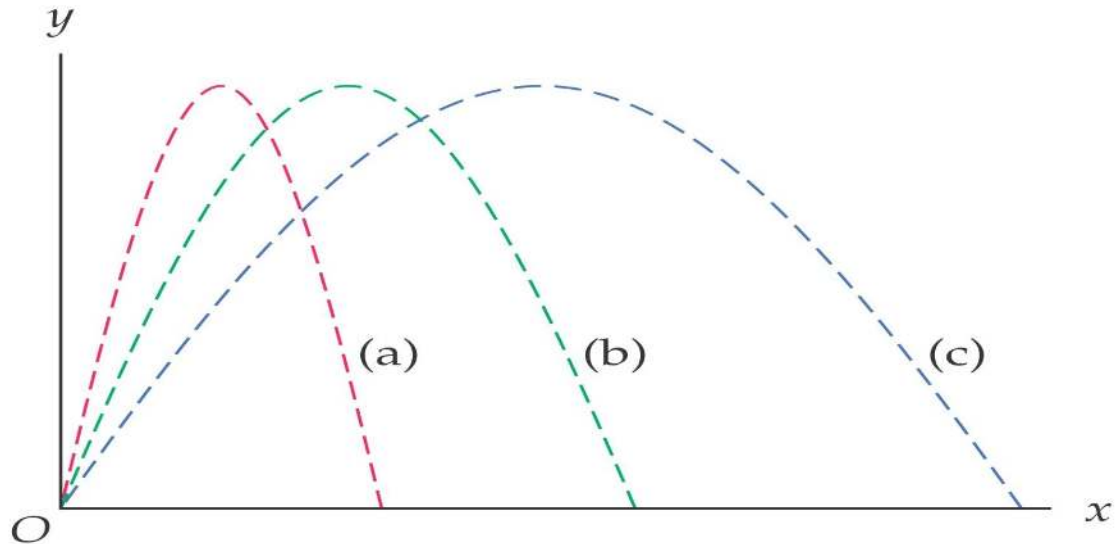


Chapter -4

Two-Dimensional Kinematics



Chapter 4

Two-Dimensional Kinematics

Equations of Motion in 2D

For x :

$$① x_f = x_i + v_{ix} t$$

$$② x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2$$

$$③ v_{fx} = v_{ix} + a_x t$$

$$④ v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x$$

For y :

$$① y_f = y_i + v_{iy} t$$

$$② y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$

$$③ v_{fy} = v_{iy} + a_y t$$

$$④ v_{fy}^2 = v_{iy}^2 + 2 a_y \Delta y$$

Units of Chapter 4

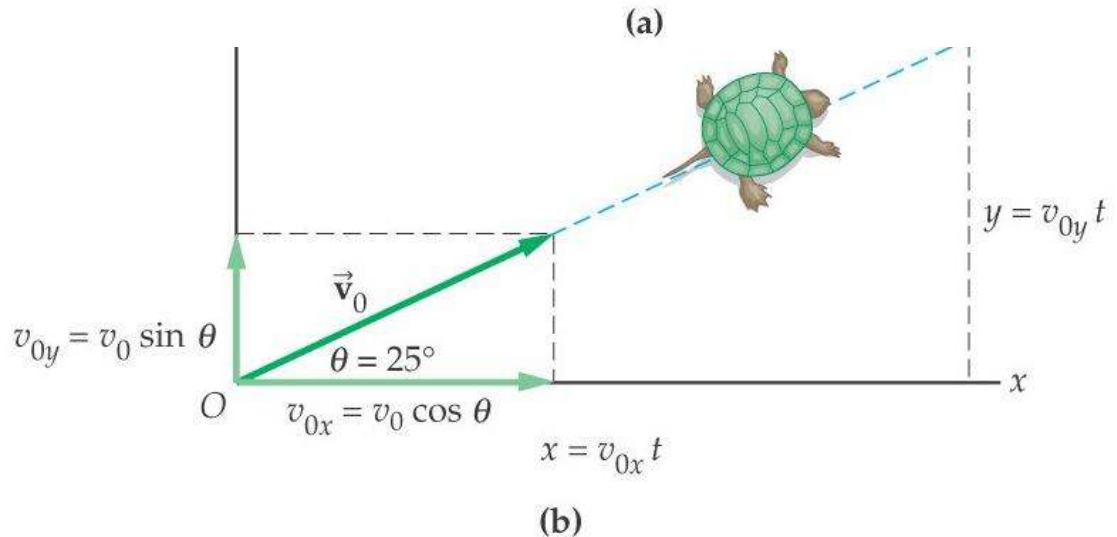
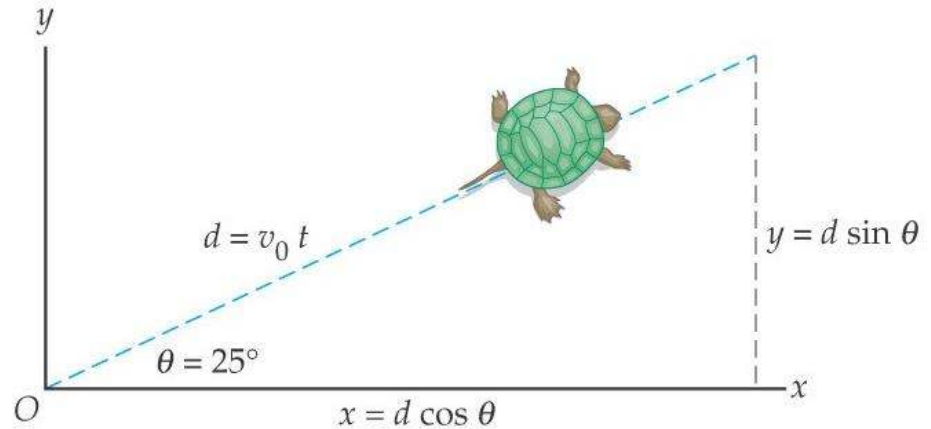
- Motion in Two Dimensions
- Projectile Motion: Basic Equations 2D Two types?
- ① • Zero Launch Angle
- ② • General Launch Angle
- Projectile Motion: Key Characteristics

4-1- Motion in Two Dimensions

Because there is an angle

it is in 2D

If velocity is constant, motion is along a straight line:



○ 4-1- Motion in Two Dimensions

$$\theta = 25^\circ$$

$$t_i = 0 \text{ s}$$

$$t_f = 50 \text{ s}$$

$$\vec{d} = ?$$

$$\vec{v} = 0.26 \text{ m/s}$$

① First Way:

$$\vec{d} = \vec{v} \times t$$



$$d = 0.26 \times 5 = 1.3 \text{ m}$$

$$dx = 1.3 \cos 25 = 1.2 \text{ m}$$

$$dy = 1.3 \sin 25 = 0.55 \text{ m}$$

② Second Way:

$$dx = v_x \times t$$

$$(|v| \cos \theta) \times t$$

$$(0.26 \cos 25) \times 0.5 = 1.2 \text{ m}$$

$$dy = v_y \times t$$

$$(|v| \sin \theta) \times t$$

$$(0.26 \sin 25) \times 0.5 = 0.55 \text{ m}$$

Example:4-1

- An eagle perched on a tree limb 19.5m above the water spots at a fish swimming near the surface. The eagle pushes off from the branch and descends towards the water. By adjusting its body in flight, the eagle maintains a constant speed of 3.10m/s at an angle of 20.0° below the horizontal.
- A) How long does it take for the eagle to reach the water? **Ans:**18.4s
- B) How far has the eagle travelled in the horizontal direction when it reaches the water? **Ans:**53.5m

Example:4-1

Speeds 3.10 m/s

$$|v| = 3.10 \text{ m/s}$$

$$y_i = 19.5 \text{ m} \quad y_f = 0$$

$$\theta = 20^\circ$$

A) $T = ?$

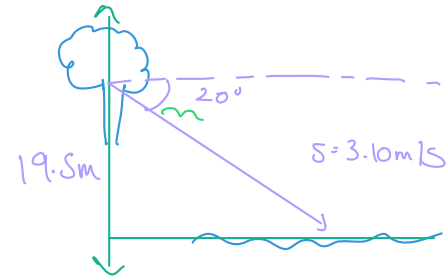
$$y_f = y_i + v_{iy} t$$

$$t = \frac{y_f - y_i}{v_{iy}} = \frac{0 - 19.5}{-(3.10 \sin 20^\circ)} = 18.4 \text{ s}$$

the falcon is going down

B) $x_f = x_i + v_{ix} t$

$$x_f = 0 + (3.10 \cos 20^\circ) 18.4$$
$$= 53.6 \text{ m}$$



4-1 -Motion in Two Dimensions

Motion in the x- and y-directions should be solved separately:

TABLE 4-1 Constant-Acceleration Equations of Motion

Position as a
function of time

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$

Velocity as a
function of time

$$v_x = v_{0x} + a_xt$$

$$v_y = v_{0y} + a_yt$$

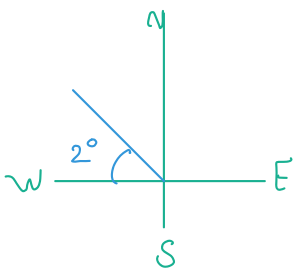
Velocity as a
function of position

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

Problem 2

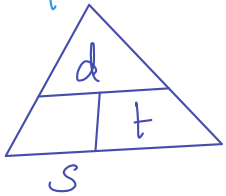
- Q2. A sails boat runs before the wind with a constant speed of 4.2m/s in the direction of 32° north of west. How far
- (a) west and
- (b) North has the sailboat travelled in 25min
- Sol:-



$$= |v_i| = 4.2 \text{ m/s}$$

$$8.25 \text{ m} \times \frac{60 \text{ s}}{1 \text{ m}} = 1500$$

$t =$



Problem 2

$$d_{\text{speed}} = \text{speed} \times t$$

$$\vec{d} = v_i \cos t$$

$$\vec{d} = -(4.2) \cos 32 \cdot 1500$$

$$= -5434 \text{ m}$$

$$d_{\text{speed}} = \text{speed}_{\text{north}} \times t$$

$$d_{\text{speed}} = \text{speed}_y \times t$$

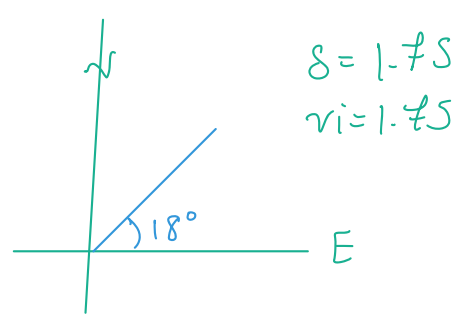
$$\vec{d} = v_i \sin t$$

$$\vec{d} = (4.2) \sin 32 \cdot 1500$$

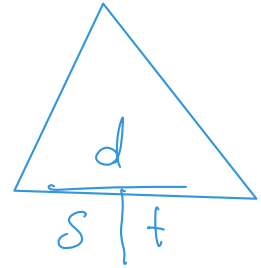
$$= 3338 \text{ m}$$

Problem 3

- Q3) As you walk to class with a constant speed of 1.75m/s , you are moving in a direction that is 18° north of east. How much time does it take to change your displacement by
 - (a) 20.0m east or
 - (b) 30.0m north
- Q4)---Assignment(pg 104)



Problem 3



a) time east = $\frac{dx}{\text{speed}_x} = \frac{20 \text{ m}}{1.75 \cos 18} = 12.5$

b) time north = $\frac{dy}{\text{speed}_y} = \frac{30 \text{ m}}{1.75 \sin 18} = 55.5 \text{ s}$

r we can use:

$x_f = x_i + v_{ix} t \rightarrow \Delta x = v_{ix} t$
 (4) --- Assignment 1 (pg 104)

$y_f = y_i + v_{iy} t \rightarrow \Delta y = v_{iy} t$

Part 1.

Motion of Objects Projected Horizontally

PROJECTILE is a body which is thrown horizontally or at an angle relative to the horizontal which follows a curved path called **trajectory**

Examples:

Ball being thrown, water coming out of the hose, a bullet fired from a gun, arrow shot from a bow, cannonballs, fountains.

y

 v_0 

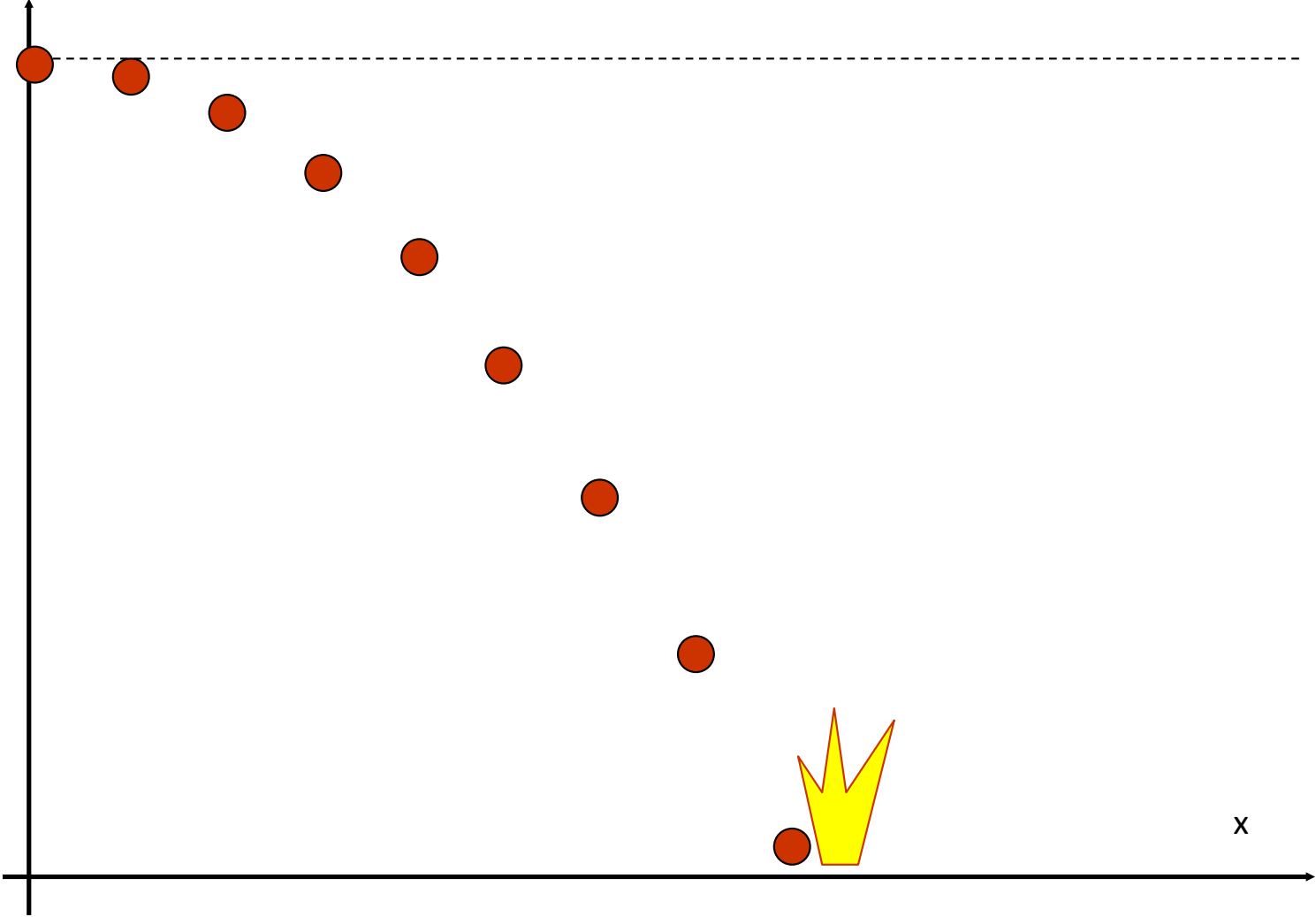
⊙ Anything that is throw horizontally, either straight or with an angle → called trajectory.

⊙ In the end their is a curved path

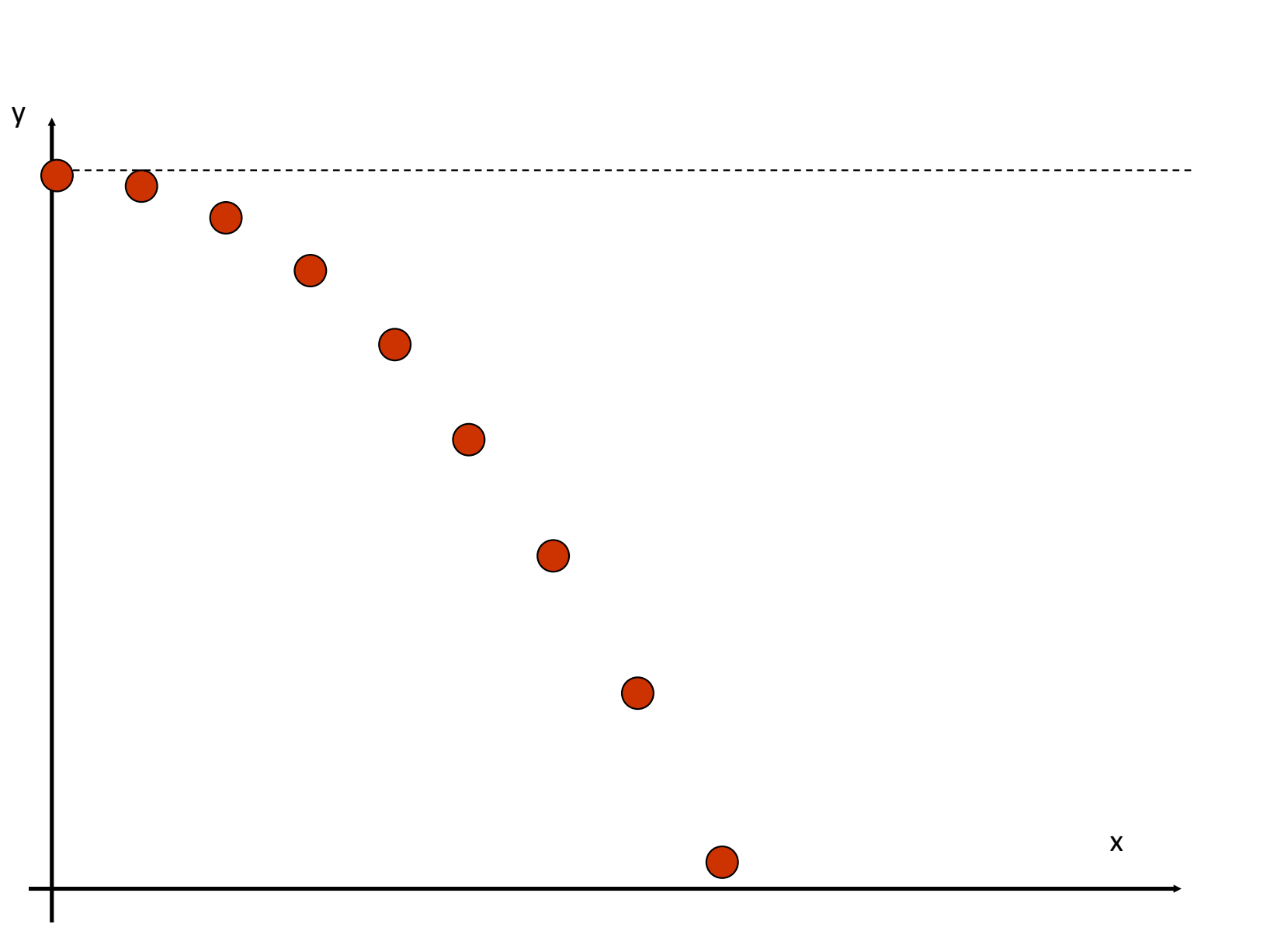
⊙ $g = 9.81 \text{ m/s}^2$ always

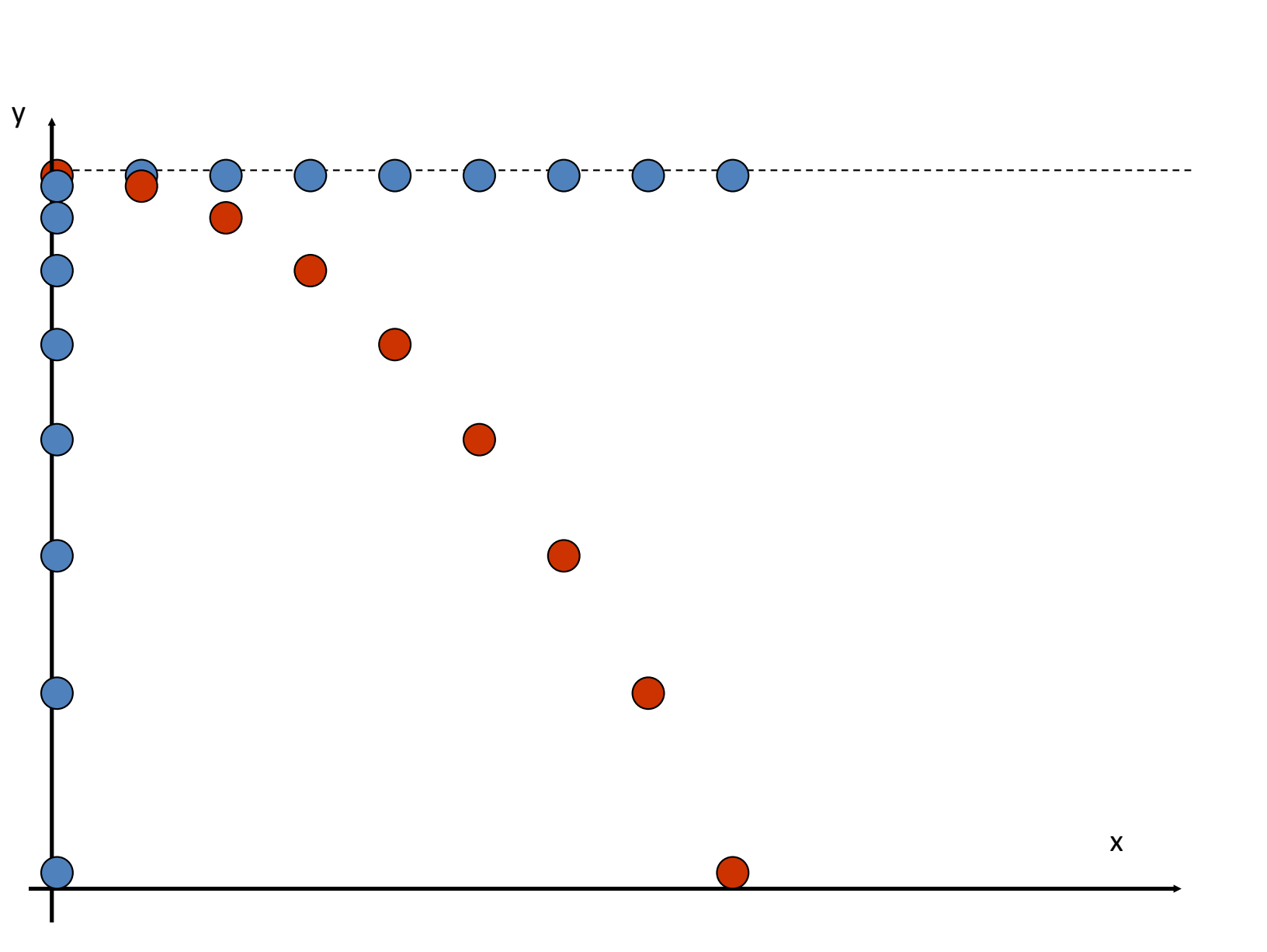
x

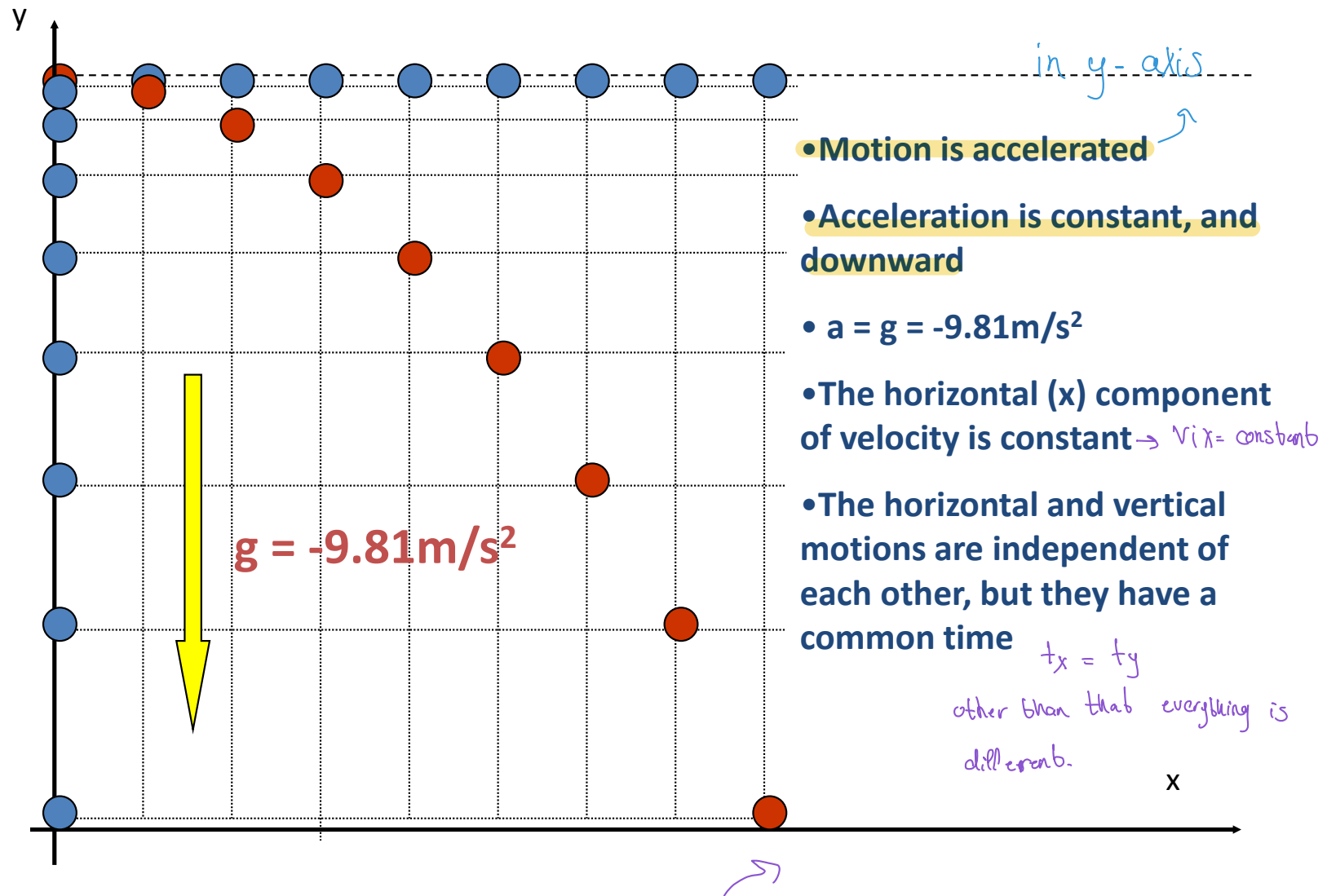
y



x







in y-axis

- Motion is accelerated

- Acceleration is constant, and downward

- $a = g = -9.81 \text{ m/s}^2$

- The horizontal (x) component of velocity is constant $\rightarrow v_{ix} = \text{constant}$

- The horizontal and vertical motions are independent of each other, but they have a common time

$$t_x = t_y$$

other than that everything is different.

x

$v_{ix} = \text{constant}, \text{ so } a_x = 0$

$v_{iy} = \text{changing}, \text{ so } a_y = -9.81 \text{ m/s}^2$

ANALYSIS OF MOTION

ASSUMPTIONS:

- **x-direction (horizontal):** $\rightarrow$ **uniform motion** $\text{constant } b = 0$
- **y-direction (vertical):** $\rightarrow$ **accelerated motion** $= -9.81 \text{ m/s}^2$
- no air resistance or earth rotation

QUESTIONS:

- **What is the trajectory?** *Curved path*
- **What is the horizontal range?** *Displacement in x axis*
- **What is the final velocity of the projectile?** $v_{fx} = ?$

$v_{fy} = ?$

2 final velocities.

4-2 -Projectile Motion: Basic Equations

Projectile is an object that is launched into motion and then allowed to follow a path determined solely by the influence of gravity.

Assumptions:

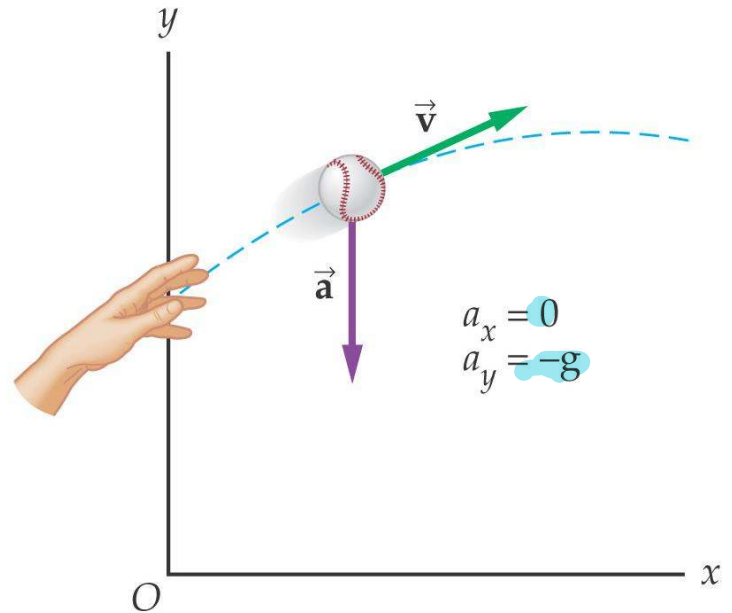
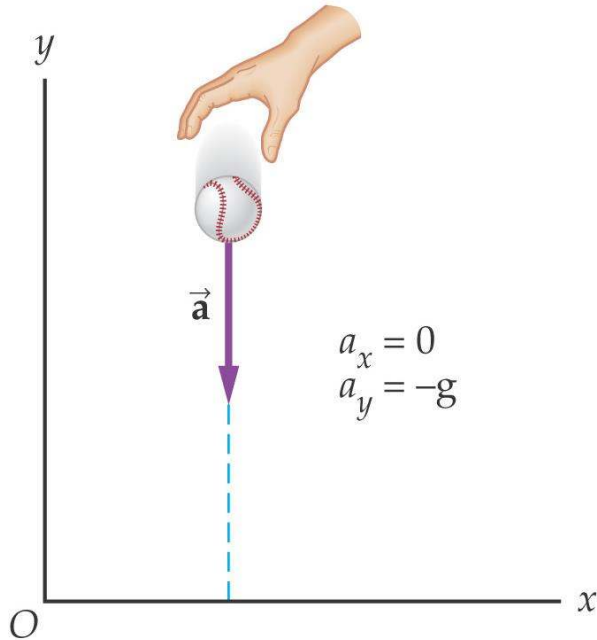
- ignore air resistance
- ignore Earth's rotation
- $g = 9.81 \text{ m/s}^2$, downward $\rightarrow a_y = -g$

If y-axis points upward, acceleration in x-direction is zero and acceleration in y-direction is -9.81 m/s^2

4-2- Projectile Motion: Basic Equations

The acceleration is independent of the direction of the

velocity: even if $a = +$, that doesn't mean velocity is positive



4-2 -Projectile Motion: Basic Equations

These, then, are the basic equations of projectile motion:

$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_x = v_{0x}$$

$$v_y = v_{0y} - gt$$

$$v_x^2 = v_{0x}^2$$

$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

4-2 -Projectile Motion: Basic Equations

X-axis :

g y
↓

$$x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

gone, why? $a=0$

$$v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x$$

gone, why? $a=0$

-y -uy 0-

Y-axis :

$$y_f = y_i + v_{iy} t + \frac{1}{2} g t^2$$

$$v_{fy} = v_{iy} + g t$$

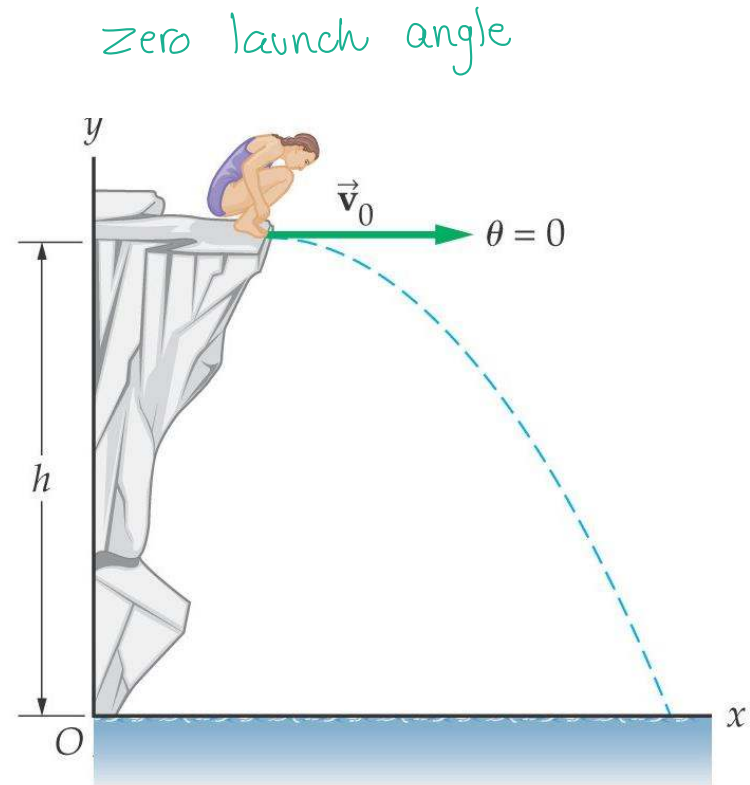
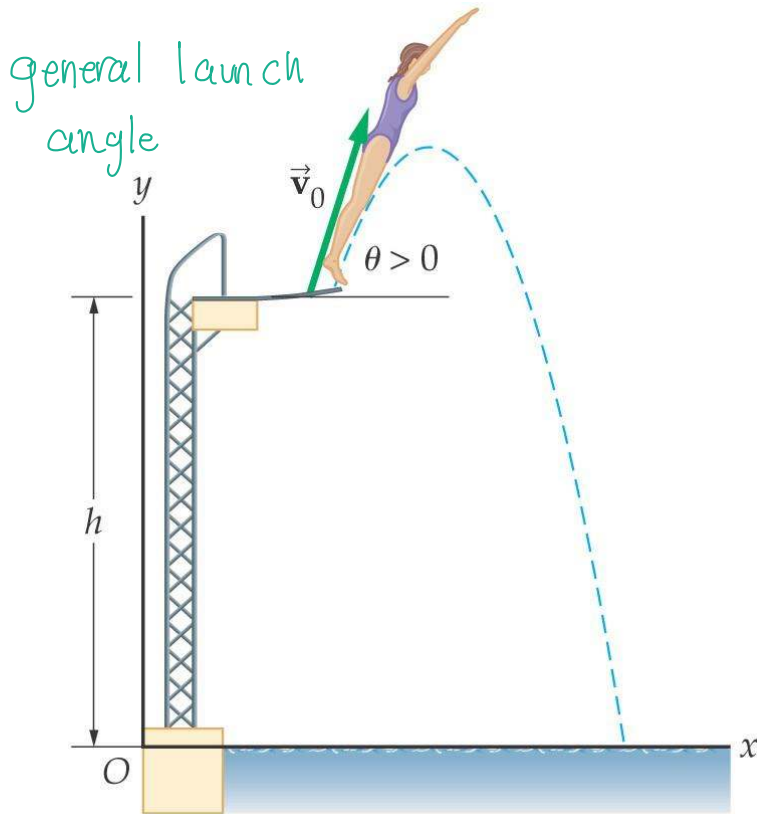
$$v_{fy}^2 = v_{iy}^2 + 2 g \Delta y$$

all a are changed to $g = 9.81 \text{ m/s}^2$

y -y -y -y

4-3 -Zero Launch Angle

Launch angle: direction of initial velocity with respect to horizontal



4-3- Zero Launch Angle

In this case, the initial velocity in the y-direction is zero. Here are the equations of motion, with $x_0 = 0$ and $y_0 = h$:

this is the difference =

• $v_{ix} = v_i \cos(\theta) = v_i(1) = v_i \rightarrow v_{ix} = v_i$

• $v_{iy} = v_i \sin(\theta) = v_i(0) = 0 \rightarrow v_{iy} = 0$

• $x_i = 0$

$$x = v_0 t$$

• $y_i = h$

$$y = h - \frac{1}{2}gt^2$$

$$v_x = v_0 = \text{constant}$$

$$v_x^2 = v_0^2 = \text{constant}$$

$$v_y = -gt$$

$$v_y^2 = -2g\Delta y$$

4-3- Zero Launch Angle

X axis :

$$x_f = v_i t$$

$$v_{fx} = v_{ix}$$

$$v_{fx}^2 = v_{ix}^2$$

$$v_y = -gt$$

y axis :

$$y_f = h + \frac{1}{2} -gt^2$$

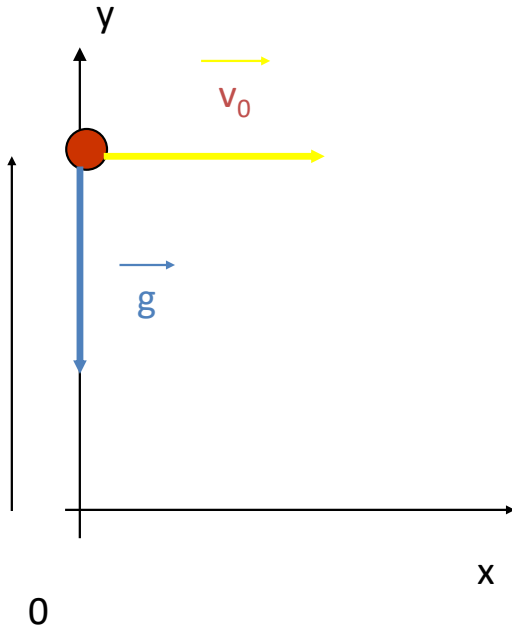
$$v_{fy} = -gt$$

$$v_{fy}^2 = 2 -g \Delta y$$

$$v_y^2 = -2g\Delta y$$

Frame of reference:

Equations of motion:

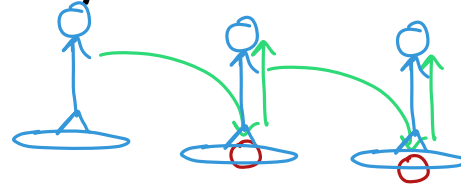


	X Uniform m.	Y Accel. m.
ACCL.	$a_x = 0$	$a_y = -g = -9.81 \text{ m/s}^2$
VELC.	$v_x = v_0$	$v_y = -g t$
DSPL.	$x = v_0 t$	$y = h - \frac{1}{2} g t^2$

Zero launch angle, why? Because there is height.

Example 4-3

- A person skateboarding with a constant speed of 1.30m/s releases a ball from a height of 1.25m above the ground. Given that $x_0 = 0$
- and $y_0 = h = 1.25\text{m}$, Find x and y for
- (a) $t = 0.250\text{s}$ and
- (b) $t = 0.500\text{s}$
- (c) Find the velocity, speed and direction of motion of ball at $t = 0.500\text{s}$
- **Sol:** (a) $x = 0.325\text{m}$, $y = 0.943\text{m}$
- (b) $x = 0.650\text{m}$, $y = 0.0238\text{m}$
- (c) $v_x = 1.30\text{m/s}$, $v_y = -4.91\text{m/s}$, Speed = 5.08m/s , Direction = -75.2°



) x/f

le

$$1.30 \times 0.25 = 0.33 \text{ m}$$

a $y_f = h + \frac{1}{2} - g t$

$$1.25 - \left(\frac{1}{2} \times 9.81 \times (0.25)^2 \right) = 0.943 \text{ m}$$

b) $x_f = v_i t$
 $= 1.30 \times 0.5 \rightarrow$

$$y_f = h + \frac{1}{2} - g t^2$$

$$1.25 - \left(\frac{1}{2} \times 9.81 \times (0.5)^2 \right)$$

=

- (c) $v_x = 1.30 \text{ m/s}$, $v_y = -4.91 \text{ m/s}$, Speed = 5.08 m/s , Direction = -75.2°

$a_x = 0$
 $a_y = -g$
 Projectile
 Zero
 launch
 $y_i = h$
 $v_{ix} = v_i$
 $v_{iy} = 0$
 $x_i = 0$

le

$$c) \quad v_{fx} = v_i \\ = 1.30 \text{ m/s}$$

$$v_{fy} = -g t \\ = -9.81 \times 0.5 = -4.905$$

$$\text{Speed} = \sqrt{(v_{fx})^2 + (v_{fy})^2} \\ = \sqrt{(1.30)^2 + (-4.9)^2} = 5.1 \text{ m/s}$$

$$\Theta_v = \tan^{-1} \left(\frac{v_{fy}}{v_{fx}} \right) = \\ = \tan^{-1} \left(\frac{-4.9}{1.30} \right) = -75^\circ \text{ fourth Quarter}$$

- (c) $v_x = 1.30 \text{ m/s}$, $v_y = -4.91 \text{ m/s}$, $\text{Speed} = 5.08 \text{ m/s}$, $\text{Direction} = -75.2^\circ$

$a_x = 0$
 $a_y = -g$
 Projectile
 Zero
 launch
 $y_i = h$
 $v_{ix} = v_i$
 $v_{iy} = 0$
 $x_i = 0$

4-4- General Launch Angle

In general, $v_{0x} = v_0 \cos \vartheta$ and $v_{0y} = v_0 \sin \vartheta$

This gives the equations of motion:

$$x_i = y_i = 0$$

$$v_i x = v_i \cos \theta$$

$$v_i y = v_i \sin \theta$$

$$x = (v_0 \cos \theta)t$$

$$y = (v_0 \sin \theta)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos \theta$$

$$v_x^2 = v_0^2 \cos^2 \theta$$

$$v_y = v_0 \sin \theta - gt$$

$$v_y^2 = v_0^2 \sin^2 \theta - 2g\Delta y$$

4-4- General Launch Angle

In general, $v_{ix} = v_i \cos \theta$ and $v_{iy} = v_i \sin \theta$

$$x_i = y_i = 0$$

$$v_{ix} = v_i \cos \theta$$

$$v_{iy} = v_i \sin \theta$$

$$v_y = v_0 \sin \theta - gt \qquad v_y^2 = v_0^2 \sin^2 \theta - 2g\Delta y$$

General $\rightarrow$ because it starts from the origin

Exercise: 4-1

+ there is an angle

- A projectile is launched from the origin with an initial speed of 20.0m/s at an angle of 35.00° above the horizontal. Find the x and y positions of the projectile at times

- (a) $t = 0.500\text{s}$ **Sol:-** $x = 8.19\text{m}$, $y = 4.51\text{m}$ Just replace time
- (b) $t = 1.00\text{s}$ **Sol:-** $x = 16.4\text{m}$, $y = 6.57\text{m}$ $\nearrow$ time
- (c) $t = 1.50\text{s}$ **Sol:-** $x = 24.6\text{m}$, $y = 6.17\text{m}$ Just replace time $\rightarrow$ time

2D $\rightarrow$ Projectile $\rightarrow a_x = 0$, $a_y = g$

General launch $\rightarrow x_i = y_i = 0$, $v_{ix} = v_i \cos(\theta)$, $v_{iy} = v_i \sin(\theta)$

$$a) \quad x_f = (v_i \cos \theta) t$$

$$= 20 \cos 35^\circ \times 0.5 \rightarrow 8.2 \text{ m}$$

$$y_f = (v_i \sin \theta) t - \frac{1}{2} g t^2$$

$$= 20 \sin 35^\circ \times 0.5 - \left(\frac{1}{2} \times 9.81 \times (0.5)^2 \right) \rightarrow 4.5 \text{ m}$$

$$b) \quad x_f = (v_i \cos \theta) t$$

$$= 20$$

$$- (c) \quad t = 1.505 \quad \text{301.7} \quad x = 24.0111, \quad y = 0.1711$$

Exercise 4-2

- Referring to ex-4-1 ,find the velocity of the projectile at times
- (a) $t = 0.500\text{s}$ **Sol:-** $\mathbf{V} = (16.4\text{m/s})\mathbf{i} + (6.57\text{m/s})\mathbf{j}$
- (b) $t = 1.00\text{s}$ **Sol:-** $\mathbf{v} = (16.4\text{m/s})\mathbf{i} + (1.66\text{ m/s})\mathbf{j}$
- (c) $t = 1.50\text{s}$ **Sol:-** $\mathbf{v} = (16.4\text{m/s})\mathbf{i} + (-3.24\text{ m/s})\mathbf{j}$



unit vector

notation

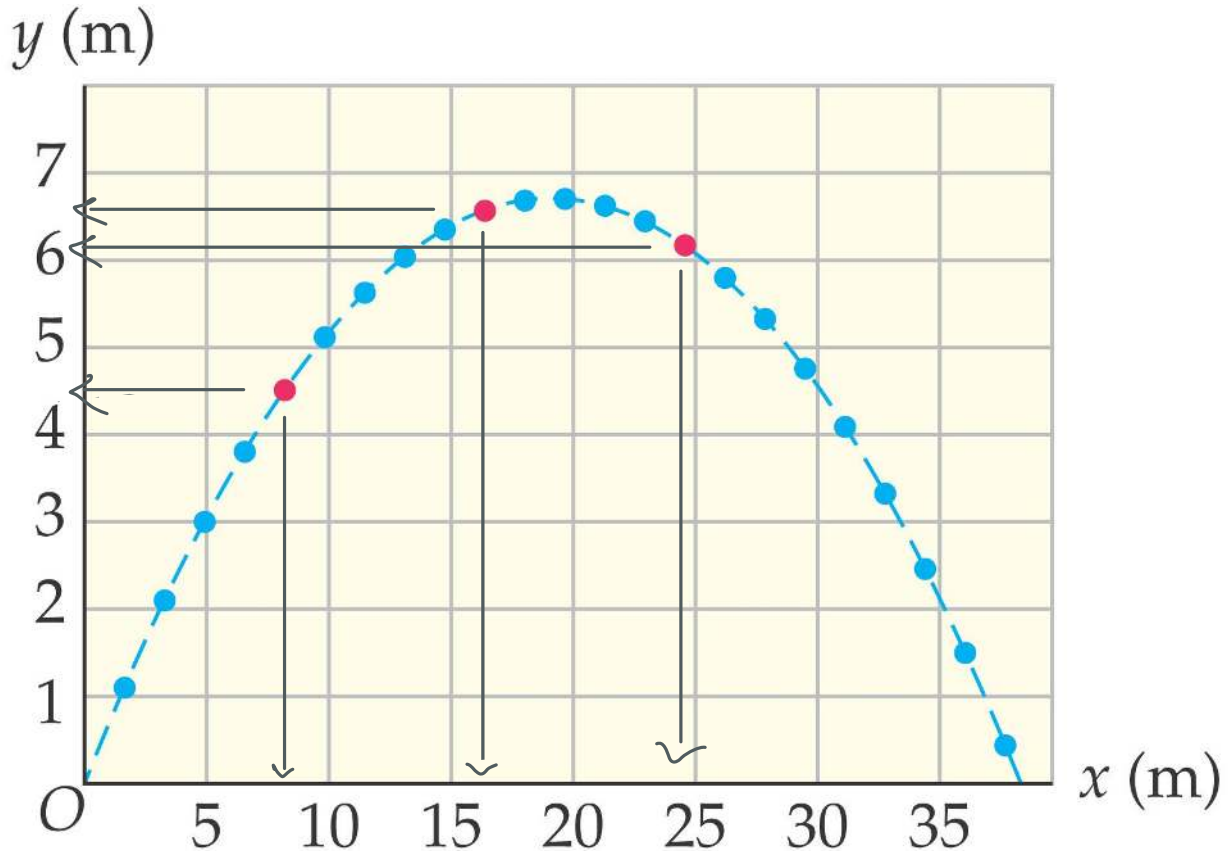
Exercise 4-2

$$\begin{aligned} a) \quad v_x &= v_i \cos \theta \\ &= 20 \cos 35 = (16.4 \text{ m/s}) \hat{x} \end{aligned}$$

$$\begin{aligned} v_y &= v_i \sin \theta - g t \\ &= 20 \sin 35 - (9.81)(0.5) = (6.6 \text{ m/s}) \hat{y} \end{aligned}$$

4-4- General Launch Angle

Snapshots of a trajectory; red dots are at $t = 1$ s, $t = 2$ s, and $t = 3$ s



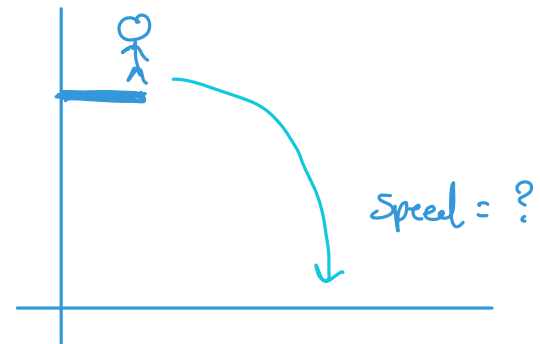
2D $\rightarrow$ projectile $\rightarrow$ zero launch, because there is h . $\rightarrow$

Problem: 12

$$x_i = 0$$
$$y_i = h$$

$$v_{ix} = v_i$$
$$v_{iy} = 0$$

- A diver runs horizontally off the end of diving board with an initial speed of 1.85m/s. If the diving board is 3.00m above the water, what is the diver's speed just before she enters the water?
- Sol: $V_y^2 = 58.9\text{m}^2/\text{s}^2$
- Speed = 7.89m/s



$$v_{fx} = v_i$$

$$\therefore v_{fx} = \sqrt{v_i^2} \rightarrow v_{fx} = \sqrt{1.85^2} = 1.85 \text{ m/s}$$

$$v_{fy}^2 = -2g(y_f - y_i)$$

$$\therefore v_{fy} = \sqrt{(-2)(9.81)(0 - 3)} = 7.7 \text{ m/s}$$

$$\text{Speed} = \text{magnitude} = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

$$\therefore \text{Speed} = \sqrt{(1.85)^2 + (7.7)^2}$$

$$= \sqrt{(1.85)^2 + (7.7)^2} = 7.9 \text{ m/s}$$

General launch, because there is an angle

$$x_i = y_i = 0$$

Problem 31

$$v_{ix} = v_i \cos \theta =$$

$$v_{iy} = v_i \sin \theta =$$

- A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30m in 1.25s , what was its initial speed.

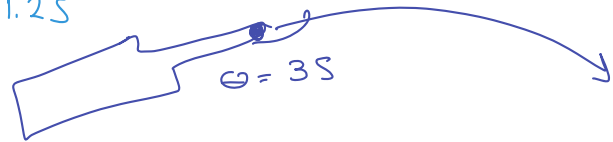
$$V_x = 1.04 \text{ m/s}$$

$$V_0 = 1.27 \text{ m/s}$$

Speed $|v_i| =$

$$\Delta x = 1.30 \text{ m}$$

$$t = 1.25$$



$$\bullet \quad \frac{x_f}{\cos \theta t} = \frac{v_i \cos \theta t}{\cos \theta t} = \dots$$

$$v_i = \frac{x_f}{\cos \theta t} = \frac{1.30}{\cos 35^\circ \times 1.2 \text{ s}} = 1.3 \text{ m/s}$$

$$v_0 = 1.27 \text{ m/s}$$

$$r_{fy} = 0$$

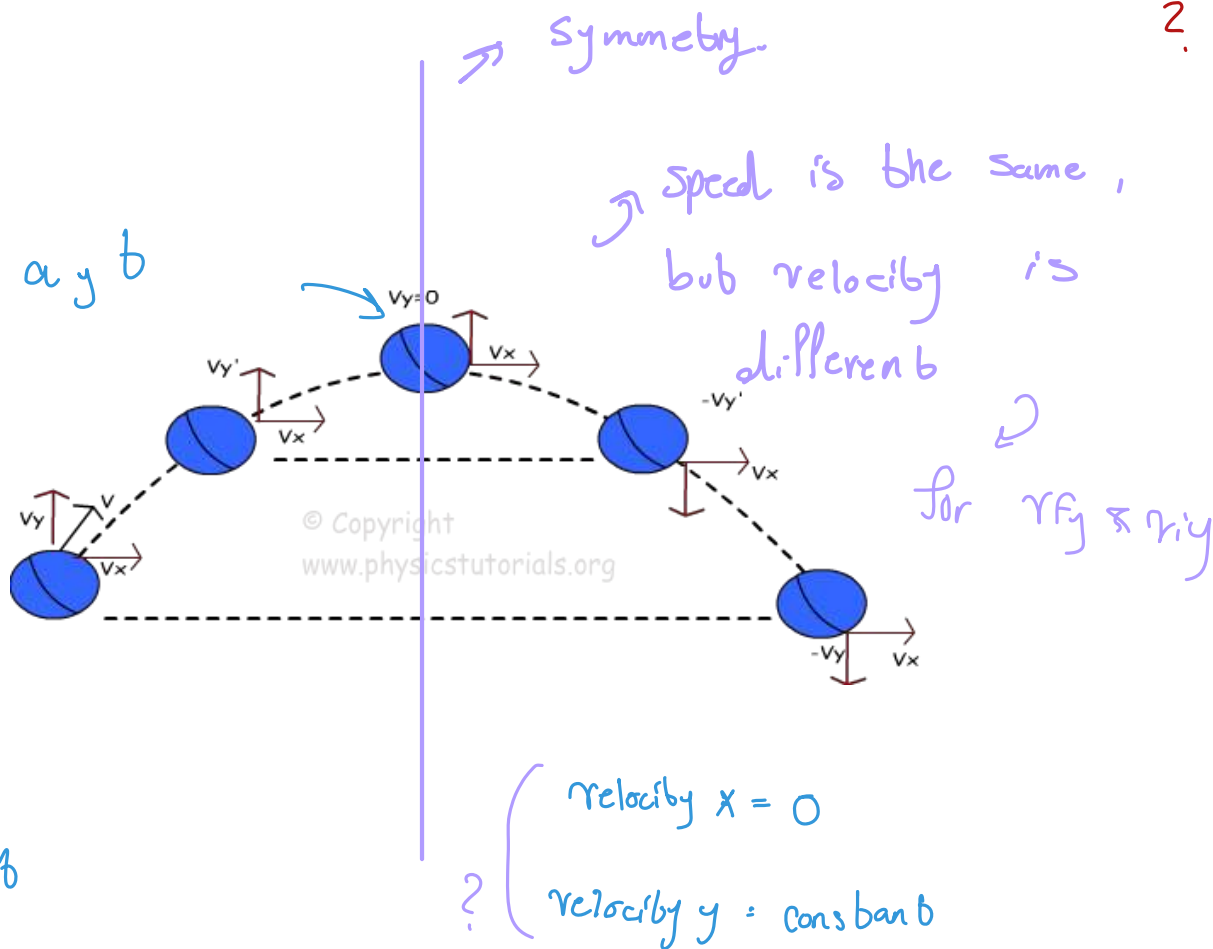
$$r_{fy} = r_{iy} + a_y t$$

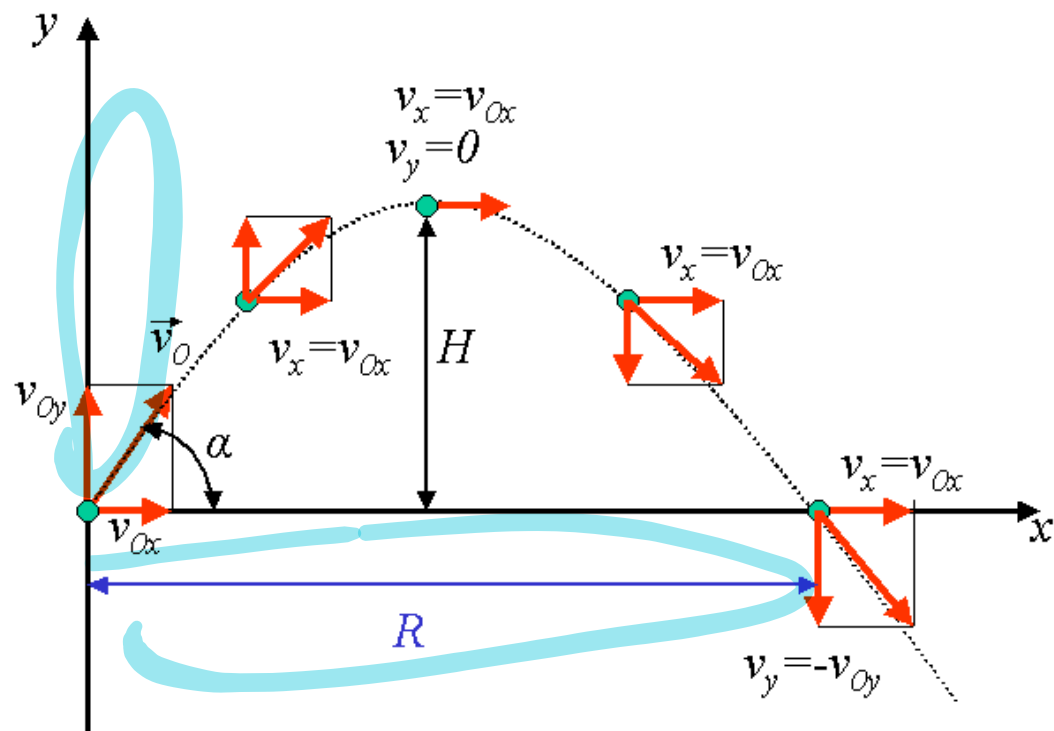
$$r_{iy} = r_i \sin \theta$$

$$r_{fy} = r_i \sin \theta - g t$$

$$0 = r_i \sin \theta - g t$$

$$\frac{g t}{g} = \frac{r_i \sin \theta}{g} \rightarrow t = \frac{r_i \sin \theta}{g} \leftarrow \text{half time} \rightarrow \text{to prove the half time.}$$





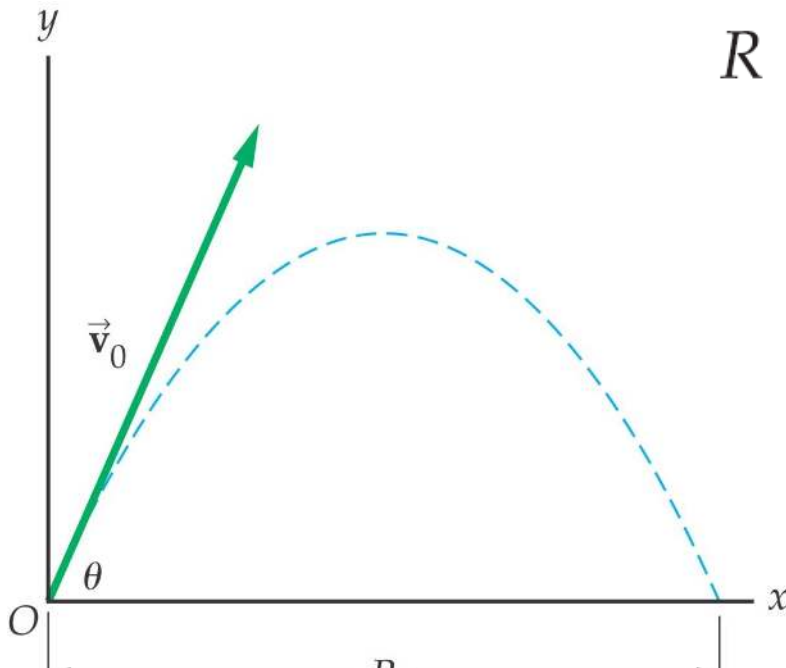
RANGE is the horizontal displacement of the projectile (dx)

MAXIMUM HEIGHT is the vertical displacement of the projectile (dy)

4-5 -Projectile Motion: Key Characteristics

- **Range:** the horizontal distance a projectile travels
- If the initial and final elevation are the same:
- Range of projectile launched from origin with an initial speed v_0 and a launch angle θ is

$$R = \left(\frac{v_0^2}{g} \right) \sin 2\theta$$



$$1) T_{total} : \frac{2v_i \sin \theta}{\frac{1}{2}g}$$

$$2) T_{halP} : \frac{v_i \sin \theta}{\frac{1}{2}g}$$

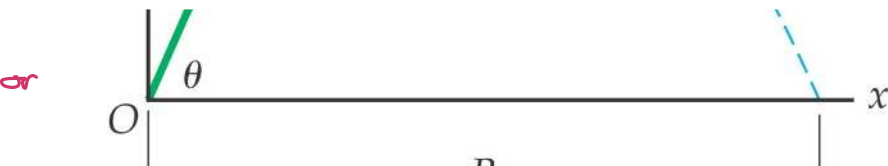
$$3) Range : x_f = 0 + (v_i \cos \theta) \frac{2v_i \sin \theta}{g}$$

or

$$3) x_f = \frac{v_i^2 \sin \theta}{g}$$

$$4) y_{max} : \frac{v_i \sin \theta^2}{2g}$$

max
height



30°



Initial velocity: v_0 Horizontal distance: x Time of flight: t Maximum height: y_{max}

MAXIMUM RANGE

45°



Initial velocity: v_0 Horizontal distance: x Time of flight: t Maximum height: y_{max}

60°

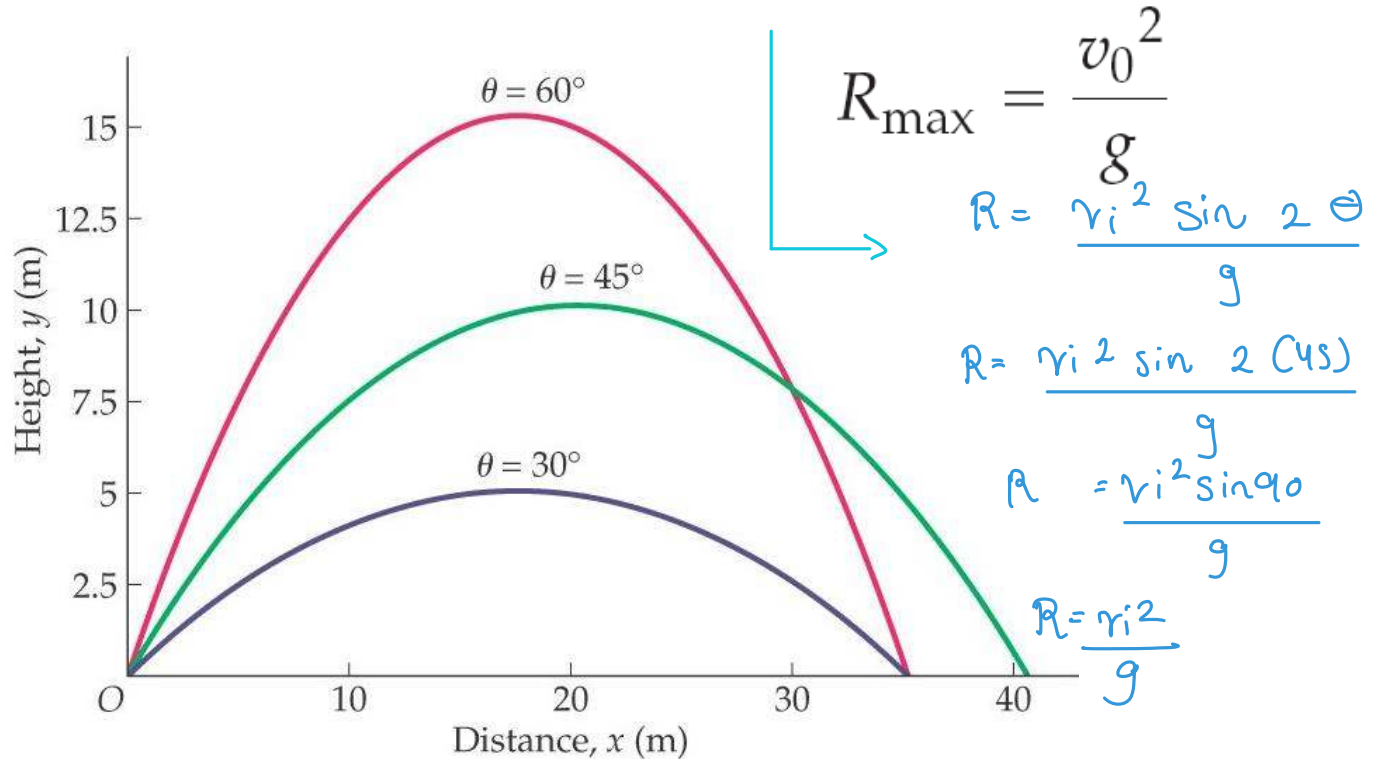


Initial velocity: v_0 Horizontal distance: x Time of flight: t Maximum height: y_{max}

4-5 -Projectile Motion: Key Characteristics

The range is a maximum when $\theta = 45^\circ$:

Don't memorize, just explaining

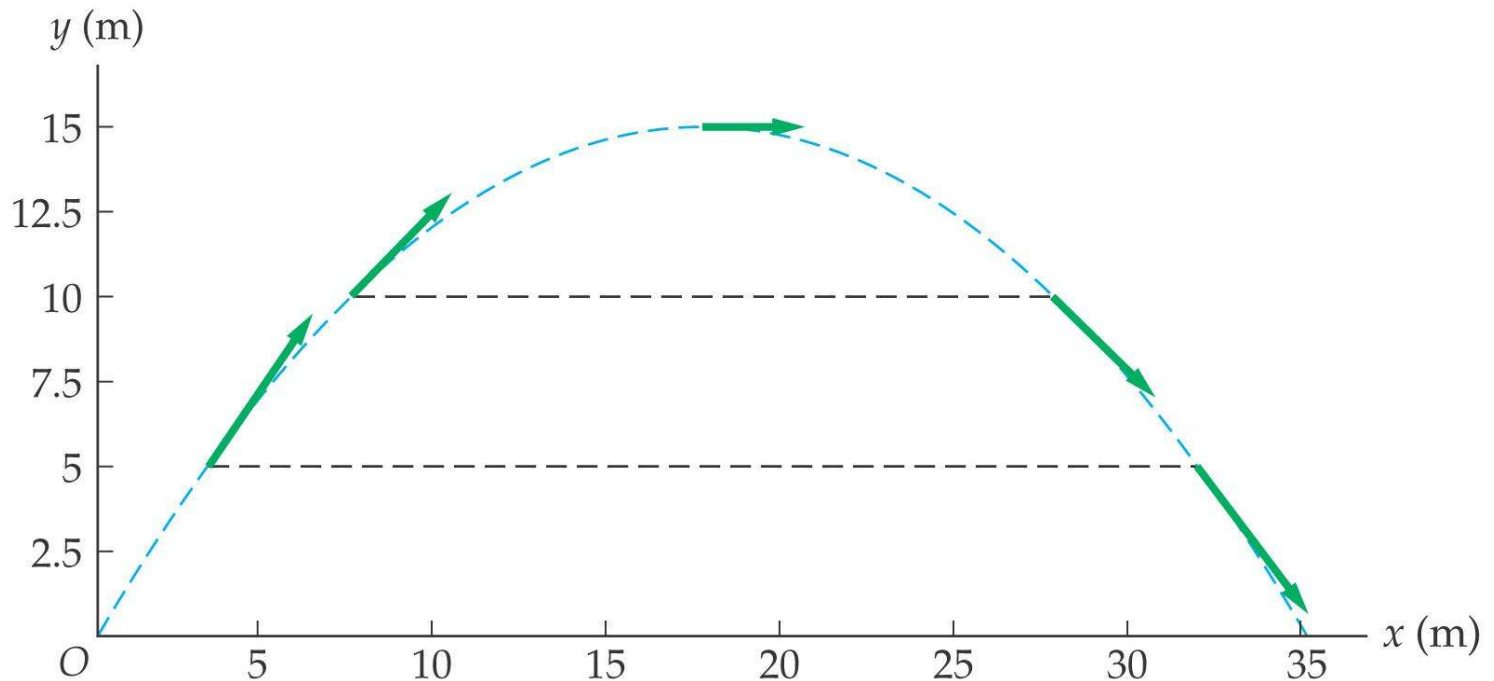


$$x = \frac{1}{2} R \rightarrow (y_{\max})$$

(b)

4-5- Projectile Motion: Key Characteristics

Symmetry in projectile motion:



Time of flight

1-Time of Flight : The time taken by the projectile from when launched and when it lands is

$$t = (2 v_o / g) \sin\theta$$

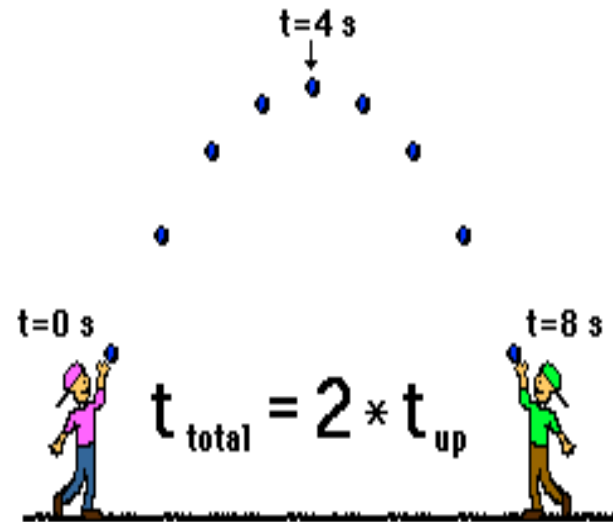
Time of flight

- **Determination of the Time of Flight**

- $t_{total} = T = 2v_o \sin\theta / g$

- **Time at peak**

- $t_{half} = v_o \sin\theta / g$



If it takes a projectile 4 seconds to rise to its peak, then it will take a total of 8 seconds to move through the air from start to finish.

Maximum height

- Maximum height of a projectile above its launch site is

$$y_{max} = (v_0 \sin \theta)^2 / 2g$$

where,

y_{max} = displacement (m)

θ = angle of projection (degrees)

V_0 ...initial speed in m/s

Problems

- **Question 1:** A body is projected and landed at the same level with a velocity of 20 ms^{-1} at 50° to the horizontal. Find
 -
 - (i) Maximum height reached----- **Ans.= 11.97 m.**
 - (ii) Time of flight and----- **Ans.= 3.126 s.**
 - (iii) Range of the projectile----- **Ans.= 40.196 m.**
- **Question 2:** John is on top of the building and jack is down. If john throws a ball at an angle of 60° and with initial velocity 20 m/s . At what height will the ball reach after 2 s ?
- Vertical distance, $y = V_{oy} t - \frac{1}{2} gt^2$
- **Ans.= 15.04 m**

Problems

- **Question 1:** A body is projected and landed at the same level with a velocity of 20 ms^{-1} at 50° to the horizontal. Find

(i) Maximum height reached----- **Ans.= 11.97 m.**

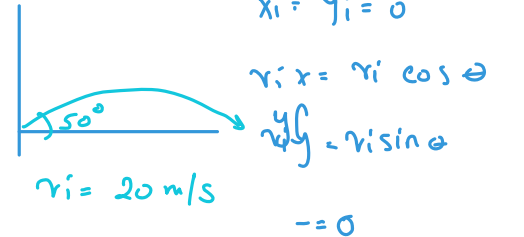
Total ← (ii) Time of flight and----- **Ans.= 3.126 s.**

$$\begin{aligned} \text{(i): Max H} &= \frac{v_i \sin \theta^2}{2g} \\ &= \frac{20 \sin 50^\circ^2}{2(9.81)} = 12 \text{ m} \end{aligned}$$

$$\text{(ii): } T_{\text{Total}} = \frac{2 v_i \sin \theta}{g}$$

• **Ans.= 15.04 m**

$$= \frac{2(20 \sin 50)}{9.81} = 3.1 \text{ s}$$

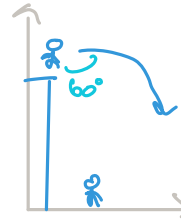


$$\begin{aligned} \text{(iii): Range} &= \frac{v_i^2 \sin 2\theta}{g} \\ &= \frac{20^2 \sin (2 \times 50)}{9.81} = 40 \text{ m} \end{aligned}$$

9.81

Problems

- Question 2:** John is on top of the building and Jack is down. If John throws a ball at an angle of 60° and with initial velocity 20 m/s. At what height will the ball reach after 2 s?



$$v_i = 20 \text{ m/s}$$

$$t = 2.0 \text{ s}$$

General launch

$$x_i = y_i = 0$$

$$r_{ix} = r_i \cos \theta$$

$$r_{iy} = r_i \sin \theta$$

- Vertical distance, $y = v_{oy} t - \frac{1}{2} g t^2$

$$\text{eg } y = v_i \sin \theta t - \frac{1}{2} g t^2$$

$$H = h_b + y = (20 \sin 60^\circ) t - \left(\frac{1}{2} \times 9.81 \times 2^2 \right)$$

$$= 15$$

Ans. = 15.04 m

2D (x, y)

in text book

Curved $\rightarrow$ projectile : $a_x = 0$, $a_y = -g$

Solved

General launch :

Example-4-5

$$x_i = y_i = 0$$

$$v_{ix} = \cos \theta$$

$$v_{iy} = \sin \theta$$

$$y_f = 0$$

- Chipping from the rough, a golfer sends a ball over a 3m high tree that is 14m away. The ball lands at the same level from which it was struck after traveling a horizontal distance of 17.8m on green of course.
- (a) if the ball left the club 54.0 above the horizontal and landed on the green 2.24s later, what was its initial speed?
- (b) How high was the ball when it passed over the tree?



$$\cancel{x_f} - \cancel{x_i} = v_i \cos \theta \cdot t \quad \rightarrow \quad v_i = \frac{17.8}{\cos(54^\circ) \times 2.24} = 13.5 \text{ m/s}$$

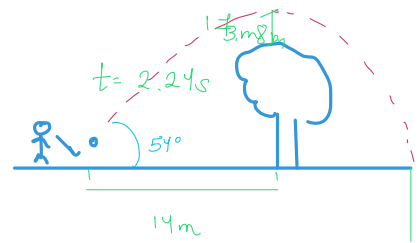
$$= \cancel{y_i} + \cancel{v_i \sin \theta} \cdot t - \frac{1}{2} g t^2$$

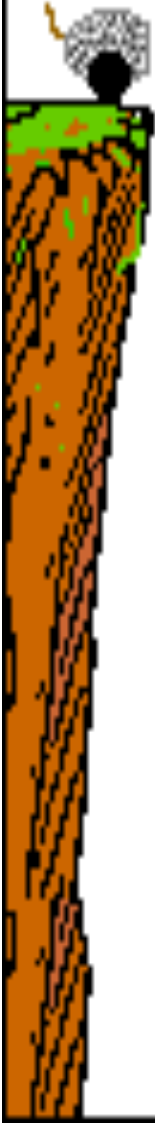
$$\cos \quad : \quad v_i \cos \theta \cdot t \rightarrow t = \frac{x_f}{v_i \cos \theta} = \frac{14}{13.5} = 1.04 \text{ s}$$

$$y_f = (13.5) \sin 54^\circ \cdot 1.04 - \frac{1}{2} (9.81) (1.04)^2 = 2$$

x_f

the tree?





PROJECTILE I

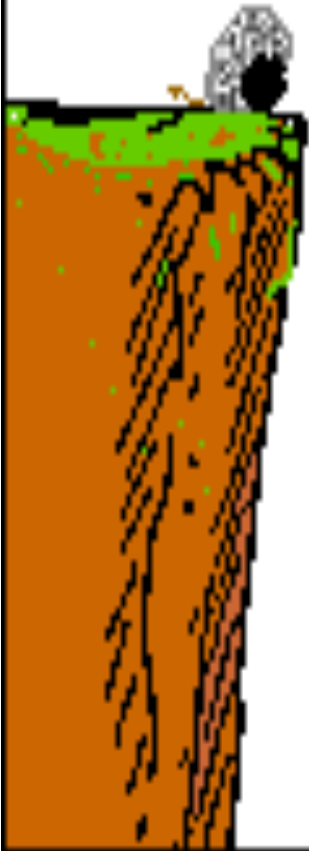
(SIMPLE PROJECTILE)

Zero
launch

$$t = \text{--} \text{ s}$$

$$v_x = \text{--} \text{ m/s} \quad v_y = \text{--} \text{ m/s}$$

PROJECTILE II (WITH ANGLE)



$$t = \text{--} \text{ s}$$

$$v_x = \text{--} \text{ m/s} \quad v_y = \text{--} \text{ m/s}$$

Summary of Chapter 4

- Components of motion in the x - and y -directions can be treated independently
- In projectile motion, the acceleration is $-g$
- If the launch angle is zero, the initial velocity has only an x -component
- The path followed by a projectile is a parabola
- The range is the horizontal distance the projectile travels