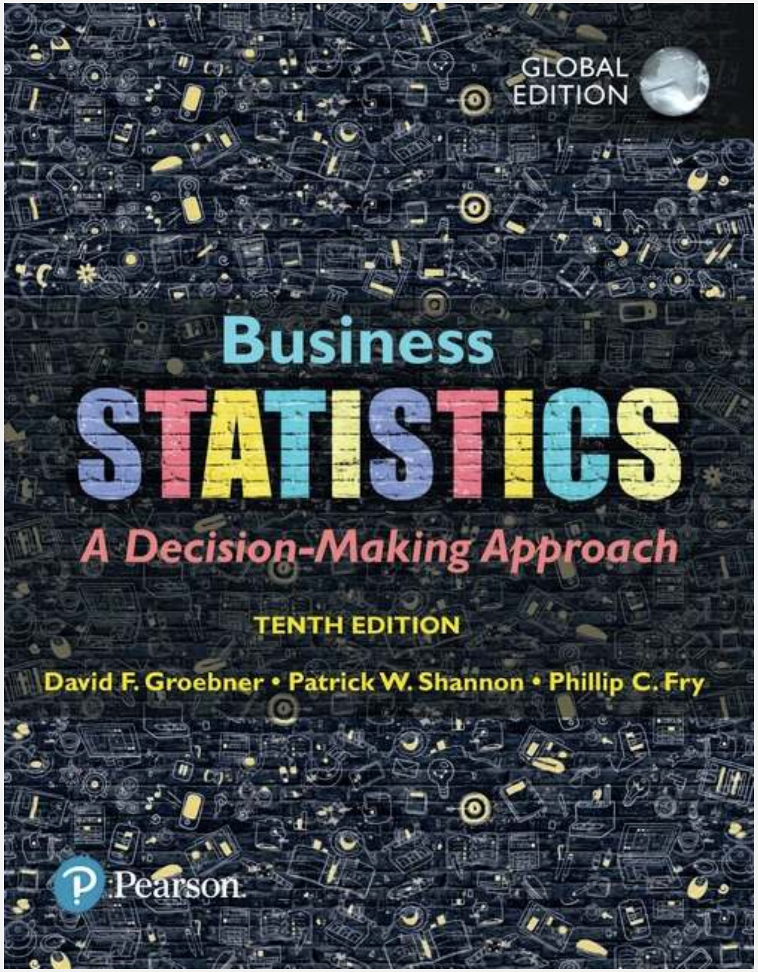
Chapter 4

Introduction To Probability



Learning Outcomes

Outcome 1. Identify situations for which each of the three approaches to assessing probabilities applies.

Outcome 2. Be able to apply the Addition Rule.

Outcome 3. Know how to use the Multiplication Rule.

Outcome 4. Know how to use Bayes' Theorem for applications involving conditional probabilities.

4.1 The Basics of Probability

- Probability
  - The chance that a particular event will occur
  - The probability value will be in the range 0 to 1.
- Experiment
  - A process that produces a single outcome whose result cannot be predicted with certainty
- Sample Space
  - The collection of all outcomes that can result from a selection, decision, or experiment

The Basics of Probability – Examples

- Probability

  Chance of a new business succeeding

  P(Succeed) = 0.25

- Experiment

  Activities associated with starting a new business

- Sample Space

  (Business Succeeds, Business Fails)

The Basics of Probability – Examples

- Probability

  Chance of a new hire quitting within 1st month on the job

  $P(\text{Quit}) = 0.10$

- Experiment

  Employee is hired and begins work

- Sample Space

  (Employee quits, Employ does not quit)

The Basics of Probability – Examples

- Probability

  Likelihood of an oil well containing oil

  P(Oil) = 0.60

- Experiment

  Drill an oil well

- Sample Space

  (Strike Oil, Do Not Strike Oil)

The Basics of Probability – Examples

- Probability

  Chance of producing 3 or more defects in one hour

  $P(x \geq 3) = 0.20$

- Experiment

  Produce products for one hour and inspect for defects

- Sample Space

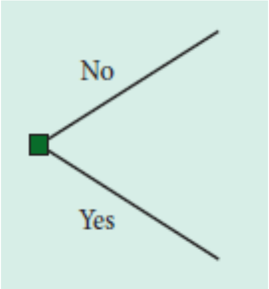
  (x = 0, 1, 2, 3, or more)

Defining the Sample Space

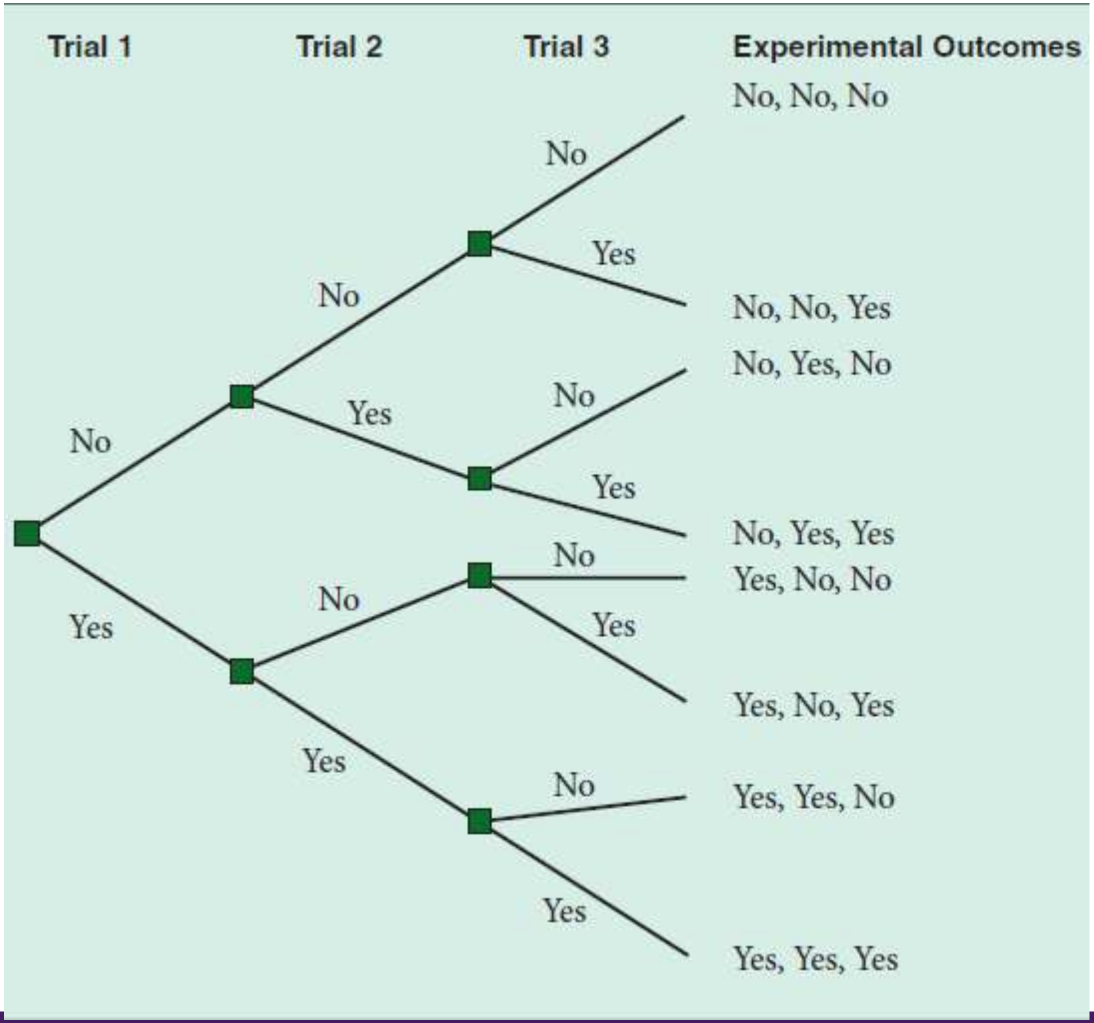
- Step 1: Define the experiment.
  - For example, the sale. The item of interest is the product sold.
- Step 2: Define the outcomes for one trial of the experiment.
  - Outcomes: $e1$ - Hamburger, $e2$ – Cheeseburger, $e3$ – Bacon Burger
- Step 3: Define the sample space (*SS*).
  - For single sale: $SS = \{e1, e2, e3\}$
  - If experiment includes two sales, there will be nine outcomes: $SS = \{o1, o2, o3, o4, o5, o6, o7, o8, o9\}$ $o1=\{e1, e1\}$, $o2=\{e1, e2\}$, $o3=\{e1, e3\}$, $o4=\{e2, e1\}$, . . . , $o9=\{e3, e3\}$

Sample Space - Tree Diagrams

- A useful way to define the sample space
- Step 1: Define the experiment.
  - Three students were asked if they like statistics
- Step 2: Define the outcomes.
  - Possible outcomes are: ‘No’ and ‘Yes.’
- Step 3: Define the sample space for three trials using a tree diagram.
  - For a single trial:



Sample Space - Tree Diagram



Types of Events

- Mutually Exclusive Events
  - Two events are mutually exclusive if the occurrence of one event precludes the occurrence of the other event.
- Independent Events
  - Two events are independent if the occurrence of one event in no way influences the probability of the occurrence of the other event.
- Dependent Events
  - Two events are dependent if the occurrence of one event impacts the probability of the other event occurring.

Types of Events - Example

- Mutually Exclusive Events

  Two events are mutually exclusive if the occurrence of one event precludes the occurrence of the other event

Event A – Oil well produces oil

Event B – Oil well does not produce oil

If Event A occurs, then Event B <u>can not also occur</u> – mutually exclusive events

Types of Events - Example

- Independent Events
  - Two events are independent if the occurrence of one event in no way influences the probability of the occurrence of the other event.

Event A – Business is started by a person in North Carolina

Event B – Business is started by a person in Oregon

Whether or not the business in Oregon (Event B) is successful is in no way influenced by whether the North Carolina business (Event A) is successful. – Events are Independent

Types of Events - Example

- Dependent Events
  - Two events are dependent if the occurrence of one event impacts the probability of the other event occurring.

A Company has 10 employees – 3 male and 7 female. Two employees are selected at random to attend a conference.

Event A – First person selected is female.
Event B – Second person selected is male.

Events A and B are dependent because the chance of a male selected second depends on whether a male or female is selected first.

Assigning Probability

Methods of Assigning Probability

- **Classical Probability Assessment**
- **Relative Frequency Assessment**
- **Subjective Probability Assessment**

P(*Ei*) = Probability of event *Ei* occurring

Classical Probability Assessment

- The method is based on the ratio of the number of ways an outcome or event of interest can occur to the number of ways *any* outcome or event can occur when the individual outcomes are equally likely.

$$P(E_i) = \frac{\text{Number of ways } E_i \text{ can occur}}{\text{Total number of possible outcomes}}$$

Classical Probability Assessment - Example

- Company with 10 employees – 7 female and 3 male. Assess the probability of a female employee being selected at random to travel to a convention.

$$P(E_i) = \frac{\text{Number of ways } E_i \text{ can occur}}{\text{Total number of possible outcomes}}$$

Event

Classical Probability Assessment - Example

- New car dealer has 5 GM, 6 Ford, 3 Toyota, 8 Nissan, and 2 BMW cars. If one car is selected to be placed on sale, what is the probability that it is a Nissan?

$$P(E_i) = \frac{\text{Number of ways } E_i \text{ can occur}}{\text{Total number of possible outcomes}}$$

Event

Relative Frequency Probability Assessment

- The method defines probability as the number of times an event occurs divided by the total number of times an experiment is performed in a large number of trials.

$$P(E_i) = \frac{\text{Number of times } E_i \text{ occurs}}{N}$$

Ei - The event of interest
N - Number of trials

Relative Frequency Probability Assessment - Example

- A local pub has tracked beer purchases. Of the past 500 purchases, 120 have been for “light” beers. What is the probability that the next beer ordered will be “light”?

$$P(E_i) = \frac{\text{Number of times } E_i \text{ occurs}}{N}$$

Relative Frequency Probability Assessment - Example

- A basketball player has taken 90 free throws during games and made 68. What is the probability that the next free throw will be made?

$$P(E_i) = \frac{\text{Number of times } E_i \text{ occurs}}{N}$$

Subjective Probability Assessment

- The method defines probability of an event as reflecting a decision maker’s state of mind regarding the chances that the particular event will occur.
- Represents a person’s belief that an event will occur.

Subjective Probability Assessment

- A manager is asked to assess the chances that a shipment from a new supplier will arrive on time.

Taking into account such factors as:

- Supplier's reputation
- Weather conditions
- Size of the order
- Etc.

She might assess the probability as:

Subjective Probability Assessment

- A jury member in a civil case must assess the probability that the defendant is at fault in the case.

Taking into account such factors as:

- Defendant’s testimony
- Plaintiff’s testimony
- Physical evidence
- Etc.

The juror might assess the probability as:

4.2 The Rules of Probability

Probability Rules

- Possible Values
- Summation of Possible Values
- Addition Rule for Individual Outcomes
- Complement Rule
- Addition Rule for Any Two Events
- Addition Rule for Mutually Exclusive Events
- Conditional Probability for Any Two Events
- Conditional Probability for Independent Events
- Multiplication Rule for Any Two Events
- Multiplication Rule for Independent Events
- Bayes' Theorem

Possible Values and Summation Rules

Probability Rule 1

Probability of event occurring is always between 0 and 1.

For any event E_i
$0 \le P(E_i) \le 1$ for all i

0 = no chance of occurring
1 = 100% chance of occurring

Probability Rule 2

Sum of the probabilities of all possible outcomes is 1.

$$\sum^{k}$$

k - Number of outcomes in the sample
e_i - ith outcome

Probability Rules 1 and 2 - Example

A sales manager has placed three products on display in a store. The following shows the probabilities associated with the number of these products that sell in the first 30 minutes on display.

x = number sold	P(x)
0	0.2
1	0.3
2	0.4
3	0.1

Rule 1: For any event E_i
$0 \le P(E_i) \le 1$ for all i

Rule 2: Sum of probabilities is 1.00.

Rule 3: Addition Rule for Individual Outcomes

The probability of an event Ei is equal to the **sum** of the probabilities of the individual outcomes forming Ei .

For example, if $Ei = \{e1, e2, e3\}$ then

$$P(Ei) = P(e1) + P(e2) + P(e3).$$

Addition Rule for Individual Outcomes - Examples

Roll a 6 sided die. What is the probability that the value will be an <u>odd number</u>? E = (1,3,5)

P(E) = 1/6 + 1/6 + 1/6 = 3/6=1/2

A sales manager has placed three products on display in a store. The following shows the probabilities associated with the number of these products that sell in the first 30 minutes on display. What is the probability that <u>2 or more</u> products will sell?

x = number sold	P(x)
0	0.2
1	0.3
2	0.4
3	0.1

E = (2,3)

P(E) = 0.4 + 0.1 = 0.5

Addition Rule for Individual Outcomes - Example

Real Estate Investment

x = Return	P(x)
-$100,000	0.10
$0	0.20
+$20,000	0.50
+$500,000	0.20

What is the probability of making at least $20,000?

E = [$20,000, $500,000]

P(E) =0.50 + 0.20

P(E) = 0.70

Complement Rule

- The complement of an event *E* is the collection of all possible outcomes not contained in event *E.*

> The probability of the complement of event *E* is 1 minus the probability of event *E.*
>
> $$P(\bar{E}) = 1 - P(E)$$

- This rule is corollary to Probability Rules 1 and 2.

Complement Rule - Example

An inspector checks 7 products to see if they are defective. The possible number of defects and their probabilities are given. What is the probability of 1 or more defects?

x =defects	P(x)
0	0.03
1	0.12
2	0.23
3	0.27
4	0.20
5	0.10
6	0.04
7	0.01

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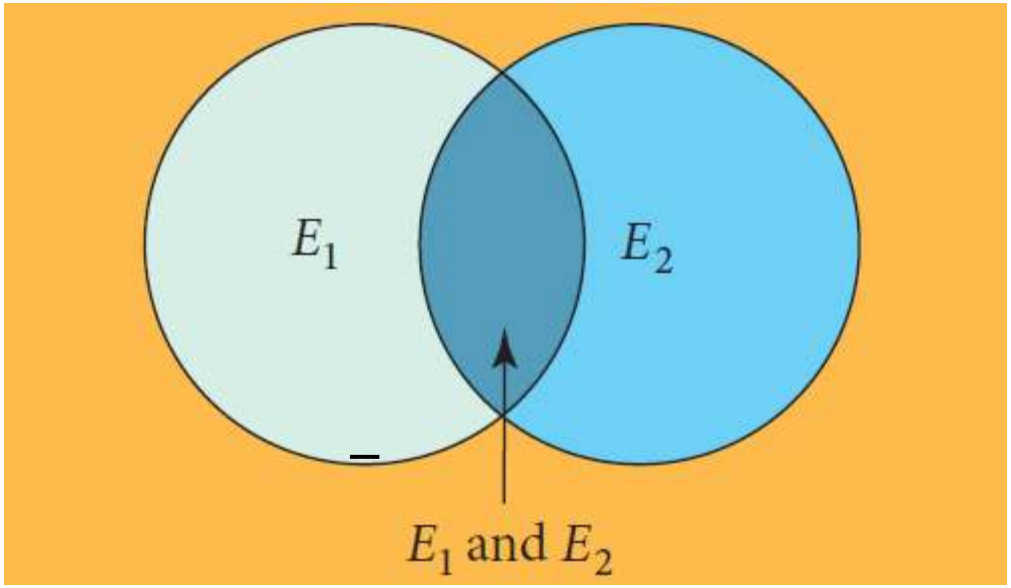
Using the Complement Rule

Rule 4: Addition Rule for <u>Any</u> Two Events – Keyword "or" Means Addition



$$P(E1 \text{ or } E2) = P(E1) + P(E2) - P(E1 \text{ and } E2)$$

Addition Rule for <u>Any</u> Two Events - Example

Drawing Cards from a Deck of 52 Cards

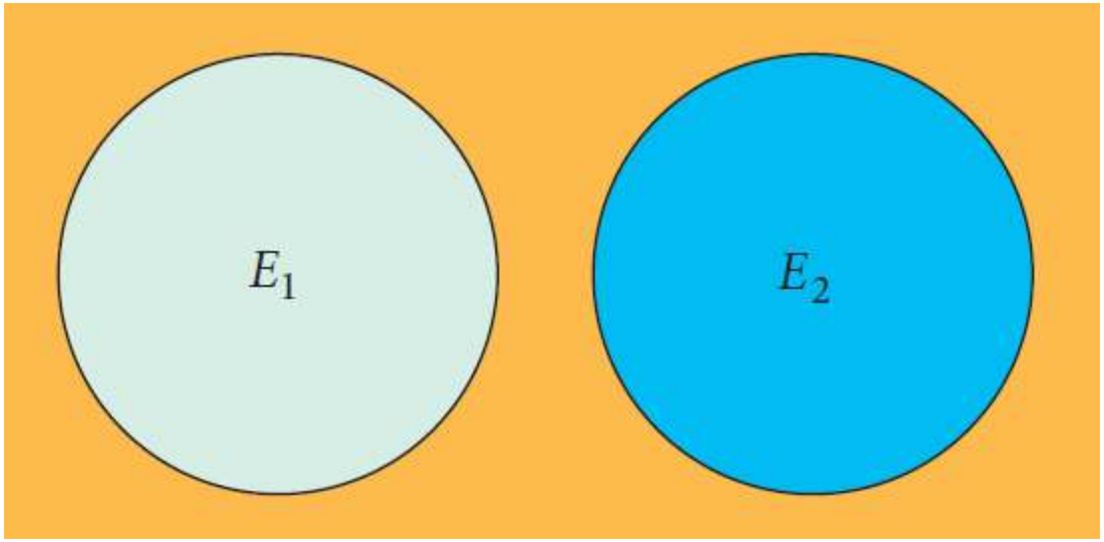
***P*(Red or Ace) = *P*(Red) + *P*(Ace) - *P*(Red and Ace)**

***P*(Red or Ace) = 26/52 + 4/52 - 2/52 = 28/52= 7/13**

Type	Color		Total
	Red	Black	
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Don't count the two red aces twice!

Rule 5: Addition Rule for Mutually Exclusive Events – No Overlap



$$P(E1 \text{ or } E2) = P(E1) + P(E2)$$

Both events cannot occur at the same time: $P(E1 \text{ and } E2) = 0$

Addition Rule for Mutually Exclusive Events – Example

A retail clothing store has three styles of work boots. They currently have 6 pairs of style A, 12 pairs of style B and 2 pairs of style C. If a store clerk randomly picks one pair to put on display, what is the probability that boot is style A or style C?

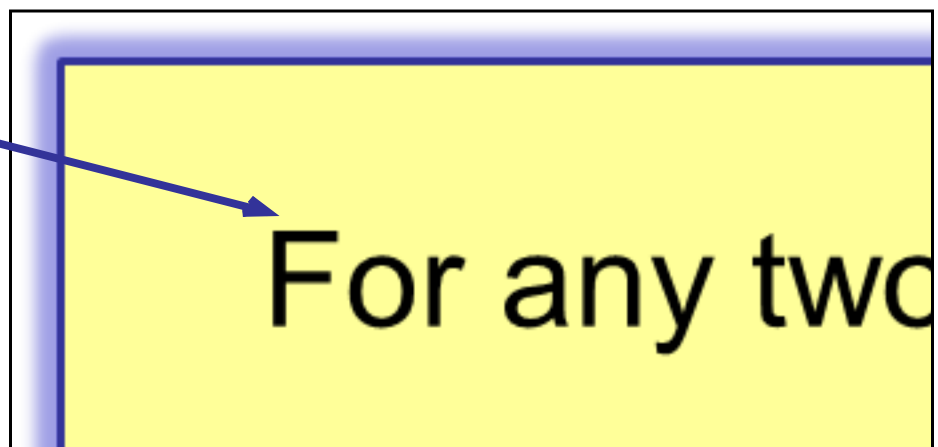
Because if one style boot is selected, another style cannot also be selected, the events are mutually exclusive.

Rule 6: Conditional Probability for Any Two Events

- Conditional Probability
  - The probability that an event will occur *given* that some other event has already happened

Probability Rule 6

Given that



Conditional Probability Example

Of the cars on a used car lot, **70%** have air conditioning (AC), **40%** have a CD player (CD), and **20%** of the cars have both.

What is the probability that a car has a CD player (E_1), given that it has AC (E_2)?

$$\boldsymbol{P(\mathrm{CD}|\mathrm{AC}) = P(E_1|E_2) = ?}$$

	CD	No CD	Total
AC	**0.2**	**0.5**	**0.7**
No AC	**0.2**	**0.1**	**0.3**
Total	**0.4**	**0.6**	**1.0**

$$P(CD|AC) = \frac{P(CD\ and\ AC)}{P(AC)} = \frac{0.2}{0.7} = 0.2857$$

Conditional Probability Example

A company has 500 employs. Of these 300 hold college degrees. Suppose two employees are to be randomly selected to receive a front row parking space. **What is the probability that the second person selected has a college degree, given that the first person selected also has a college degree?**

Event C1 – First Person has College Degree
Event C2 – Second Person selected has a College Degree

If C1 has occurred, 299 College Degree employees remain out of 499.

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Rule 7: Conditional Probability for Independent Events

Probability Rule 7

For independent events $E1$, $E2$

$P(E1|E2) = P(E1) \qquad P(E2) > 0$

$P(E2|E1) = P(E2) \qquad P(E1) > 0$

The conditional probability of one event occurring, given a second independent event has already occurred, is simply the probability of the first event occurring.

Conditional Probability for Independent Events - Example

The time an employee is with the company is independent of the employee's gender.

		Length of Time With Company					
		Under 1 Year	1-3 Years	4-10 Years	10-20 Years	Over 20 Years	Total
	Male	5	10	8	12	15	50
Gender	Female	15	30	24	36	45	150
	Total	20	40	32	48	60	200

Check for Independence:

Rule 8: Multiplication Rule for Any Two Events

- If we do not know the joint relative frequencies, the multiplication rule for two events can be used.

Probability Rule 8

For two events $E1$ and $E2$

$$P(E1 \text{ and } E2) = P(E1)\,P(E2|E1)$$

Multiplication Rule for <u>Any</u> Two Events-Example

A small boutique hotel has 20 rooms. Five of the rooms are suites with a living room and bedroom while the other 15 are traditional hotel rooms. What is the probability that first two rooms booked during the week of October 13-19 are both suites if the bookings are assumed to be random?

Event: S = Suite T = Traditional

Multiplication Rule – Tree Diagram Example

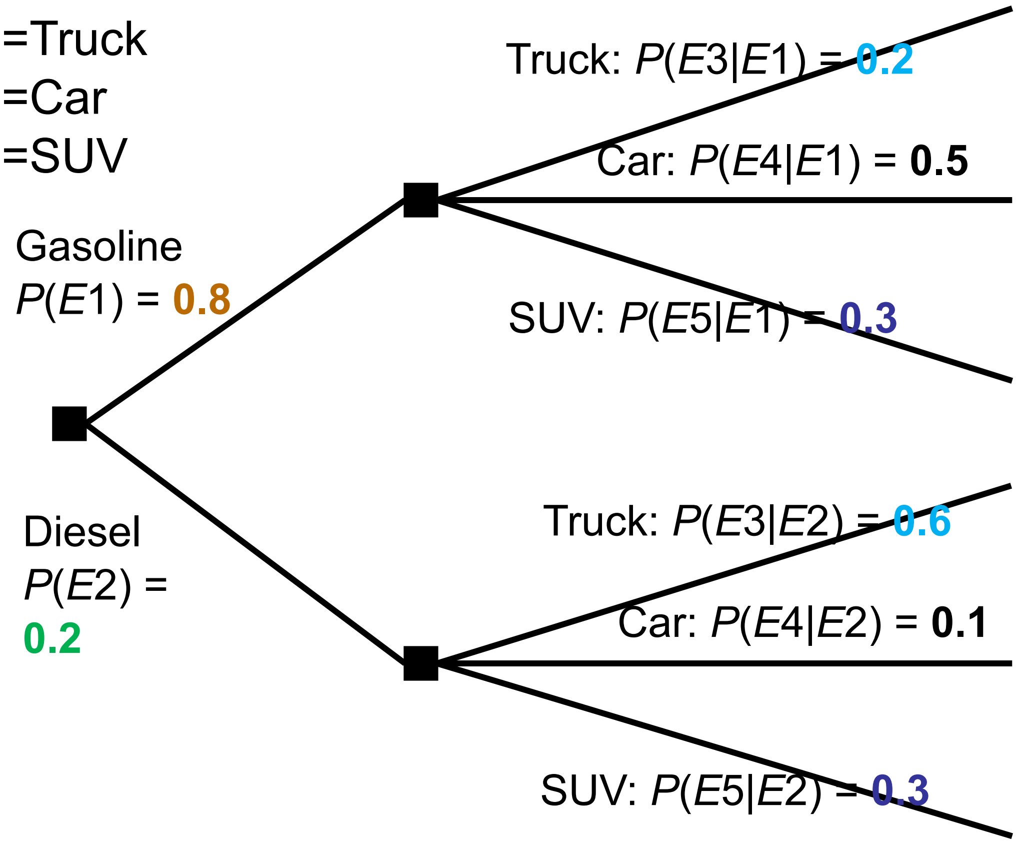
Vehicles use one of two types of fuel; gasoline or diesel. The types of vehicles are; trucks, cars, and SUVs. Assume that the following events and probabilities are given.

E1=Gasoline
E2=Diesel
E3=Truck
E4=Car
E5=SUV

Determine the probabilities for each of the following:

Multiplication Rule – Tree Diagram Example

E1=Gasoline
E2=Diesel
E3=Truck
E4=Car
E5=SUV



$P(E1 \text{ and } E3) = 0.8 \times 0.2 = \mathbf{0.16}$

$P(E1 \text{ and } E4) = 0.8 \times 0.5 = \mathbf{0.40}$

$P(E1 \text{ and } E5) = 0.8 \times 0.3 = \mathbf{0.24}$

$P(E2 \text{ and } E3) = 0.2 \times 0.6 = \mathbf{0.12}$

$P(E2 \text{ and } E4) = 0.2 \times 0.1 = \mathbf{0.02}$

$P(E2 \text{ and } E5) = 0.2 \times 0.3 = \mathbf{0.06}$

Rule 9: Multiplication Rule for Independent Events

- The joint probability of two independent events is simply the product of the probabilities of the two events.

Probability Rule 9

For independent events $E1$, $E2$

$$P(E1 \text{ and } E2) = P(E1)\,P(E2)$$

Multiplication Rule for Independent Events - Example

A small mid-western city has three major airlines that serve the airport. Suppose that 60% of the passengers fly United Airlines, 30% fly Delta, and 10% fly Southwest. Further, suppose that 70% of the fliers are flying for pleasure and 30% fly for business. Assuming type of trip and the airline used are <u>independent</u>, what is the probability that a flier uses United and is a business traveler?