Scaler is just adding #s

mention the direction

Vectors

Chapter 3

Units of Chapter 3

3-1: Scalars Versus Vectors

3-2: The Components of a Vector مكونات المتجهات

3-3: Adding and Subtracting Vectors

3-4: Unit Vectors

3-5: Position , Displacement , Velocity and Acceleration Vectors.

x & y is either one of them

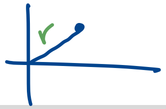
Scalars Versus Vectors

- **Scalar**: number with units   Just numbers
  - Examples: time, temperature

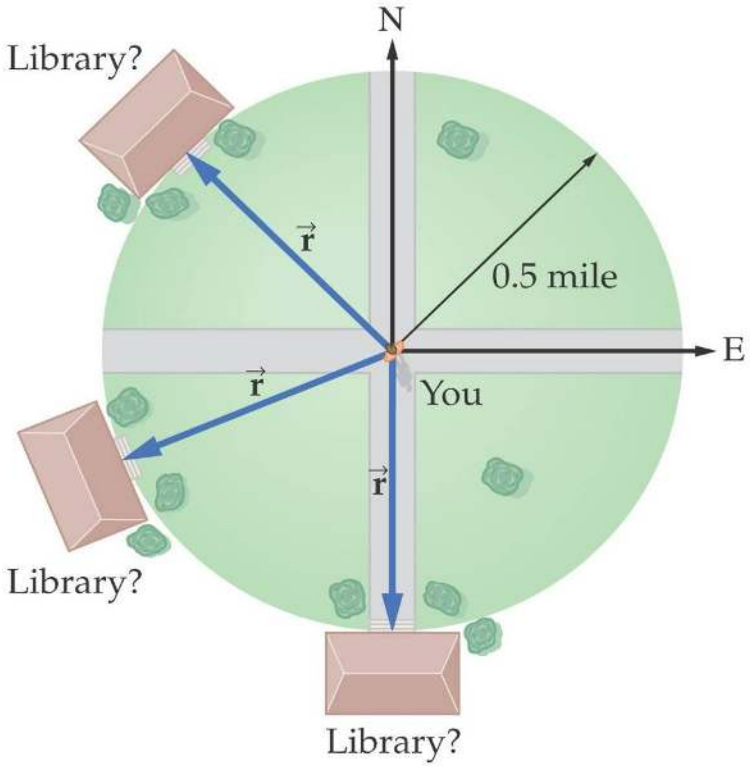
displace velocity, acceleration   # 0.5 mile   south of west

- **Vector**: quantity with **magnitude and direction**
  - How to get to the Library? You need to know how far and in which direction
  - Examples: position, displacement, velocity
  - $\vec{r}$ represents a vector of magnitude r (number) or $|\vec{r}|$

يمثل متجهًا للحجم

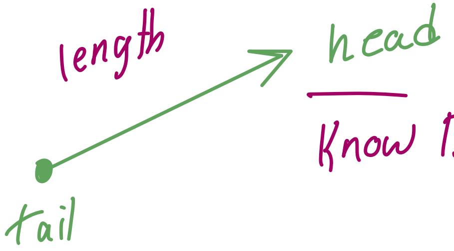


The position in 2 Dimension



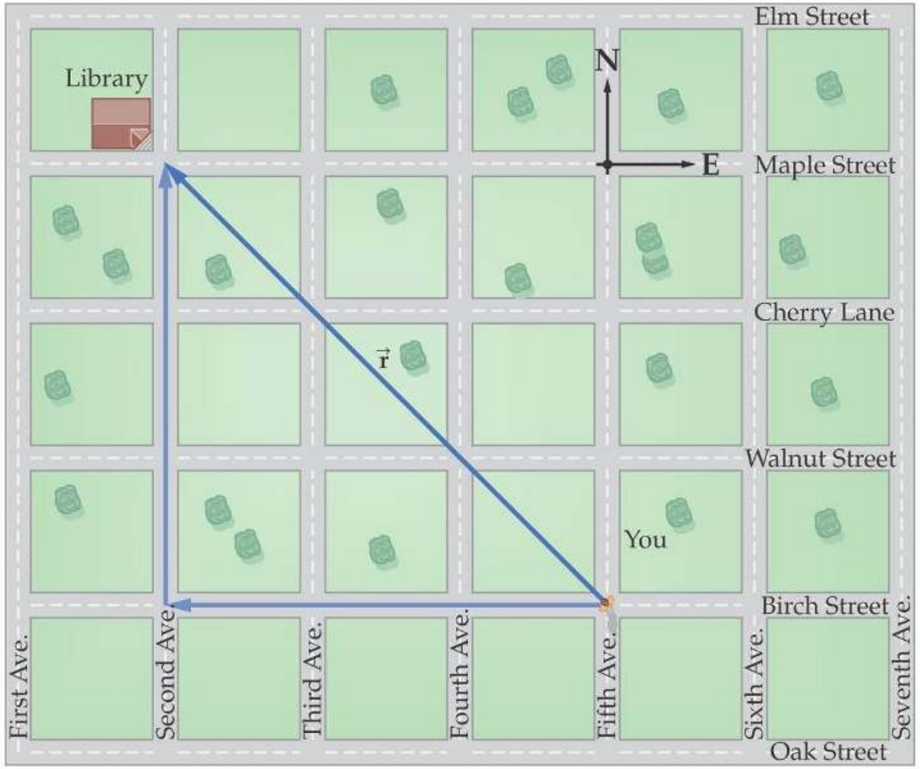
Describe vector either by component or magnitude & Direction

Vector Components



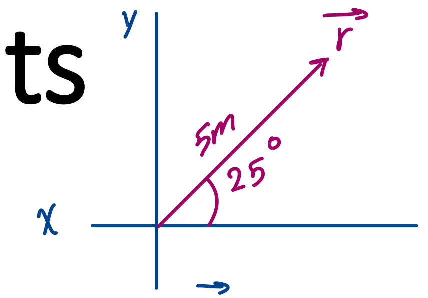
Know Direction by head.

- Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line!
- Adding the components of $\vec{r}$ leads to the same vector $\vec{r}$] means vector OR with Bold font r **r**
- To describe a vector, one needs:
  - It's length and direction (angle), OR
  - It's components (here two components; along the x-axis and the y-axis). For 3D vectors you need 3 components.

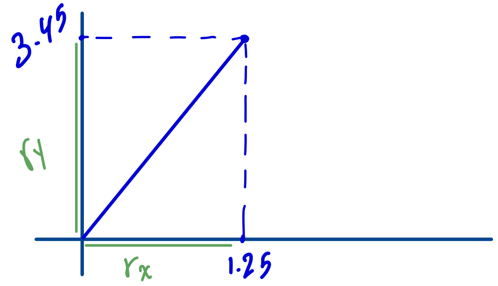


Vector Components

To present vector



given: $\vec{r} = 5\,m$
$\theta_r = 25°$



given $r_x = 1.25m$
$r_y = 3.45m$

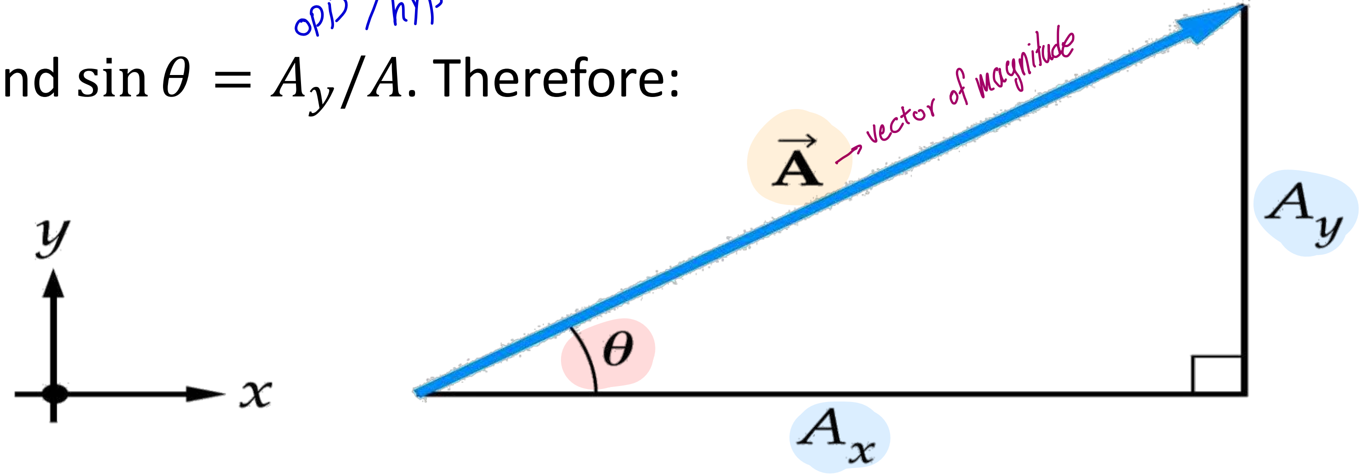
of r
hypotenuss

I. If you know the length (magnitude) and the angle, you can calculate the components using trigonometry. r_x & r_y

adj / hyp   opp / hyp

Note that $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$. Therefore:

- $A_x = A\cos\theta$
  adjesent
- $A_y = A\sin\theta$
  oppisite

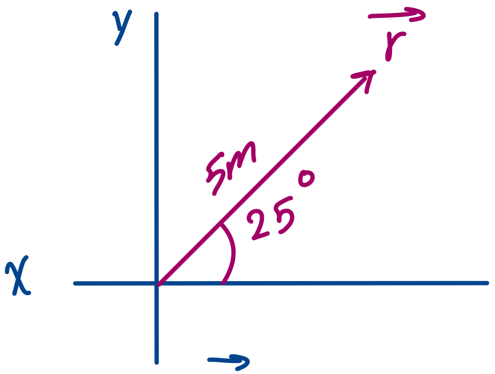


II. On the other hand, if you know the components you can calculate the length and the angle: (Pythagorean theorem)

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{|A_y|}{|A_x|}$$

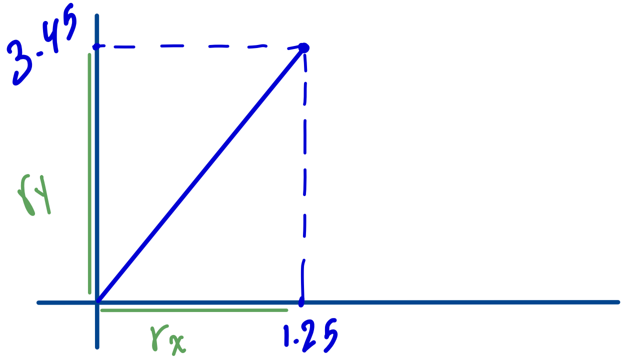
length

To present vector



given: $\vec{r} = 5m$

$\theta_r = 25^\circ$



given $r_x = 1.25m$

$r_y = 3.45m$

$$hyp^2 = adj^2 + opp^2$$

$$|r|^2 = r_x^2 + r_y^2$$

$$|r| = \sqrt{r_x^2 + r_y^2}$$

$$\tan\theta = \frac{opp}{adj} = \frac{r_y}{r_x}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

find

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad (1)$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad (2)$$

By magnitude

$$A_x = |A| \cdot \cos\theta \quad (3)$$

$$A_y = |A| \sin\theta \quad (4)$$

By component

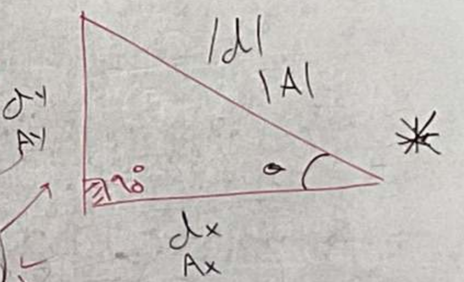
Quiz 5%
next week
on Tuesday 19/9/2023
Chapters 1 & 2

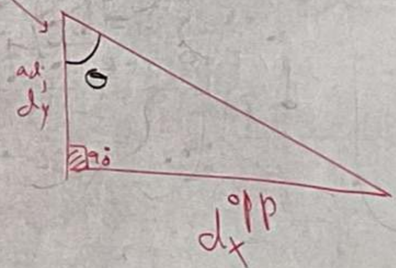
$$|\vec{d}| = \sqrt{(d_x)^2 + (d_y)^2}$$

$$\theta = \tan^{-1}\left(\frac{d_y^{\,opp}}{d_{x\,adj}}\right)$$

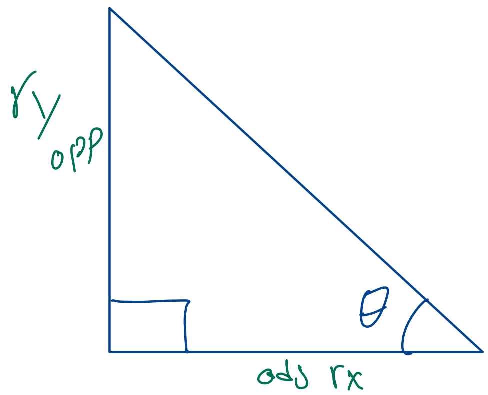
$$d_x^{adj} = |d| \cos\theta$$

$$d_y^{opp} = |d| \sin\theta$$



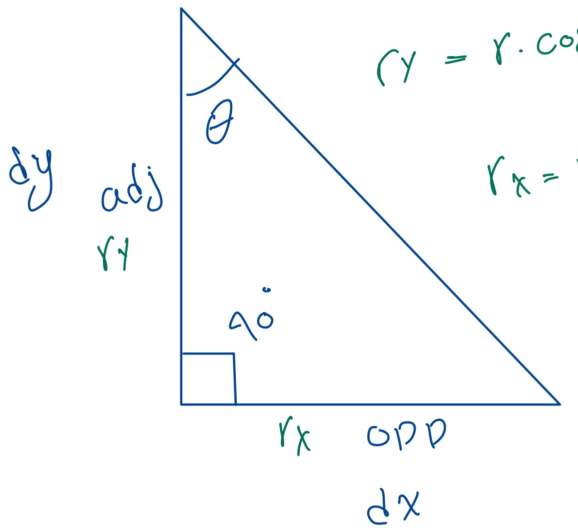


$$|\vec{d}| = \sqrt{dx^2 + dy^2}$$



$r_y = r \cdot \sin\theta$

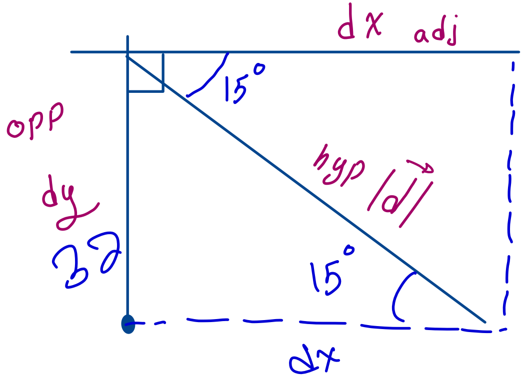
$r_x = r \cdot \cos\theta$



$r_y = r \cdot \cos\theta$

$r_x = r \cdot \sin\theta$

Problems



$\tan\theta = \dfrac{dy}{dx}$

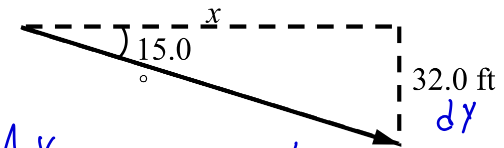
$dx = \dfrac{dy}{\tan\theta} = \dfrac{32}{\tan 15} = 119.4 = 119\ ft$

3 sf

Problem 5
The Press box at a baseball park is 32.0 ft above the ground. A reporter in the press box looks at an angle of 15.0° below the horizontal to see second base. What is the horizontal distance from the press box to second base?

$A_x = A \cdot \cos\theta$

$A_y = A \sin\theta$



$A = \dfrac{A_y}{\sin\theta} = 123.63\ ft$

$A_x = 123.63 \times \cos 15$

$= 119.4$

$= 119\ ft$

Answer: 119 ft

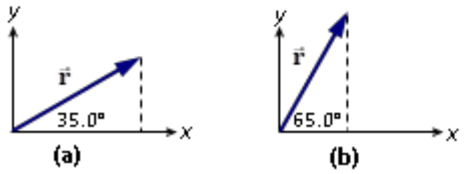
Problem 8
Find the x and y components of a position vector **r** of magnitude r = 75 m, if its angle relative to the x axis is (a) 35.0° and (b) 65.0°.

Answer: (a) r_x

(a) $r_x = |r| \cos\theta_a = 75 \times \cos 35 = 61 m$

$r_y = |r| \sin\theta_a = 75 \times \sin 35 = 43 m$

(a) $r_x = 61\ m$; $r_y = 43\ m$ (b) $r_x = 32\ m$; $r_y = 68\ m$

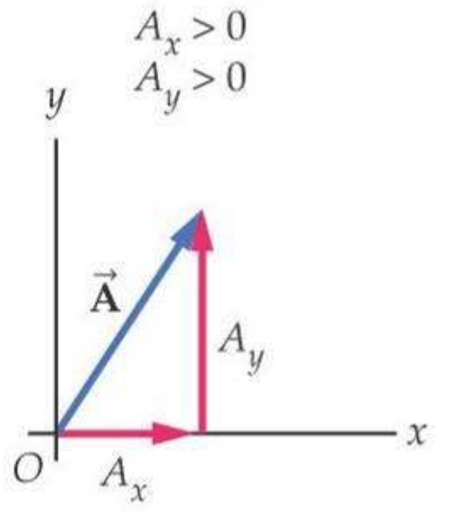


multiple choic

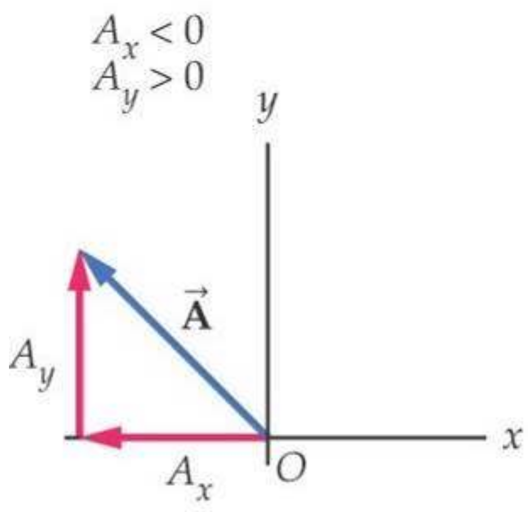
& problems.

Vector Components

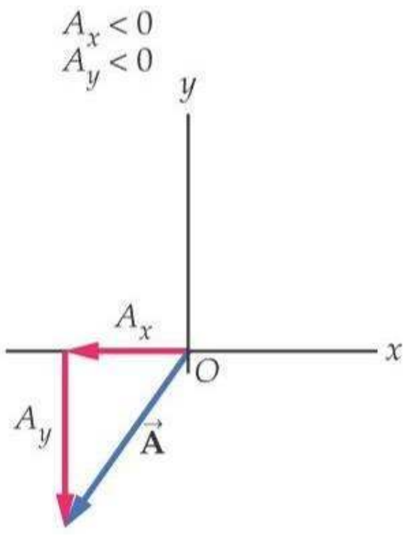
- Direction angle: the full angle with the positive *x*-axis counterclockwise. The signs of components will come automatically if you use the **full angle.** You can use the small angle (with +ve/−ve axes) but be careful with the signs.
- Signs of vector components.



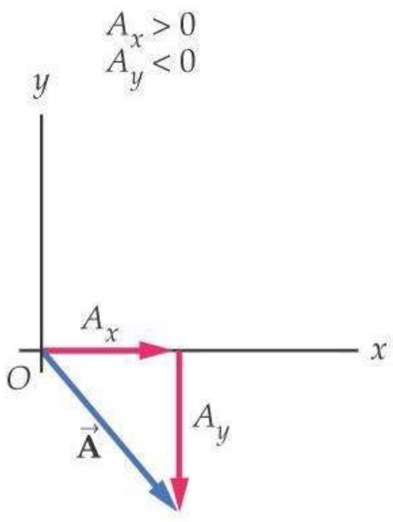
1st quadrant



2nd quadrant



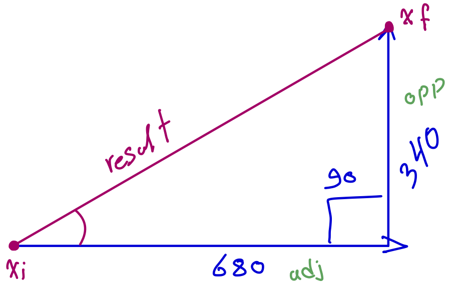
3rd quadrant



4th quadrant

$r_y = r\sin\theta = (150\text{ m})\sin(20.0°) = \boxed{51\text{ m}} = 0.14\text{ km}$

Problems



$|\vec{d}| = \sqrt{dx^2 + dy^2}$

displacment

$= \sqrt{680^2 + 340^2} = 760\text{ ft}$

Direction:

$\theta = \tan^{-1}\left(\frac{340}{680}\right) =$

$\frac{opp}{adj}$

Problem 14

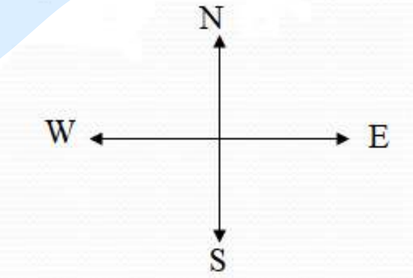
You drive a car 680 ft to the east, then 340 ft to the north. (a) What is the magnitude of your displacement? (b) estimate the direction of your displacement with a numerical calculation.

The Sign of component Show the quarter/

Answer: $r = 760\ ft$ $\theta = 27^0\ north\ of\ east$

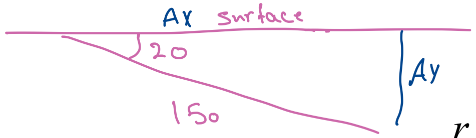
North of East

To know the place of θ



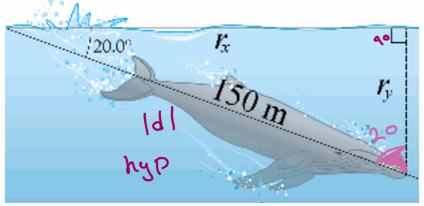
Problem 17

A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal. If the whale continues in a straight line for 150 m, (a) how deep it is, and (b) how far has it travelled horizontally?



Answer: The depth is given by r_y: $r_y = r\sin\theta = (150\text{ m})\sin(20.0°) = \boxed{51\text{ m}}$

(b) The horizontal travel distance is given by r_x: $r_x = r\cos\theta = (150\text{ m})\cos(20.0°) = \boxed{140\text{ m}} = 0.14\text{ km}$



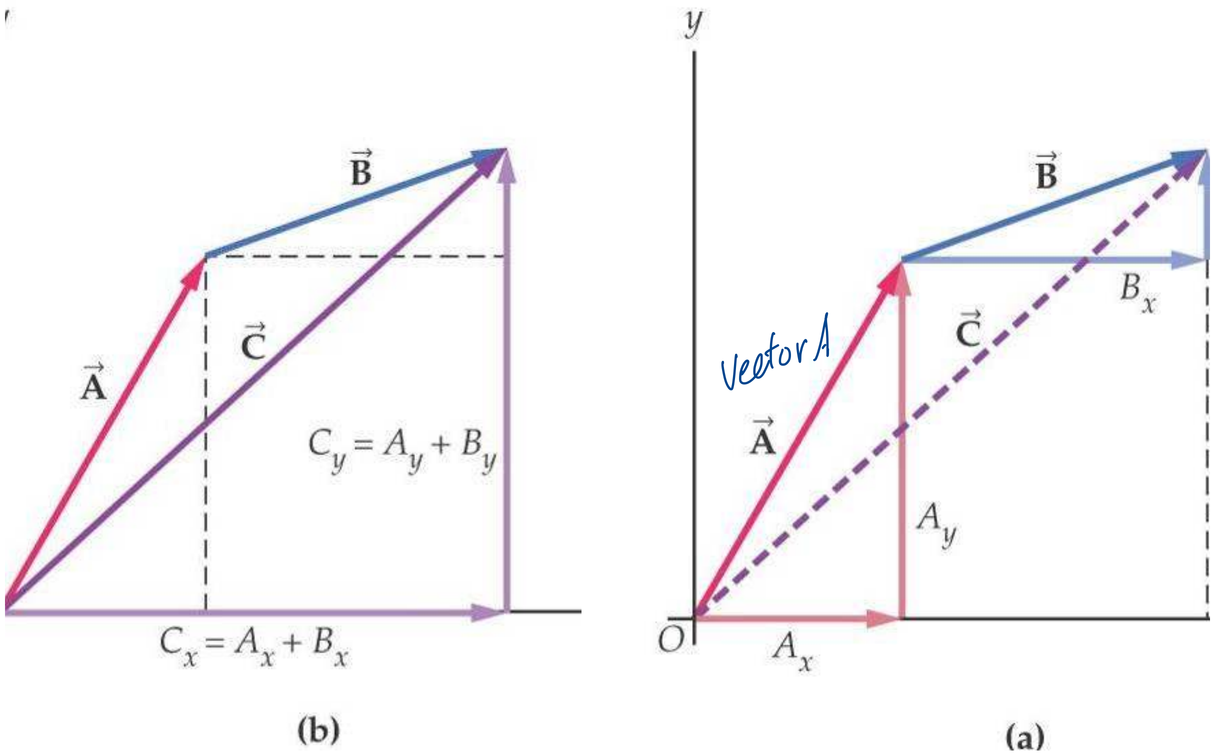
$dx = 150 \cdot \sin 20$

$dy = 150 \cdot \cos 20$

$\cdot r = \cdot \sin 20$ $A_y =$

Adding Vectors Analytically/Mathematically

- Vectors are added by components:
- If $\vec{C} = \vec{A} + \vec{B}$ then $C_x = A_x + B_x$ and $C_y = A_y + B_y$

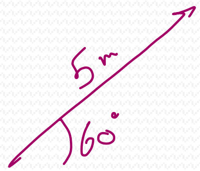


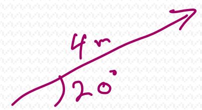
3-3 Adding and Subtracting Vectors

Adding Vectors Using Components:

1. Find the components of each vector to be added.
2. Add the *x*- and *y*-components separately.
3. Find the resultant vector.

Example





$|\vec{C}| = \sqrt{Cx^2 + Cy^2}$

$\sqrt{6.25^2 + 5.66^2} = 8.43$ m

- The vector $\vec{A}$ has a magnitude of 5.0^{o} m and the direction angle of 60^{o};the vector $\vec{B}$ has a magnitude of 4.00m and a direction angle of 20^{o}.

$C_x = A_x + B_x$

$C_y = A_y + B_y$

1) Find (a) magnitude of resultant vector '$\vec{C} = \vec{A}+\vec{B}$

(b) and, its direction angle

$A_x = A \cdot \cos\theta = 5 \cdot \cos 60 = 2.5$m

$A_y = A \cdot \sin\theta = 5 \cdot \sin 60 = 4.3$m

$B_x = B \cdot \cos\theta = 4 \cdot \cos 20 = 3.75$m

$B_y = B \cdot \sin\theta = 4 \cdot \sin 20 = 1.36$m

2) Find (a) magnitude of resultant vector $\vec{D} = \vec{A}-\vec{B}$

(b) and, its direction angle

$C_x = 2.5 + 3.75 = 6.25$ m

$C_y = 4.3 + 1.36 = 5.66$ m

± found The component

Solution: Ax = 2.50m, Ay = 4.33m , Bx= 3.76m, By= 1.37m

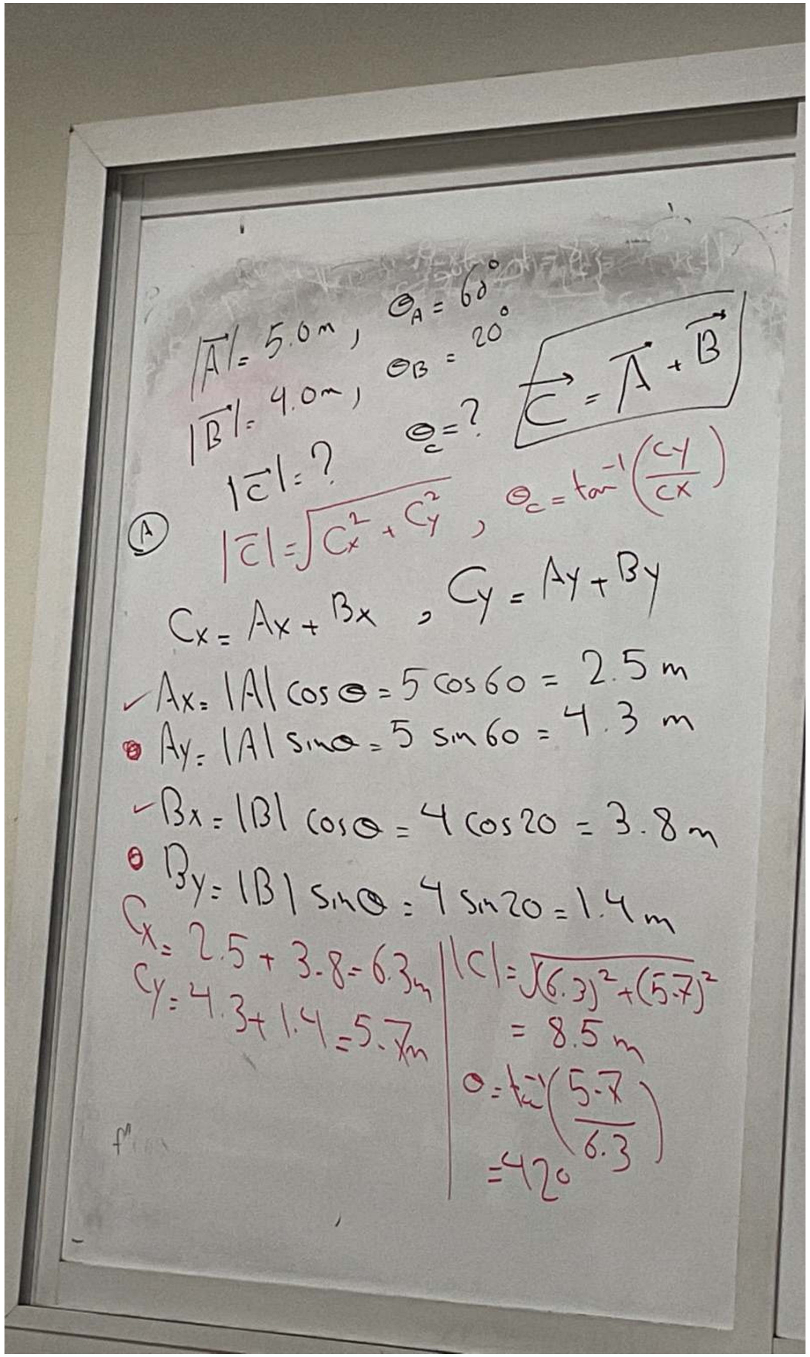
1) Cx = 6.26m, Cy= 5.70m

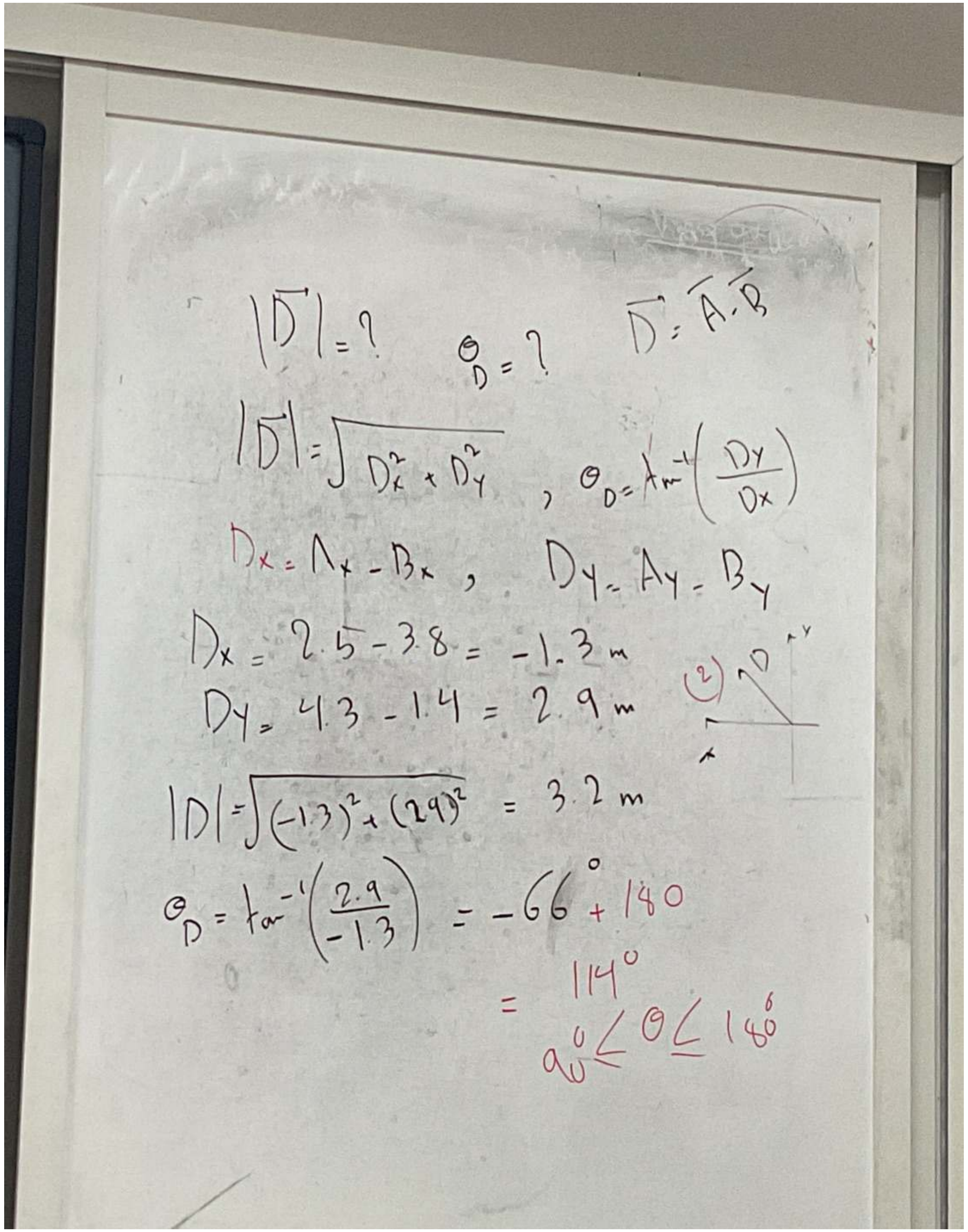
2° a1) $|C|$ =8.47m, a1) Θ= 42.3^{o}

2) Dx = -1.26 m, Dy= 2.96 m

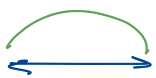
a2) $|D|$ = 3.22 m , a2) Θ= - 67^{o} = 113^{o}

+180 = 113°





negative vectors



are opp by 180
but, same length

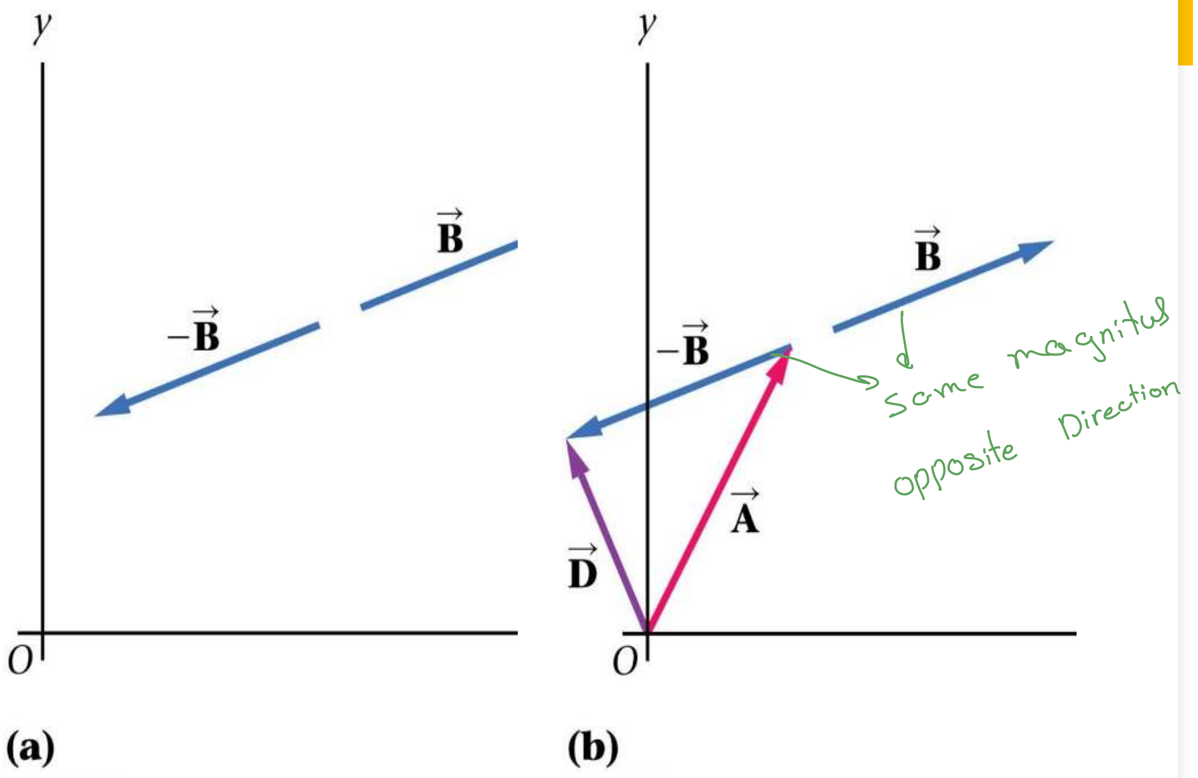
equal
same length & Direction

Subtracting Vectors

- Subtracting vectors works similar to adding vectors.

$\vec{D} = \vec{A} - \vec{B}$ is the same as $\vec{D} = \vec{A} + (-\vec{B})$.

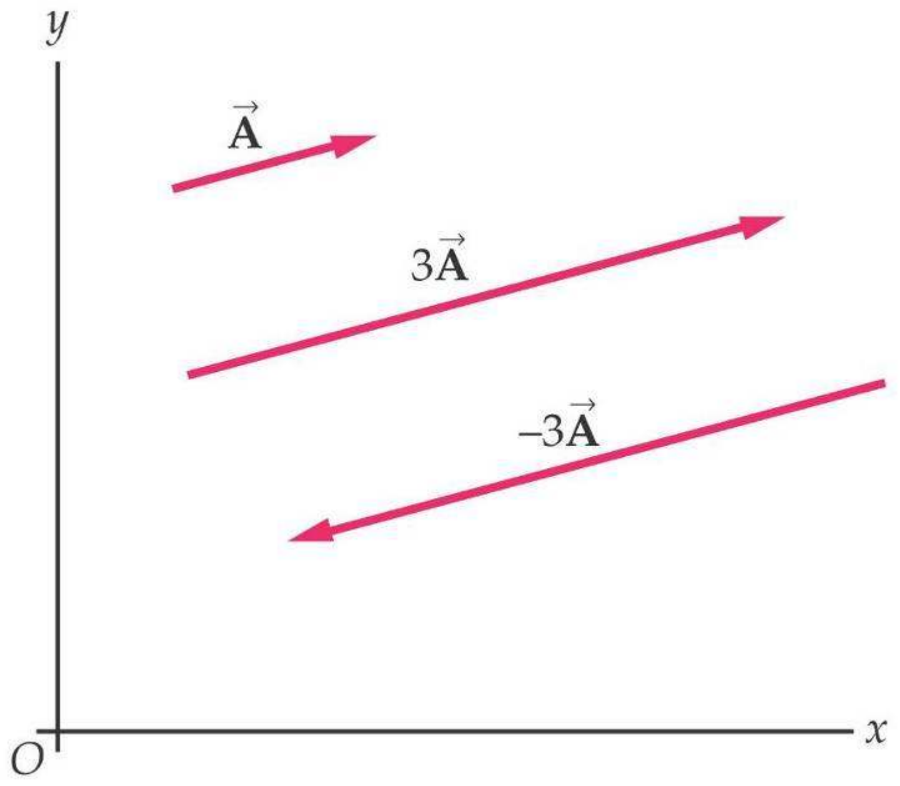
Note that $-\vec{B}$ **(negative of vector B)** has the same magnitude as $\vec{B}$, but with opposite direction



Multiplying Vectors by Scalars

Multiplying vectors

- The scalar multiplier changes the vector length, and the scalar sign can change the direction.
- This allows vectors to be factored into a unit vector and a multiplier. The unit vector specifies *only* the direction, while the multiplier provides the magnitude and units of the vector.



Note for multiples

We have 4 type of multiplication

multiply num with vector or ÷

① No $\times \vec{A}$ — Change of the Magnitut (length)

θ by 180° ← negative : change length & Direction

positive : only change in length

(increas * − Decreas ÷)

multiply unit vector with num (scaler)

② unit vector × No — change scaler to vector component

③ $\vec{A} \cdot \vec{B}$ (dot)

vector • vector

④ $\vec{A} \times \vec{B}$ (cross)

vector X Vector

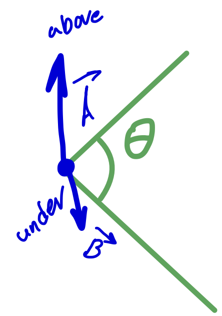
if the coefficient is positive — change in length

if the coefficient is negative # — chang in length & Direction by 180° (θ)

magnitued by فيثاغورس

$$\vec{A} \cdot \vec{B} = \overbrace{|A||B|}\cos\theta = \text{scaler}$$

result = just #



$$\vec{A} \times \vec{B} = \underbrace{|A||B|}_{\text{magnitued of } A \,\&\, B}\sin\theta\, \hat{n}$$

result = vector < magnitude, Direction

$\hat{n}$ عامودي Normal (vertical) → unit vector → up or under, out or iside

Righ hade resistance

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

information

②

$\hat{i} = \hat{x}$ → unit vector = 1 in magnitued

$\hat{j} = \hat{y}$ — $|\hat{x}|$, $\hat{y}$ & $\hat{z}$: its length one unit; direction: + positive, +x +y +z

$\hat{k} = \hat{z}$

There is No D-analysis

(scaler) $= A_x = (1.23\,m)\,\hat{x}$ Know it is vector

scaler $= A_y = (5.2m)\,\hat{y}$ Know it is vector

make scaler component as vector component multiply by unit vector

unit vector notation:

$$\vec{A} = \underbrace{(1.23m)\,\hat{x}}_{\text{length in } x} + \underbrace{(5.3\,m)\,\hat{y}}_{\text{length in } y}$$

&

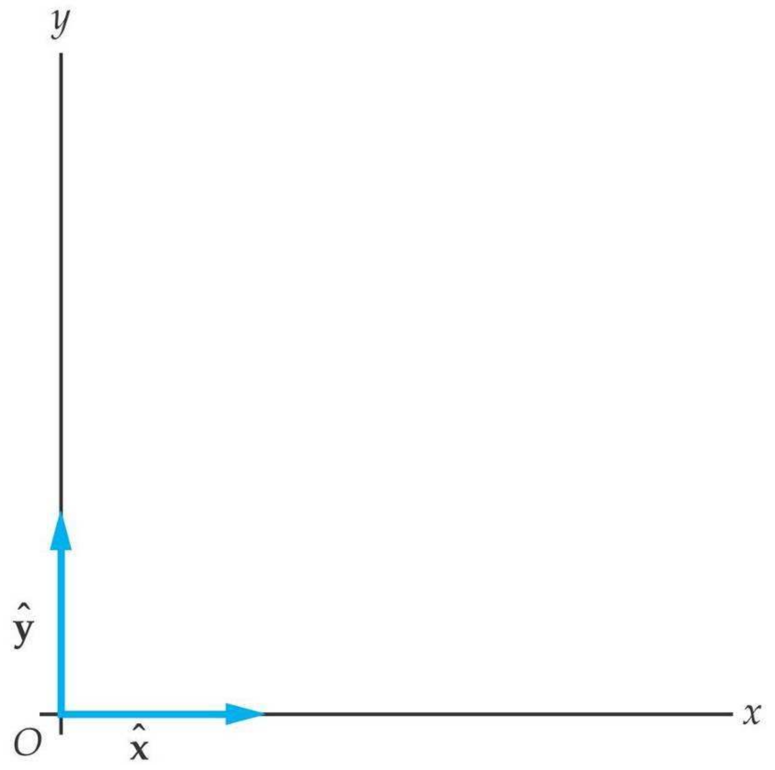
Representing Vectors Mathematically

- The vector A with an x-component of A_x and a y-component of A_y is written mathematically as:

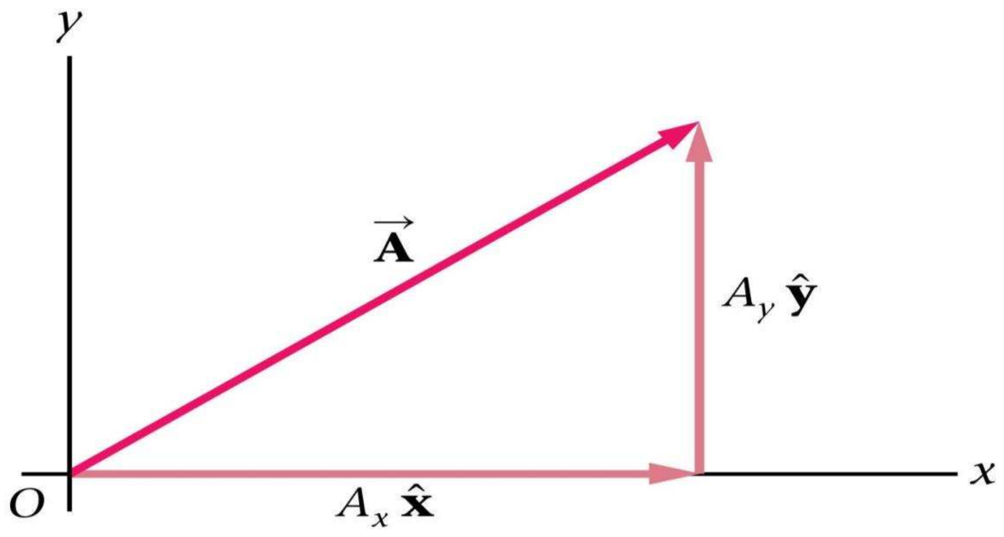
$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

- Here: $\hat{x}$ and $\hat{y}$ are the units' vectors.
- Unit vectors are dimensionless vectors of unit length (magnitude of one).

$\hat{x}$ points along the x-axis and $\hat{y}$ points along the y-axis. They are used to indicate the directions of the components.



Adding vectors using the unit vector notation



$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{x} + \left(A_y + B_y\right)\hat{y}$$
$$\vec{D} = \vec{A} - \vec{B} = (A_x - B_x)\hat{x} + \left(A_y - B_y\right)\hat{y}$$

This works for any number of vectors: add the x-components together and the y-components together

1D

$$\Delta x = x_f - x_i$$

$$\Delta y = y_f - y_i$$

2D

$$\Delta \vec{r} = \vec{r_f} - \vec{r_i}$$

$$(r_{f_x} - r_{i_x}) \;+\; \& \;(r_{f_y} - r_{i_y})$$

$$= \Delta \vec{r_x} + \Delta \vec{r_y}$$

1D

* $\Delta x = x_f - x_i$

$\Delta y = y_f - y_i$

* $\vec{V}_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$

$\vec{a} = \dfrac{\Delta V}{\Delta t} = \dfrac{V_f - V_i}{t_f - t_i}$

$\vec{r_i} = (r_{x_i})\hat{x} + (r_{y_i})\hat{y}$

$\vec{r_f} = (r_{x_f})\hat{x} + (r_{y_f})\hat{y}$

2D

$\vec{\Delta r} = \vec{r_f} - \vec{r_i}$

$= (r_{f_x} - r_{ix}) \; \overset{+}{\&} \; (r_{f_y} - r_{iy})$

$= \vec{\Delta r_x} + \vec{\Delta r_y}$

$\Delta r = r_f - r_i$

$\left(\dfrac{r_{fx} - r_{ix}}{T}\right) + \left(\dfrac{r_{fy} - r_{iy}}{T}\right)$

$=$

$\vec{V}_{avg} = \dfrac{\underset{\smile}{\Delta r}}{\Delta t}$ Displacement $= \left(\dfrac{r_{f_x} - r_{ix}}{\Delta t}\right) \; \overset{+}{\&} \; \left(\dfrac{r_{f_y} - r_{iy}}{\Delta t}\right)$

$\vec{a} = \dfrac{\Delta v}{\Delta t} = (\vec{V_x})\hat{x} \; \& \; (\vec{V_y})$

$= \dfrac{\Delta V_x}{\Delta t} + \dfrac{\Delta V_y}{\Delta t}$

$= \dfrac{V_{fx} - V_{ix}}{\Delta t} + \dfrac{V_{fy} - V_{iy}}{\Delta t} = a_x + a_y$

$r_i = (2.00\,m)\hat{x} \; \& \; (3.5\,m)\hat{y}$

$r_f = (-3.00\,m)\hat{x} + (5.50\,m)\hat{y}$

$\Delta t = 3.00\,s$

$\vec{V}_{avg} = \dfrac{\Delta r}{\Delta t} = \dfrac{r_f - r_i}{t_f - t_i}$

$\vec{V}_{avg} = \left(\dfrac{-3-2}{3}\right)\hat{x} + \left(\dfrac{5.50 - 3.5}{3}\right)\hat{y}$

$= \left(\dfrac{-5\,m}{3\,s}\right)\hat{x} + \left(\dfrac{2\,m}{3\,s}\right)\hat{y}$

$= (-1.67\,m/s)\hat{x} + (0.667\,m/s)\hat{y}$

$|\vec{V}| = \sqrt{(V_x)^2 + (V_y)^2}$

$\theta_v = \tan^{-1}\left(\dfrac{V_y}{V_x}\right)$

Displacement Vector

- The **position** vector $\vec{r}$ points from the origin to the location in question.

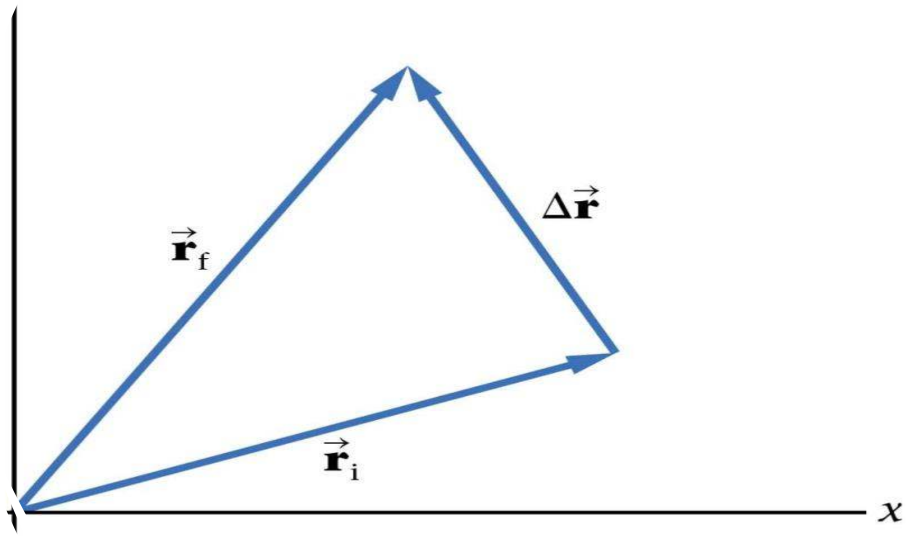
- The **displacement** vector $\Delta\vec{r}$ points from the original/initial position to the final position:

$|\vec{r}|\ \sqrt{r_x^2 - r_y^2}$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$(r_{fx} - r_{ix}) + (r_{fy} - r_{iy})$$

$$r_x + r_y$$



Velocity

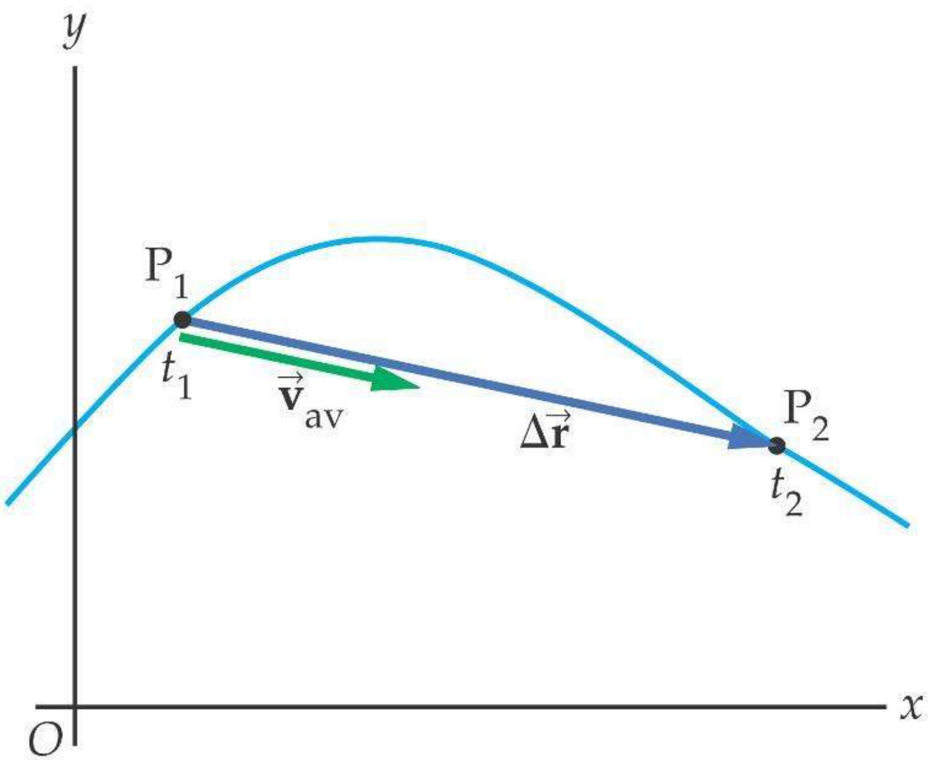
Average Velocity:

- In one dimension: $v_{av} = \dfrac{\Delta x}{\Delta t}$

- In general: $\vec{v}_{av} = \dfrac{\Delta \vec{r}}{\Delta t}$ $\dfrac{(r_{fx} - r_{ix})}{T} + \dfrac{r_{fy} - r_{iy}}{T}$

$\vec{V} = \sqrt{v_x^2 + v_i^2}$ $\qquad V_x + V_y$

$\vec{v}_{av}$ is in the same direction as $\Delta \vec{r}$

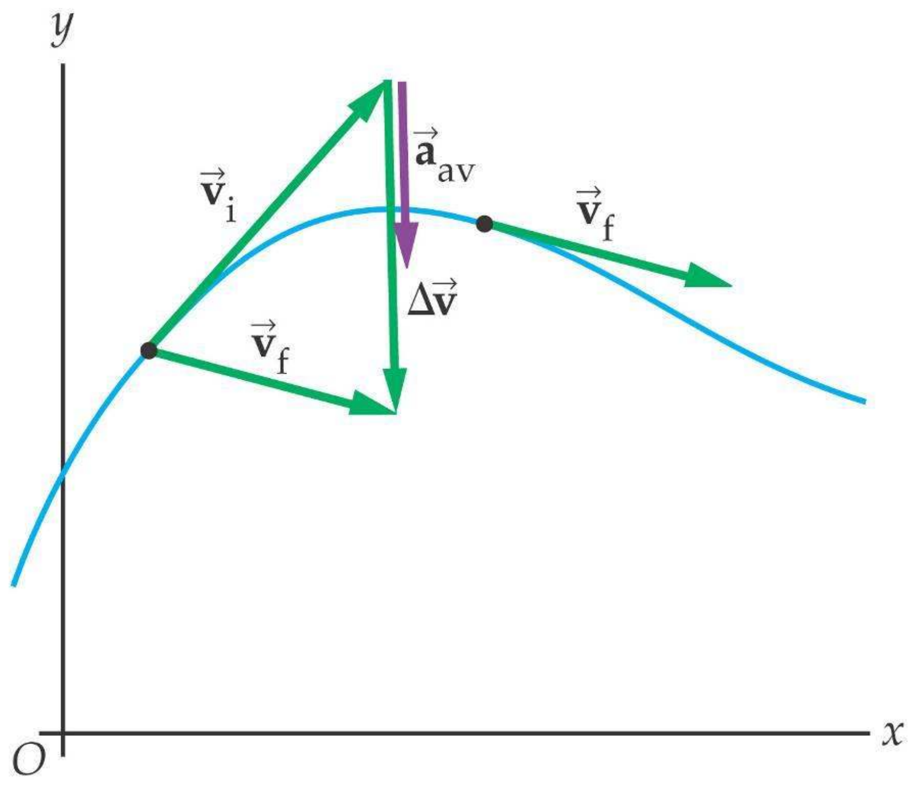


Acceleration

- Average Acceleration:

$$\vec{a}_{\mathrm{av}} = \frac{\Delta \vec{v}}{\Delta t} \quad \frac{V_{fx} - V_{ix}}{T} + \frac{V_{fy} - V_{iy}}{T}$$

$\vec{a}_{\mathrm{av}}$ is in the same direction as the change in velocity $\Delta \vec{v}$



Problems

$\vec{V} = \frac{\Delta r}{\Delta T} = \frac{r_f - r_i}{T_f - T_i} = \frac{r_{fx} - r_{ix}}{T} + \frac{r_{fy} - r_{iy}}{T}$

$|V| = \sqrt{V_x^2 + V_f^2}$

$\sqrt{(1.6)^2 + 0.66^2}$

$= \frac{-3 - 2}{3} + \frac{5.5 - 3.5}{3} = \underset{V_x}{-1.6\hat{x}} + \underset{V_y}{0.66\hat{y}}$

$\tan^{-1}\left(\frac{V_y}{V_x}\right)$

$\tan^{-1}\left(\frac{0.66}{-1.6}\right) = -22^\circ$

157°

Exercise 6: A dragonfly is observed initially at the position $\vec{r_i} = (2.00\text{ m})\,\hat{x} + (3.50\text{ m})\,\hat{y}$. Three seconds later it is at the position $\vec{r_f} = (-3.00\text{ m})\,\hat{x} + (5.50\text{ m})\,\hat{y}$. What was the dragonfly's average velocity during this time? $T = 3\text{ s}$

Exercise:7 A sailboat has coordinates ($\overset{x}{130}$ m, $\overset{y}{205}$ m) at $t_1 = 0.0$ s. Two minutes later its position is ($\overset{x}{110}$ m, $\overset{y}{218}$ m). Find its average velocity.

$1\text{ min} = 60\text{ sec}$

$2 = 120\text{ sec}$

$\vec{V} = \frac{\Delta r}{T}$ $\quad \Delta r = r_f - r_i = r_{fx} - r_{ix} + r_{fy} - r_{iy}$

$(110 - 130) + (218 - 205) = \underset{}{\overset{r_x}{-20}} \overset{r_y}{+13}$

$\vec{V} = \frac{r_{fx} - r_{ix}}{T} + \frac{r_{fy} - r_{iy}}{T} = \frac{-20}{120} + \frac{13}{120} = \underset{V_x}{-0.1\bar{6}} + \underset{V_y}{0.108}$

$-10 + 6.5$

$|\vec{V}| = \sqrt{V_x^2 + V_y^2} = \sqrt{0.16^2 + 0.10^2} = 0.1886\text{ m}$

$\tan^{-1}\left(\frac{0.10}{-0.16}\right) = -32^\circ + 180 = 147.9 = 148^\circ$

for knowledge

Scalar or Dot Product

إلغى رحلتي

- Dot (or Scalar) Product:

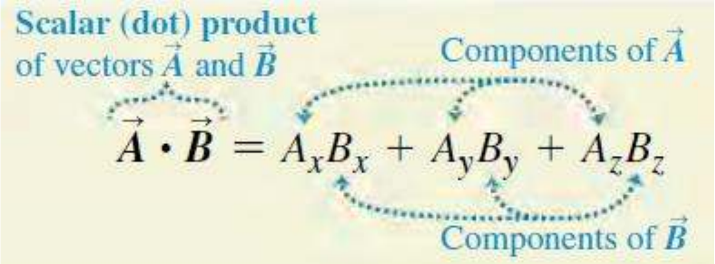
$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$$

  - θ is the angle between the two vectors.
  - The result is a scalar.
- (note: $\hat{x} \cdot \hat{x} = 1, \hat{x} \cdot \hat{y} = 0,$ ......)

The following laws are valid

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$



Vector or Cross Product

- Cross product defined as:

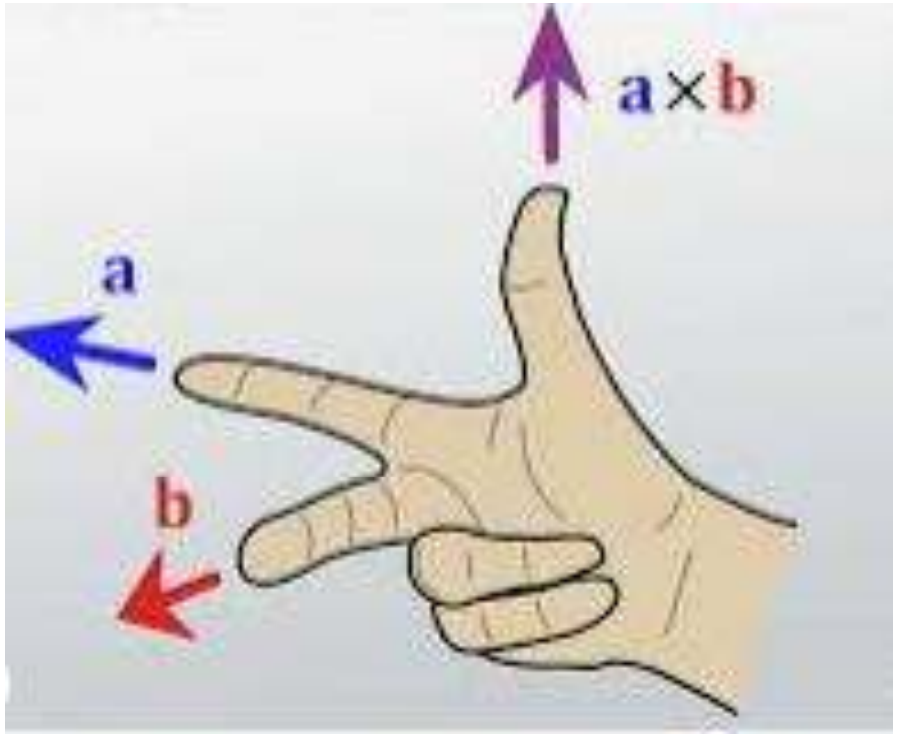
$$\vec{A} \times \vec{B} = |A|\,|B| \sin\theta\, \hat{n}$$

  - θ is the angle between the two vectors.
  - The result is a vector, perpendicular to the plane in which the two vectors exist. (for example: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{x} = 0$)
  - Where n is a unit vector indicating the direction of $\vec{A} \times \vec{B}$.

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

The following Laws are Valid:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$
$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$



Exercises

Exercise 3-7: Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{v} = (3.7\ \mathrm{m/s})\hat{x} + (-1.3\ \mathrm{m/s})\hat{y}$.

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{3.7^2 + (-1.3)^2} = 3.92 = 4\ \mathrm{m/s}$$

$$\tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-1.3}{3.7}\right) = -19.36^\circ$$ not sure

$|\hat{x}| = 1$ + X axis always positive

$|\hat{y}| = 1$ + Y axis

$\vec{A} \cdot \vec{B} = |A| \cdot |B| \cdot \cos\theta$ (maybe scaler)

$\vec{A} \times \vec{B} = |A||B| \sin\theta \, \hat{n}$ (vector)

Right hand to check the Direction of vector to know if Direction is in or out paper