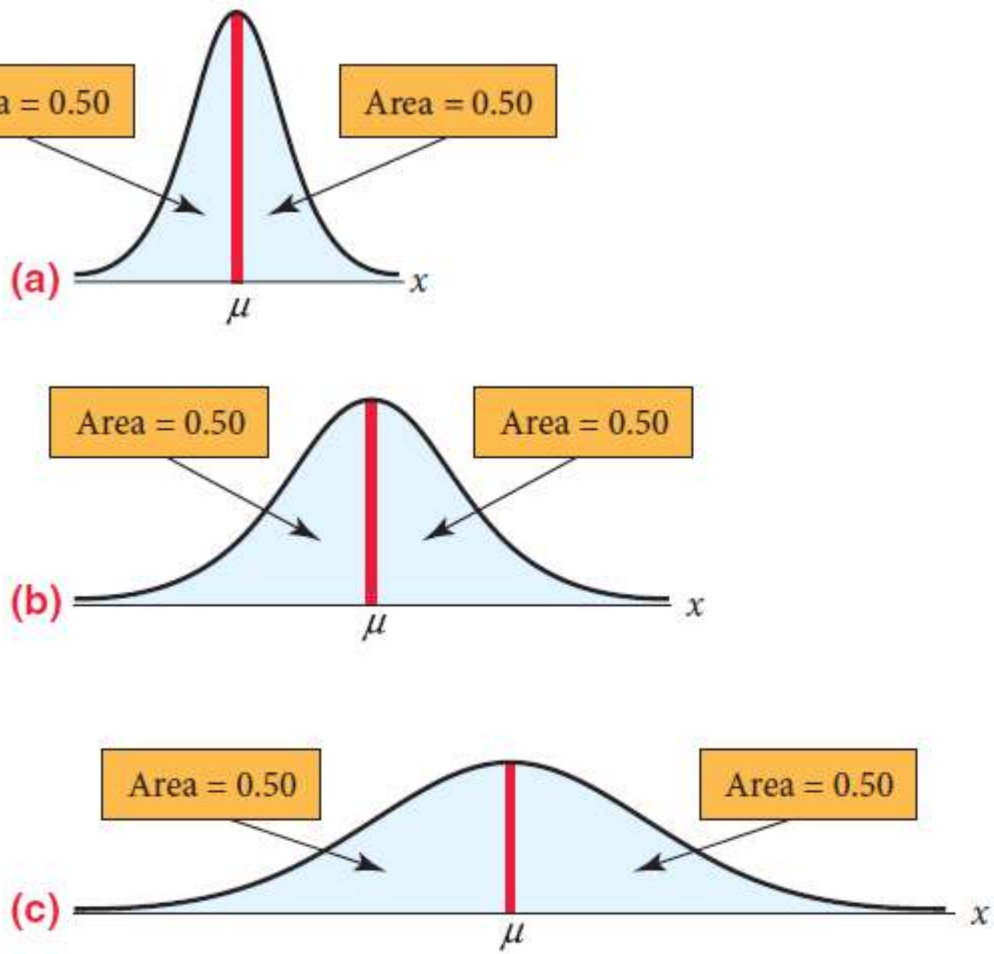
6.1 The Normal Probability Distribution

- Is a bell-shaped distribution with the following properties:
  1. It is *unimodal*; that is, the normal distribution peaks at a single value.
  2. It is *symmetrical*; this means that the two areas under the curve between the mean and any two points equidistant on either side of the mean are identical.
  3. The mean, median, and mode are equal.

The Normal Distribution Shape



Normal Distribution Variations



Different normal distributions can be obtained changing μ and σ.

Changing μ shifts the distribution left or right.

Changing σ increases
or decreases the spread.

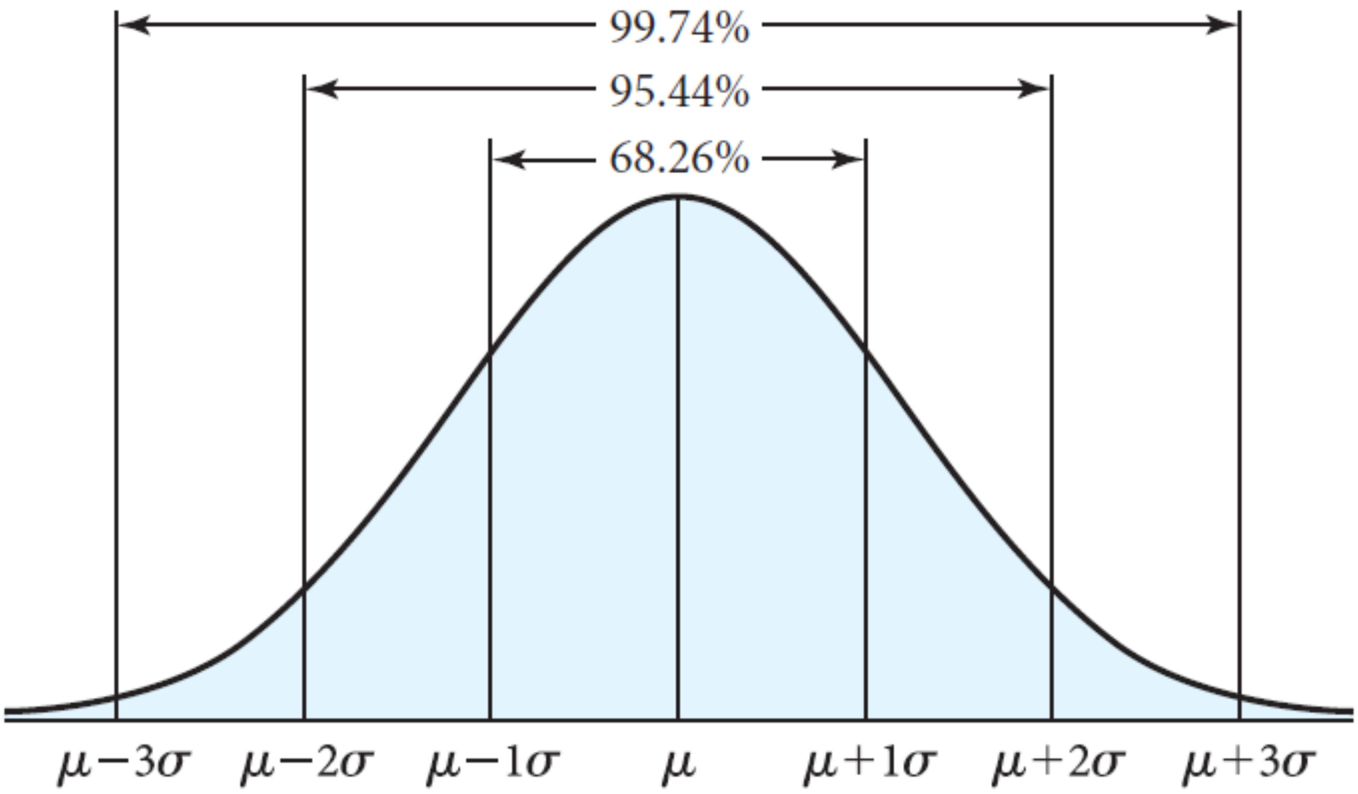
Normal Probability Density Function

$$f(x) = \quad 1$$

x - Any value of the co

σ - Population standar

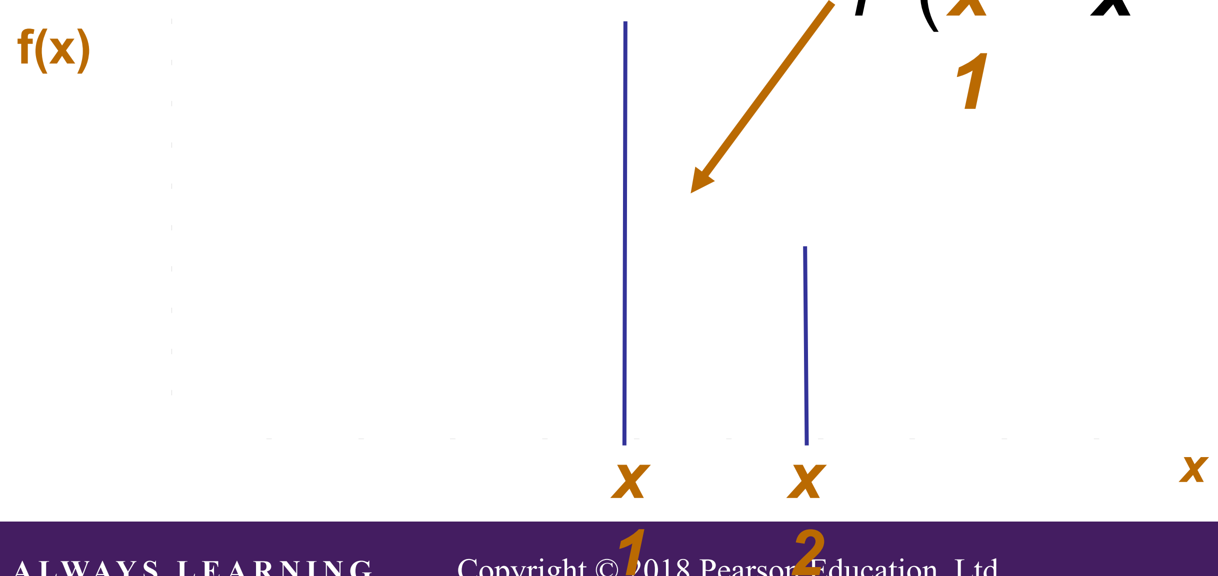
Approximate Areas under the Normal Distribution



Finding Normal Probabilities

- Because *x* is a continuous random variable, the probability, $P(x) \approx 0$ for any particular *x*
- The probability for a range of values between *x*1 and *x*2 is defined by the area under the curve between these two values.

$$P(x_1 \quad x \quad x_2)$$



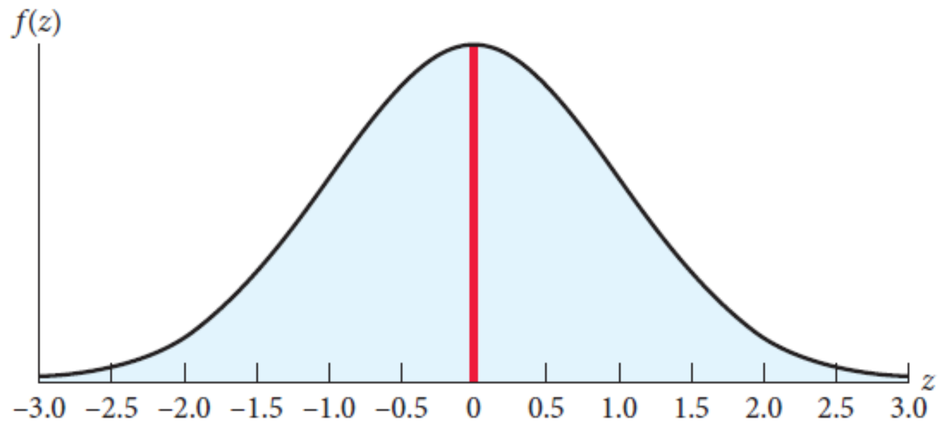
The Standard Normal Distribution

- A normal distribution that has a mean = 0.0 and a standard deviation = 1.0
- The horizontal axis is scaled in *z*-values that measure the number of standard deviations a point is from the mean
- Values above the mean have positive *z*-values
- Values below the mean have negative *z*-values

The Standard Normal Distribution

- Standardized Normal z-Value

z - Scaled valu
of standarc
point x is fr
x - Any point c



The Standard Normal Distribution

- Any normal distribution (with any mean and standard deviation combination) can be scaled into the standard normal distribution (z)
- Any specified value, x, from the population distribution can be converted into a corresponding z-value

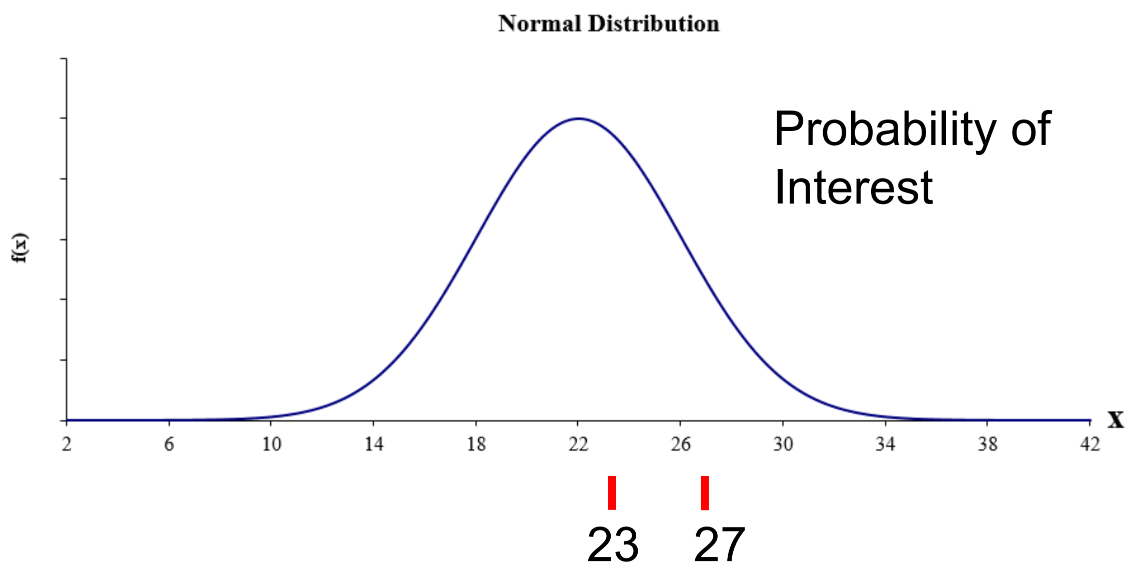
Standard Normal Distribution - Example

The gas mileage for a particular new SUV automobile is normally distributed with μ = 22 mpg and σ = 4. Find the probability of one SUV getting between 23 and 27 mpg?

$$\mu = 22$$

$$\sigma = 4$$

$$P(23 \le x \le 27) = ?$$



Need to convert to the standard normal distribution

Standard Normal Distribution – Gas Mileage Example

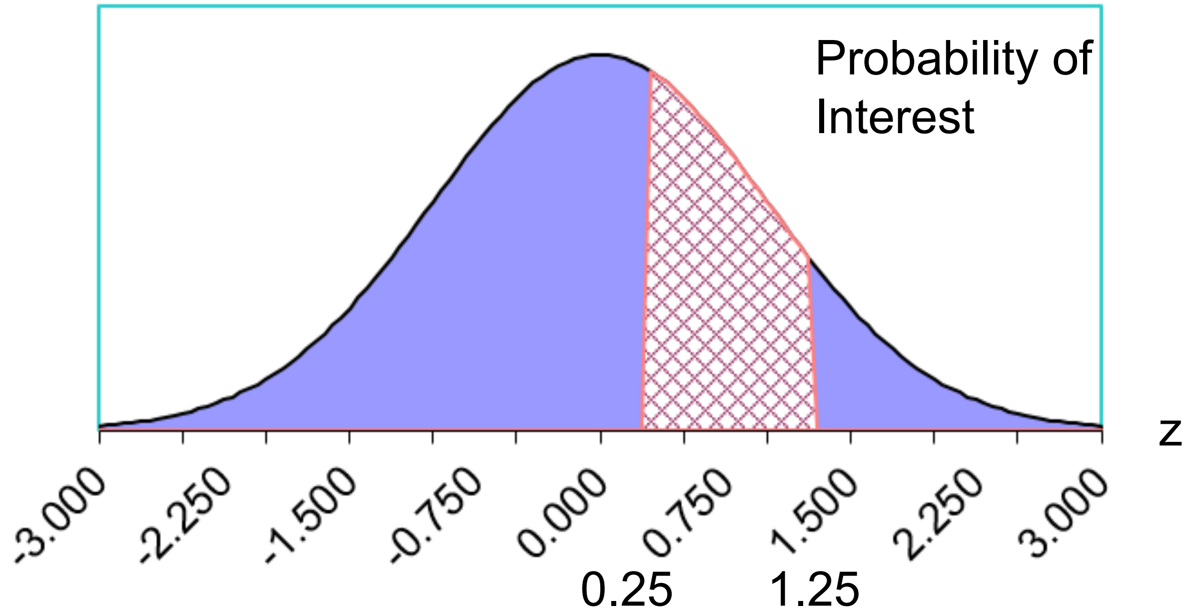
$\mu = 22$

$\sigma = 4$

$P(23 \le x \le 27) = ?$

$$z = \frac{x-\mu}{\sigma} = \frac{23-22}{4} = 0.25$$

$$z = \frac{x-\mu}{\sigma} = \frac{27-22}{4} = 1.25$$



Using The Standard Normal Distribution Table

The **column** gives the value of z to the second decimal point

z	0.00	0.01	0.02 …
0.1			
0.2			
2.0	.4772		

The **row** shows the value of z to the first decimal point

The value in the intersection gives the probability from z = 0 up to the desired z value

$P(0 < z < 2.00) = \mathbf{0.4772}$

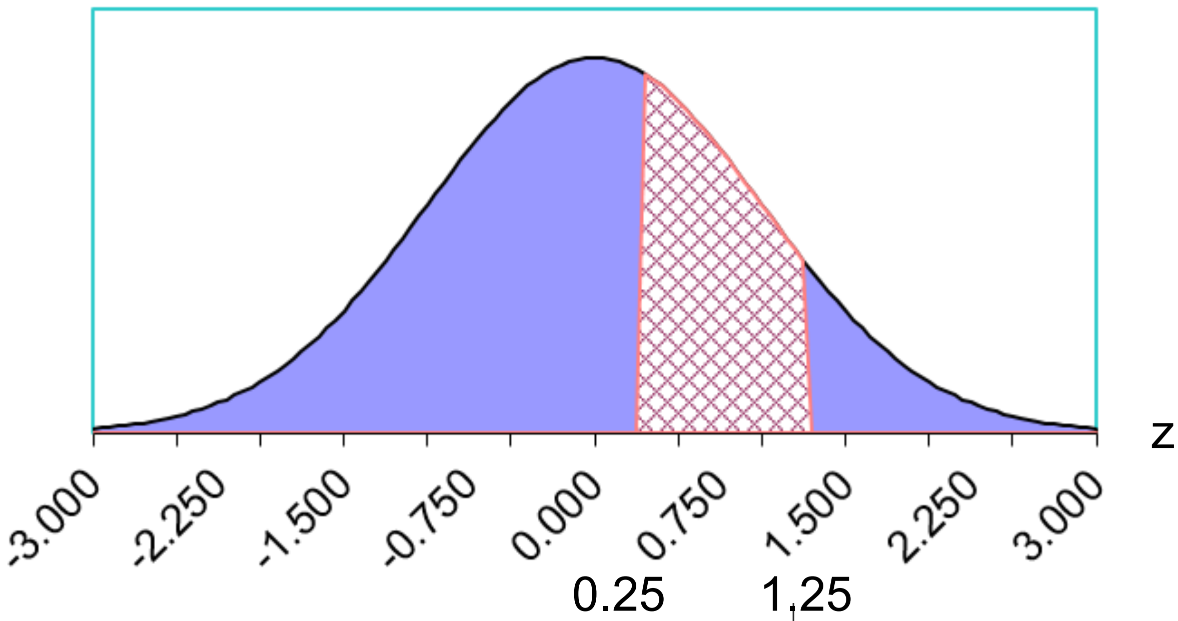
Standard Normal Distribution – Gas Mileage Example

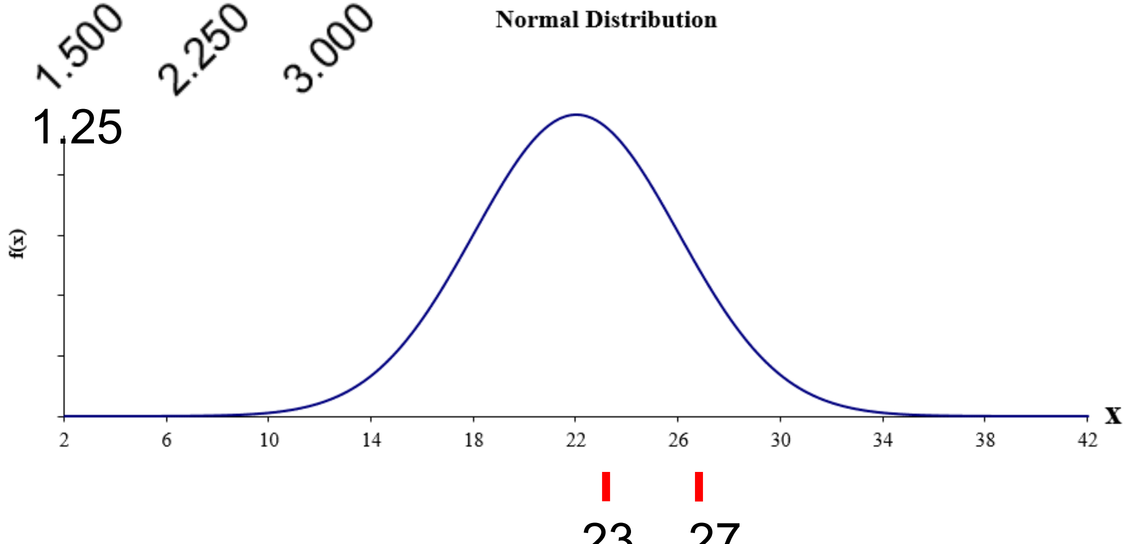
$P(0 \leq z \leq 0.25) = 0.0987$
$P(0 \leq z \leq 1.25) = 0.3944$

$P(0.25 \leq z \leq 1.25) =$
$0.3944 - 0.0987 = 0.2957$

				Standard Normal Distribution						
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Standard Normal Distribution – Gas Mileage Example





$\mu = 22$
$\sigma = 4$
$P(23 \leq x \leq 27) = 0.2957$

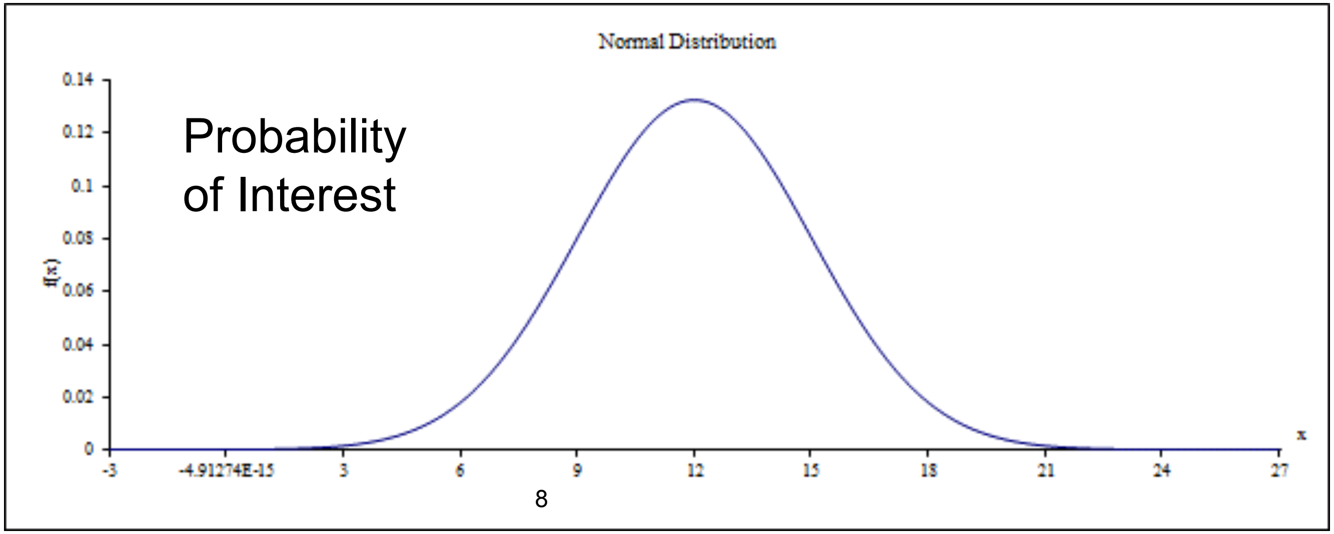
Standard Normal Distribution - Restaurant Tip Example

The tips left by customers at a local restaurant are normally distributed with a mean equal to $12.00 and a standard deviation equal to $3.00. What is the probability that a customer will leave a tip of less than $8.00?

$$\mu = 12$$

$$\sigma = 3$$

$$P(x \le 8) = ?$$



Need to convert to the standard normal distribution

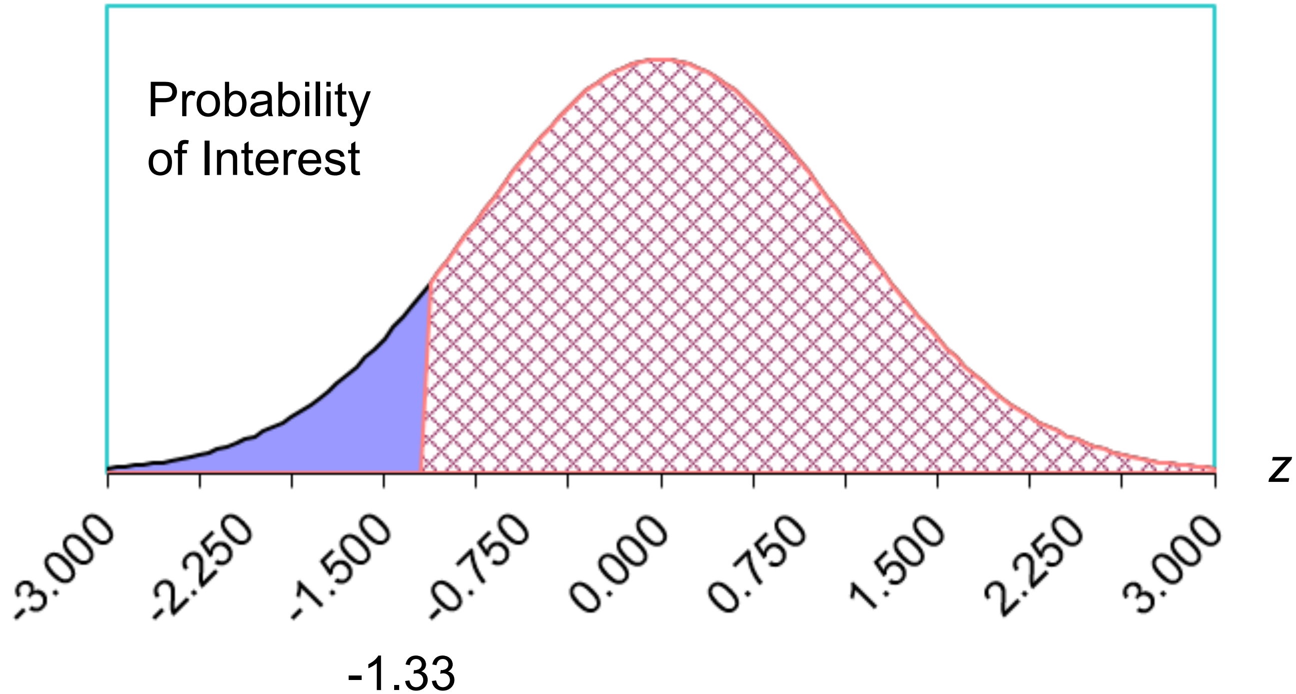
Standard Normal Distribution - Restaurant Tip Example

$$\mu = 12$$

$$\sigma = 3$$

$$P(x \le 8) = ?$$

$$z = \frac{x - \mu}{\sigma} = \frac{8 - 12}{3} = -1.33$$



Probabilities associated with negative z values equal the probabilities for the corresponding positive z values.

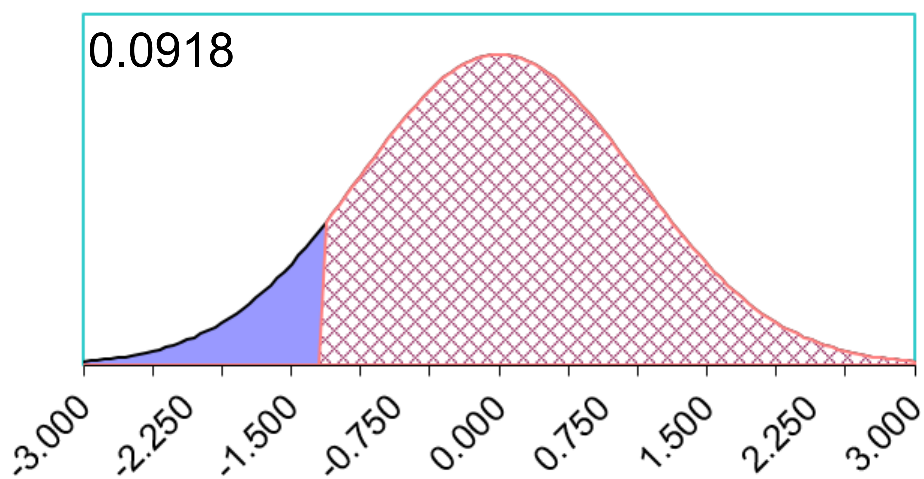
$$P(z \leq -1.33) = 0.5000 - 0.4082$$

$$P(z \leq -1.33) = 0.0918$$

$$P(x \leq \$8) = 0.0918$$

				Standard Normal Distribution						
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
		0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
		0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
		0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
		0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
		0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
		0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
		0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
		0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
		0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
		0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
		0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
		0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
		0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
		0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
		0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
		0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
		0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

0.4082



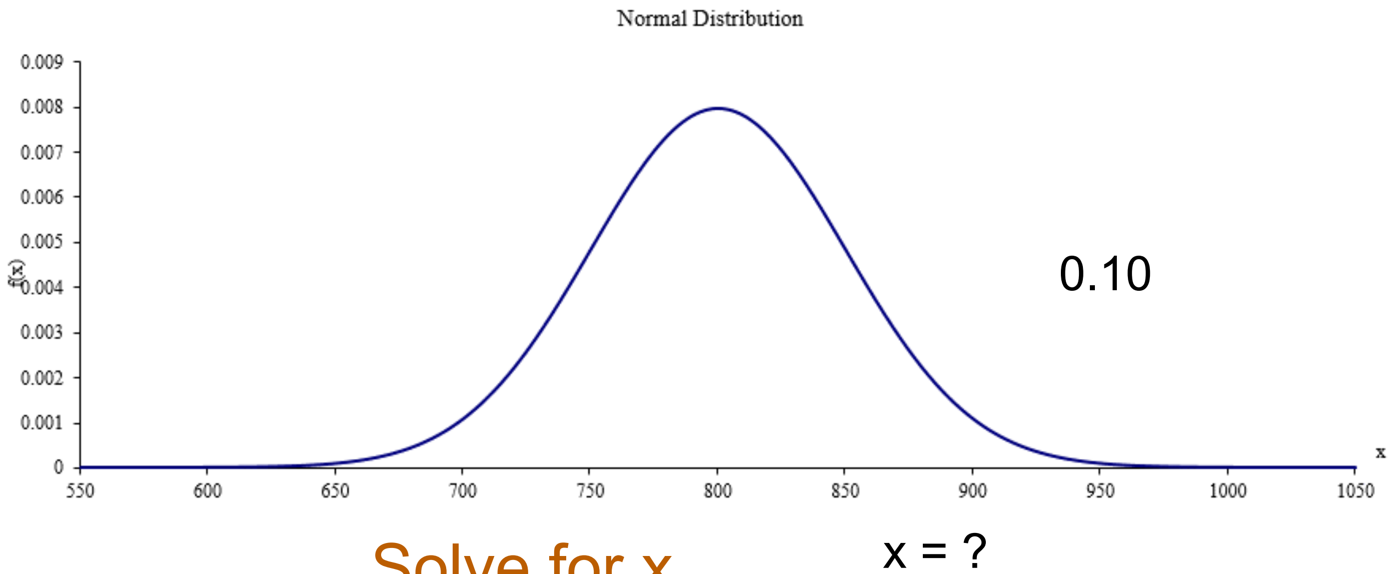
-1.33

Standard Normal Distribution – Entrance Score Example

An entrance exam for a graduate program at a major university produces scores that are normally distributed with a mean of 800 with a standard deviation equal to 50. If the school only wants to admit the top 10 percent of students, what should the cut-off score for admittance be?

$$\mu = 800$$

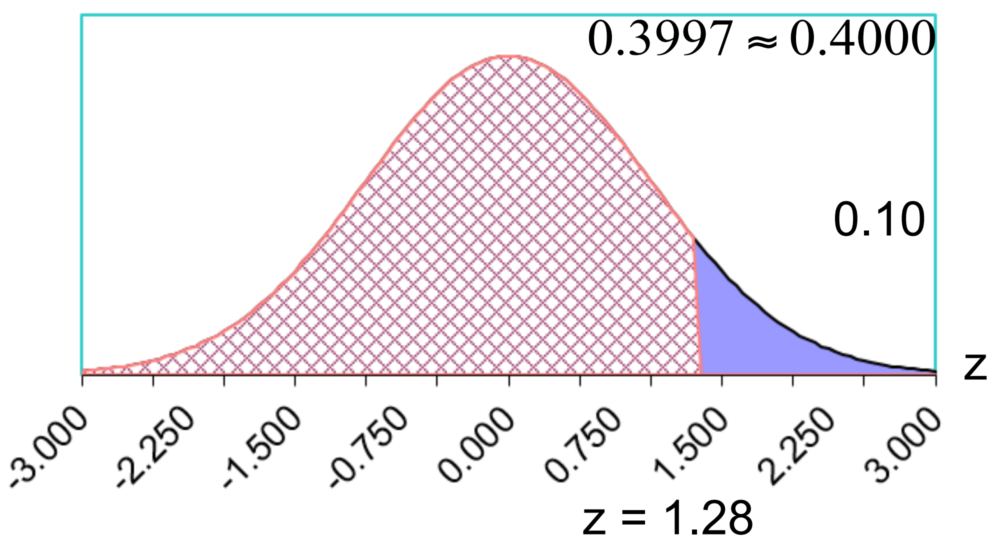
$$\sigma = 50$$



Solve for x

Standard Normal Distribution – Entrance Score Example

				Standard Normal Distribution						
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177



z corresponding to 0.3997 is 1.28

Standard Normal Distribution – Entrance Score Example

$$\mu = 800$$
$$\sigma = 50$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 800}{50}$$

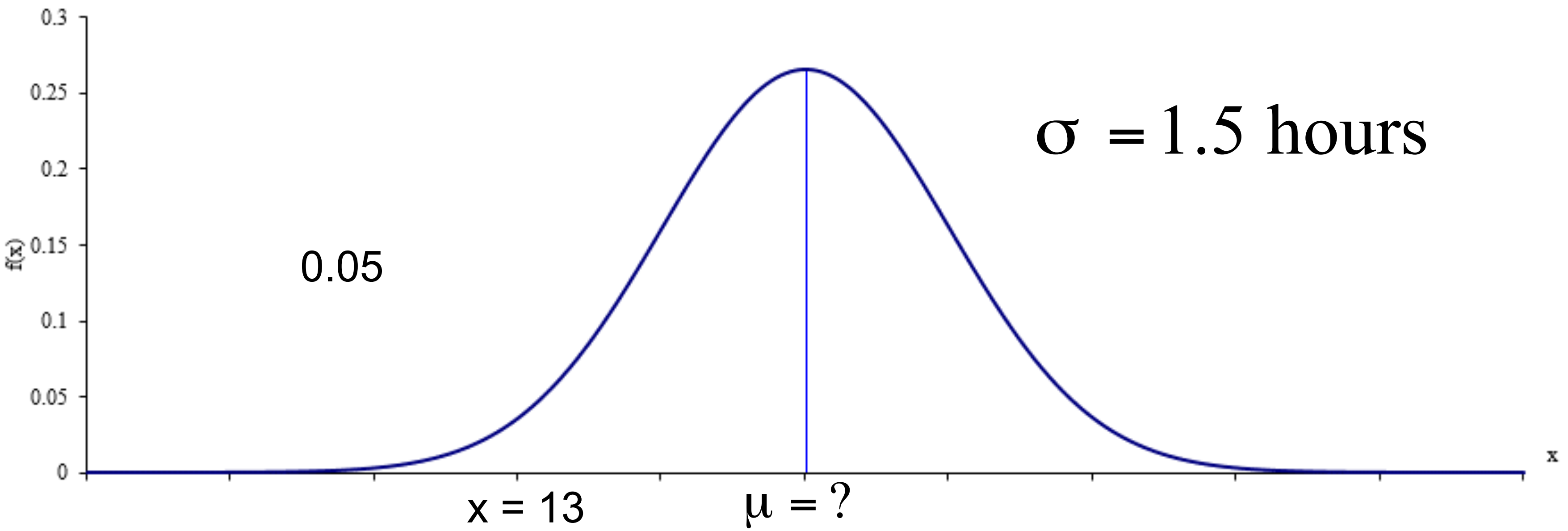
$$1.28 = \frac{x - 800}{50}$$

$$x = (1.28)(50) + 800 = 864$$

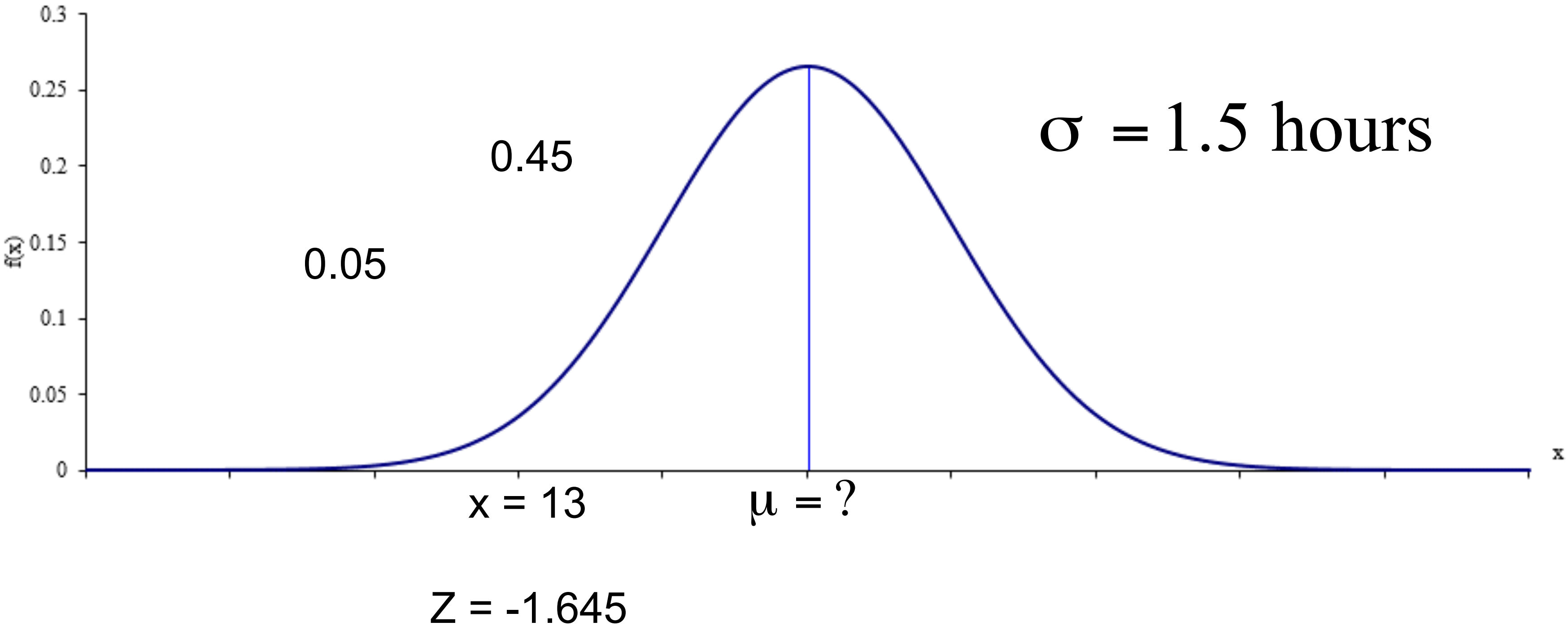
Applicants with scores of 864 or higher will be admitted

Standard Normal Distribution –Cell Phone Battery Example

A maker of cell phone batteries has determined that the time between charges is normally distributed with a standard deviation equal to 1.5 hours. If a customer wants no more than 5 percent of the batteries to require charging in 13 hours or less, what must the mean time between charges be?



Standard Normal Distribution –Cell Phone Battery Example



Z-value from the standard normal table for an area of 0.45 in the lower end of the distribution is -1.645

Standard Normal Distribution –Cell Phone Battery Example

Solving for the required population mean:

$$z = \frac{x - \mu}{\sigma}$$

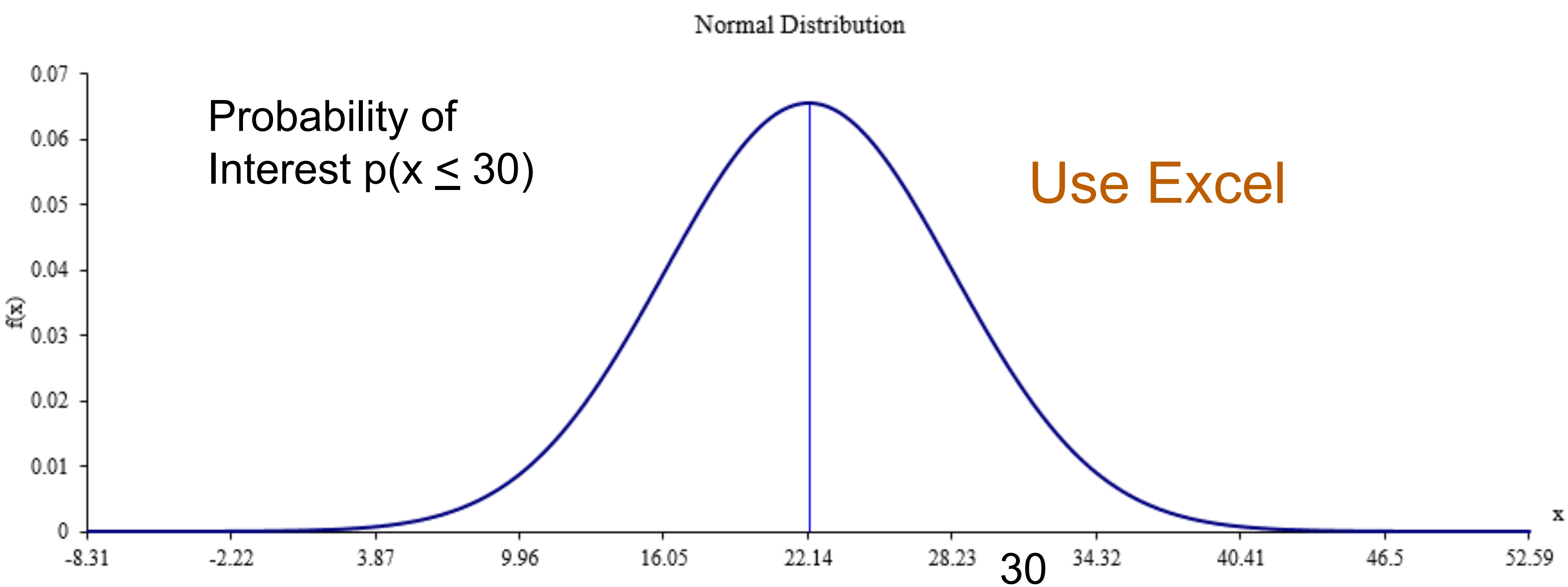
$$-1.645 = \frac{13 - \mu}{1.5}$$

$$\mu = (1.645)(1.5) + 13 = 15.47$$

In order to meet the customer requirements, the mean time between cell phone battery charges must be at least 15.47 hours.

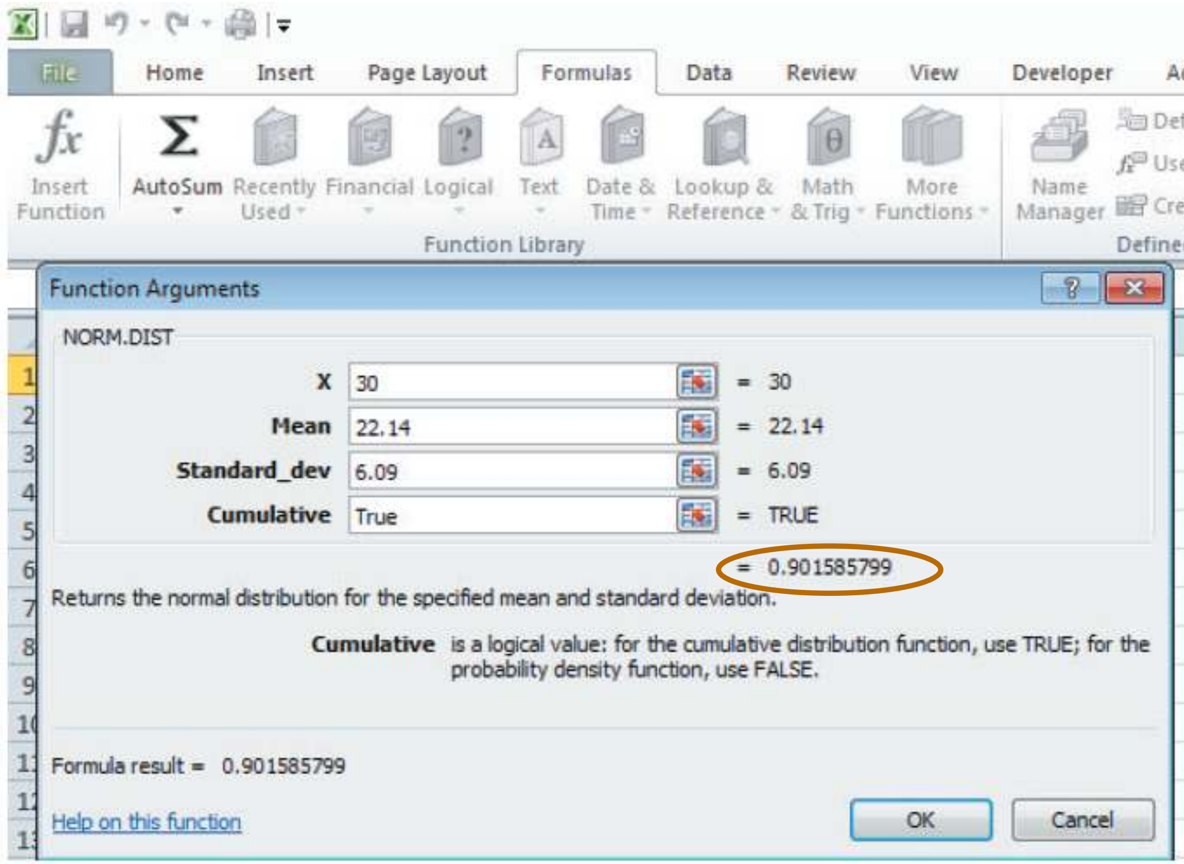
Standard Normal Distribution – Job Completion Times Example

A company that makes patio tables and chairs has determined that the time it takes to make a complete set on their automated production line is normally distributed with a mean of 22.14 minutes and standard deviation of 6.09 minutes. Given this information, what proportion of patio sets will be completed in 30 minutes or less?



How to Do It in Excel?

1. Open blank worksheet.
2. Select **Formulas**
3. Click on *fx* (*function wizard*).
4. Select **Statistical** category.
5. Select the **NORM.DIST** function.
6. Fill in the requested information in the template.
7. **True** indicates cumulative probabilities.
8. Click **OK**.



$P(x \leq 30) = .902$ Thus, slightly over 90 percent of the patio units will be completed in 30 minutes or less.