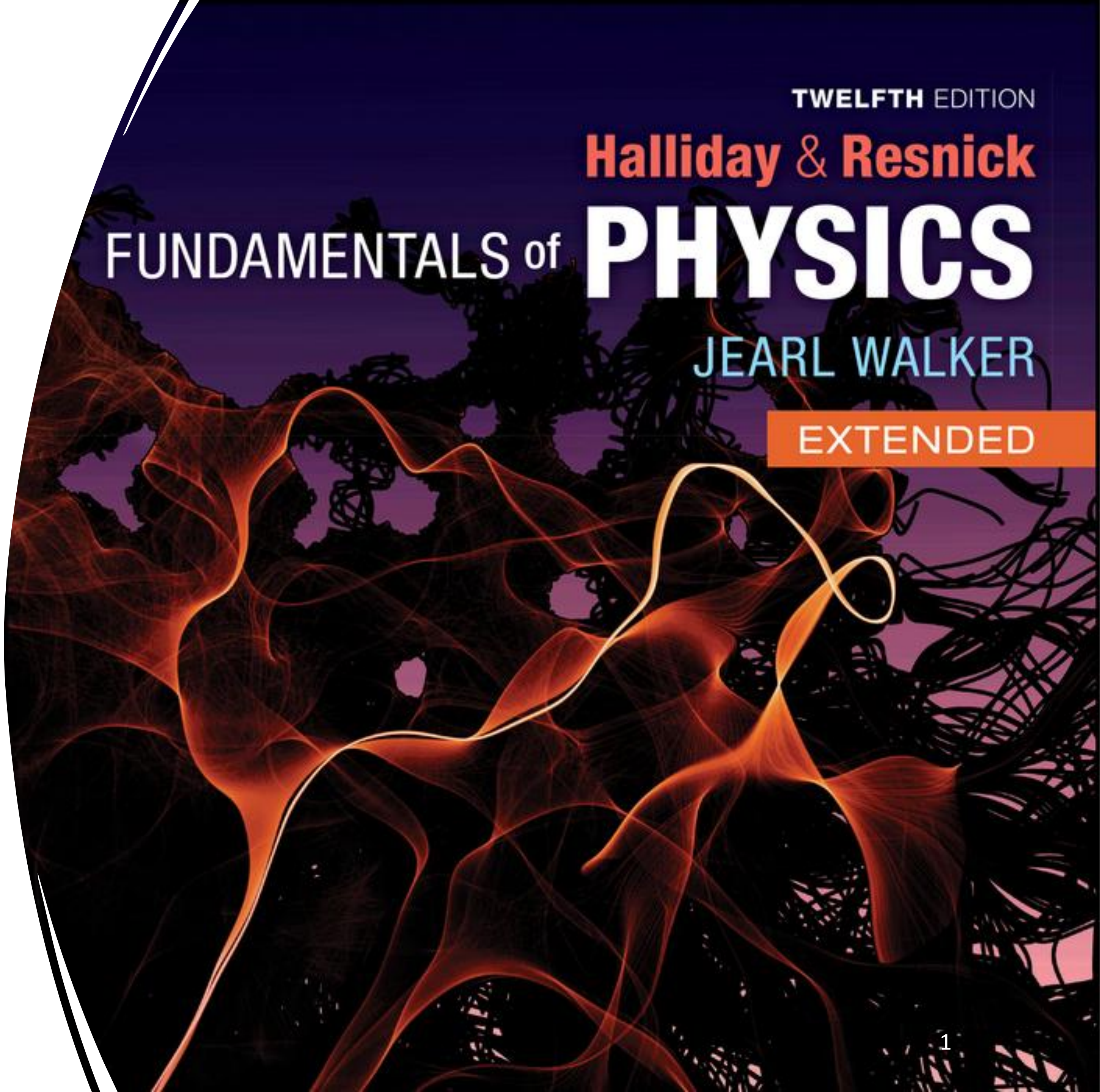
Tue 7 Apr

# Chapter 27

## Circuits

***Fundamentals of Physics*, Twelfth Edition. Halliday & Resnick, Walker**



# Chapter 27 Circuits

Generator ⇒ output : electric energy
transforming other energies
(drycell, windmills, fuel)
chemical

# Section 27.1 Single Loop Circuits

# EMF Device as "Charge Pump"

To produce a steady flow of charge, you need a "charge pump," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals. We call such a device an **emf device**, and the device is said to provide an emf $\varepsilon$ which means that it does work on charge carriers.

Figure (next slide) shows an emf device (consider it to be a battery) that is part of a simple circuit containing a single resistance $R$. The emf device keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled −). We can represent the emf of the device with an arrow that points from the negative terminal toward the positive terminal.

# EMF and Current in Single-Loop Circuits

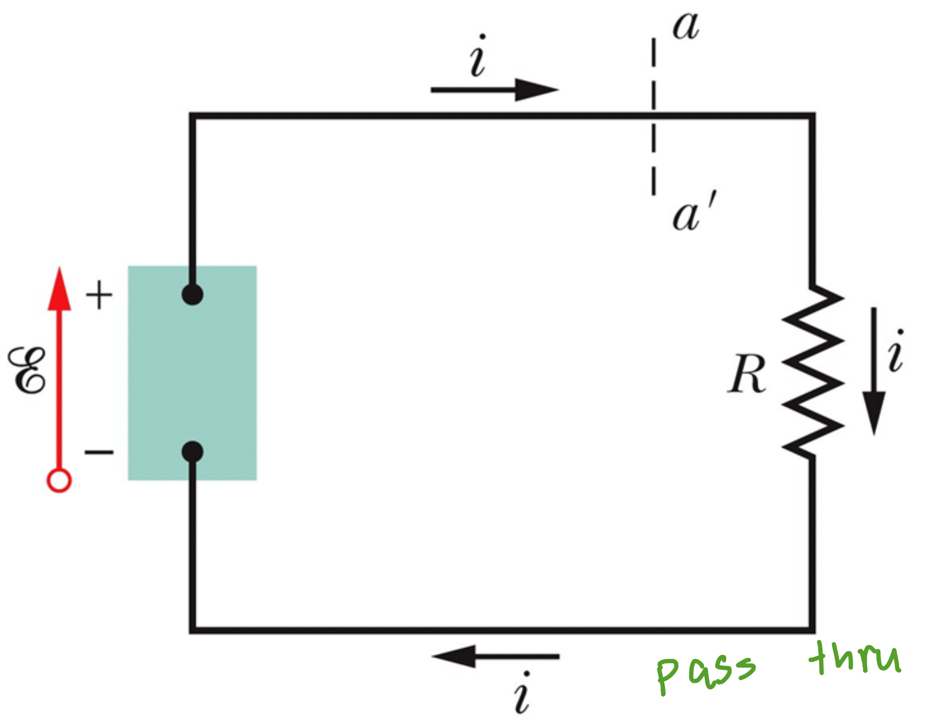
A small circle on the tail of the emf arrow distinguishes it from the arrows that indicate current direction.

Where:

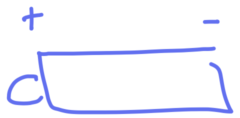
$\mathscr{E}$ = emf (voltage)

i = current

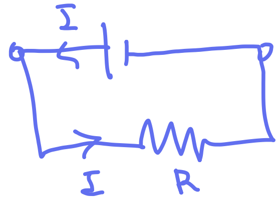
R = resistance




pass thru resistor



$\mathcal{E} = emf$




energy from: battery (generator)
given to: resistor

$E_{battery} = E_{resistor}$

$P \times \cancel{t} = P \times \cancel{t}$

$\cancel{I}\mathcal{E} = I^{\cancel{2}}R$

$\mathcal{E} = IR$

$I = \frac{\mathcal{E}}{R}$

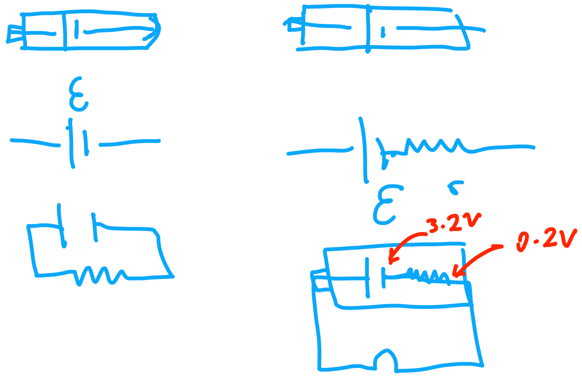
time same

time taken to give is the same as time it takes to use

that's why it gets cancelled out

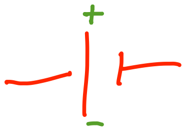
Ideal

Real

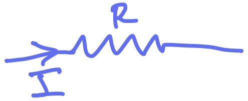



if company sells you 3V battery but it takes 0.2 in the process

they're forced to abide by their promise so they add more voltage so that net volt is correct & is the number promised



+ : high potential
- : low potential




current travel from high to low potential

# EMF as Work per Unit Charge

An emf device does work on charges to maintain a potential difference between its output terminals. If $dW$ is the work the device does to force positive charge $dq$ from the negative to the positive terminal, then the emf (work per unit charge) of the device is

$$\varepsilon = \frac{dW}{dq} (definition\ of\ \varepsilon)$$

An **ideal emf device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the emf.

A **real emf device** has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.

# Energy Method for Calculating Current in a Single-Loop Circuit

Equation, $P = i^2R$, tells us that in a time interval $dt$ an amount of energy given by $i^2R\,dt$ will appear in the resistor (shown in the figure) as thermal energy. This energy is said to be **dissipated**. (Because we assume the wires to have negligible resistance, no thermal energy will appear in them.)

During the same interval, a charge $dq = i\,dt$ will have moved through battery B, and the work that the battery will have done on this charge is

$$dW = \varepsilon dq = \varepsilon i dt$$

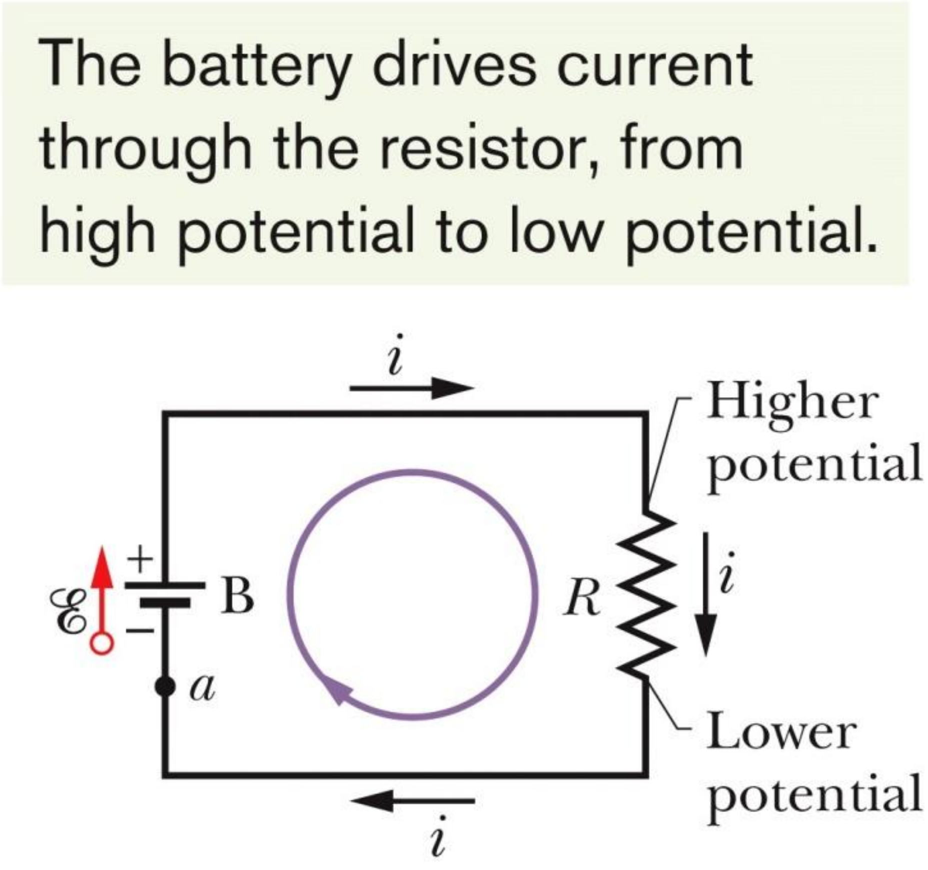
# Conservation of Energy Source vs. Circuit

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\varepsilon i dt = i^2 R dt$$

Which gives us:

$$i = \varepsilon / R$$





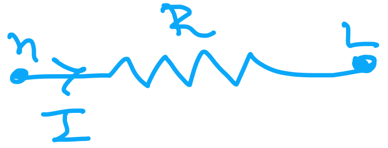
# Current Calculation Methods for Single-Loop Circuits

**Loop Rule:** The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

**Resistance Rule:** For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

**Emf Rule:** For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$, while in the opposite direction is $+\mathcal{E}$.

# Source Internal Resistance in A Circuit



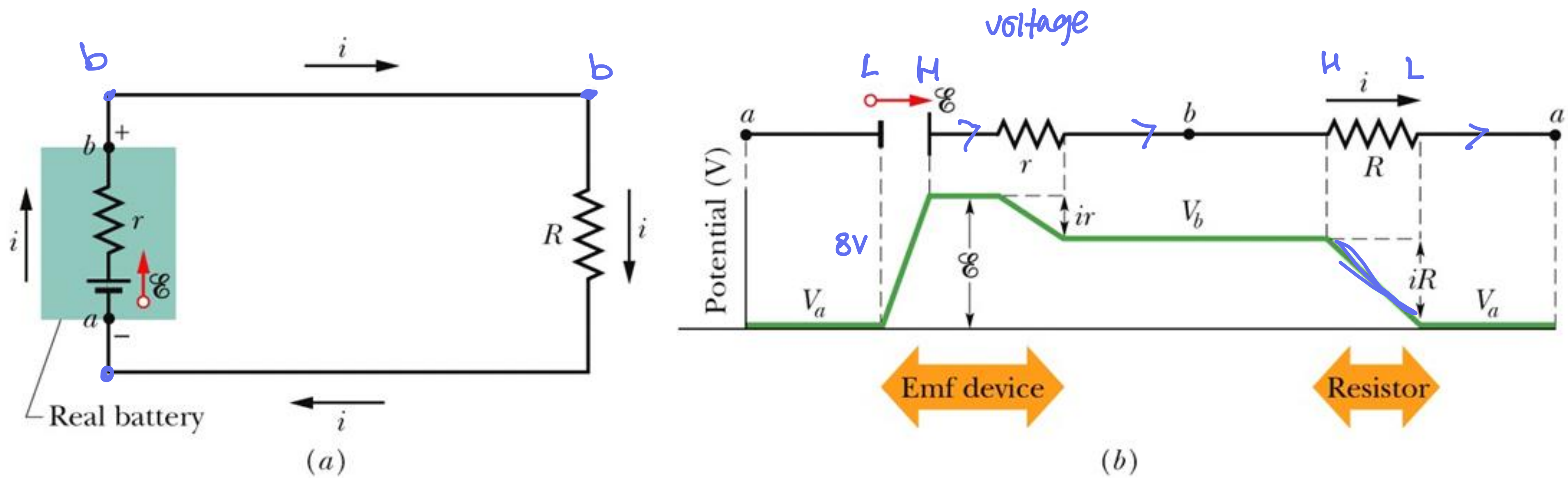
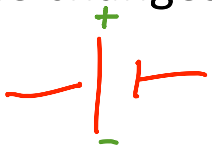





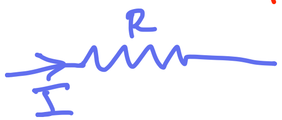

Figure (a) shows a real battery, with internal resistance *r*, wired to an external resistor of resistance *R*. The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an un removable feature of the battery. Figure (b) shows graphically the changes in electric potential around the circuit.




If we apply the loop rule clockwise beginning at point *a*, the changes in potential give us:

$\mathcal{E} - ir - iR = 0$ Solving for current, we find: $i = \dfrac{\varepsilon}{(R+r)}$

+ : high potential
− : low potential




current travel from high to low potential

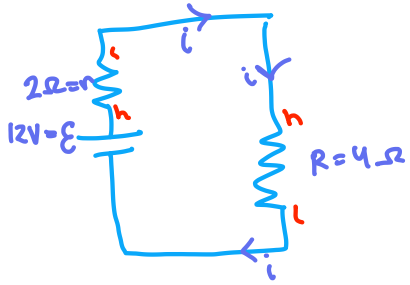
Wed 8 Apr → Came back to this slide only
rest start from Checkpoint 2

# Potential Difference Between Points in a Single Loop Circuit

Sum of potential is 0 since however much you give, you take exactly the same amount

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

potential diff either




$-iR = 0$

$12 - i \times 2 - i \times 4 = 0$

$12 - 6i = 0$

$\frac{12}{6} = \frac{\cancel{6}i}{\cancel{6}}$

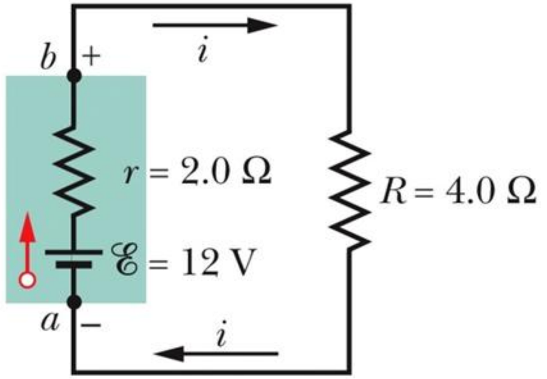
$\boxed{i = 2A}$

**Potential Difference across a real battery:** In the Figure, points *a* and *b* are located at the terminals of the battery. Thus, the potential difference $V_b - V_a$ is the terminal-to-terminal potential difference $V$ across the batter and is given by:

$$V = \varepsilon - ir$$

loop rule: $\Sigma = 0$

The internal resistance reduces the potential difference between the terminals.

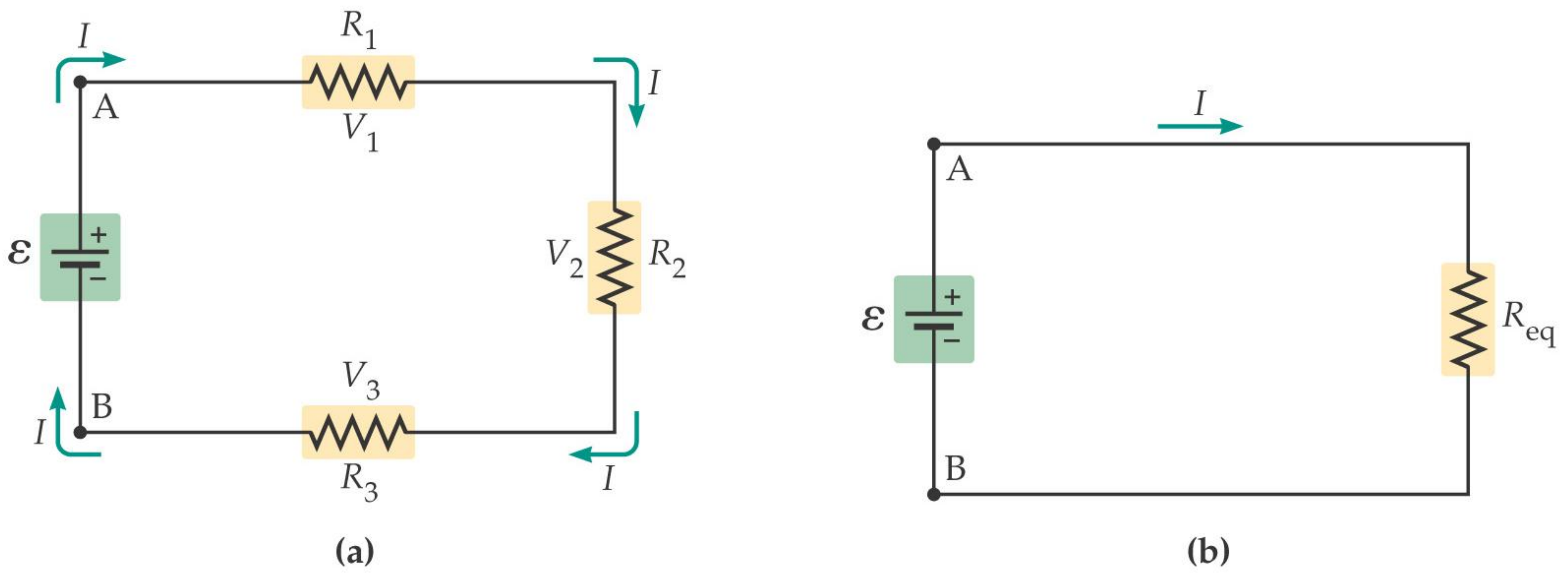


# Section 27.2 Multi-Loop Circuits

series: same current
parallel: same potential difference

# Resistors in Series

Resistors connected end to end are said to be in series. They can be replaced by a single equivalent resistance without changing the current in the circuit.



Since the current through the series resistors must be the same in each, and the total potential difference is the sum of the potential differences across each resistor, we find that the equivalent resistance is:

**Equivalent Resistance for Resistors in Series**

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R$$

SI unit: ohm, $\Omega$

# Resistors in Series

- The current in the circuit is the same for each user

$$I_{eq} = I_1 = I_2 = I_3 = \ldots$$

- The sum of the voltage drop is equal to the total voltage drop

$$V_{eq} = V_1 + V_2 + V_3 + \ldots$$

- The effective resistance is the sum of all resistors in the series

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

# Checkpoint #1

$V = iR$

same so R determines

If $R_1 > R_2 > R_3$, rank the three resistances according to:

$R_1 > R_2 > R_3$

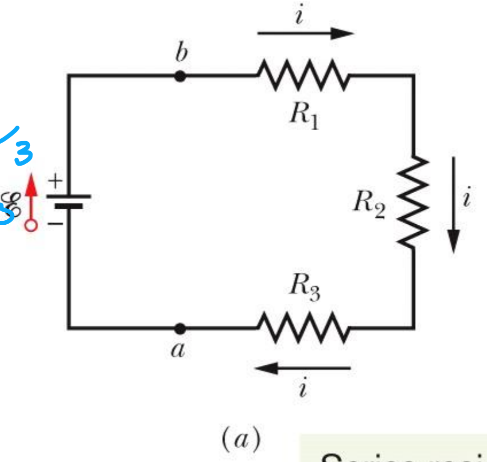
$V_1 > V_2 > V_3$

(a) the current through them and (b) the potential difference across them, greatest first.

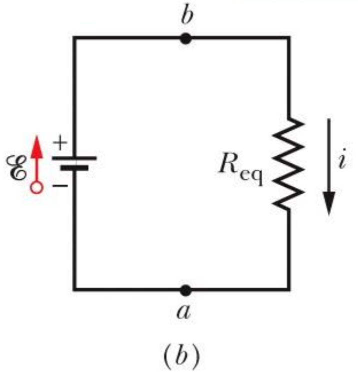
**Answer:**

(a) current is same for all resistors in series.

(b) $V_1$, $V_2$, and $V_3$ remember that current through series resistors is the same (conservation of charge), and the voltage drop over each resistor is given by Ohm's Law.

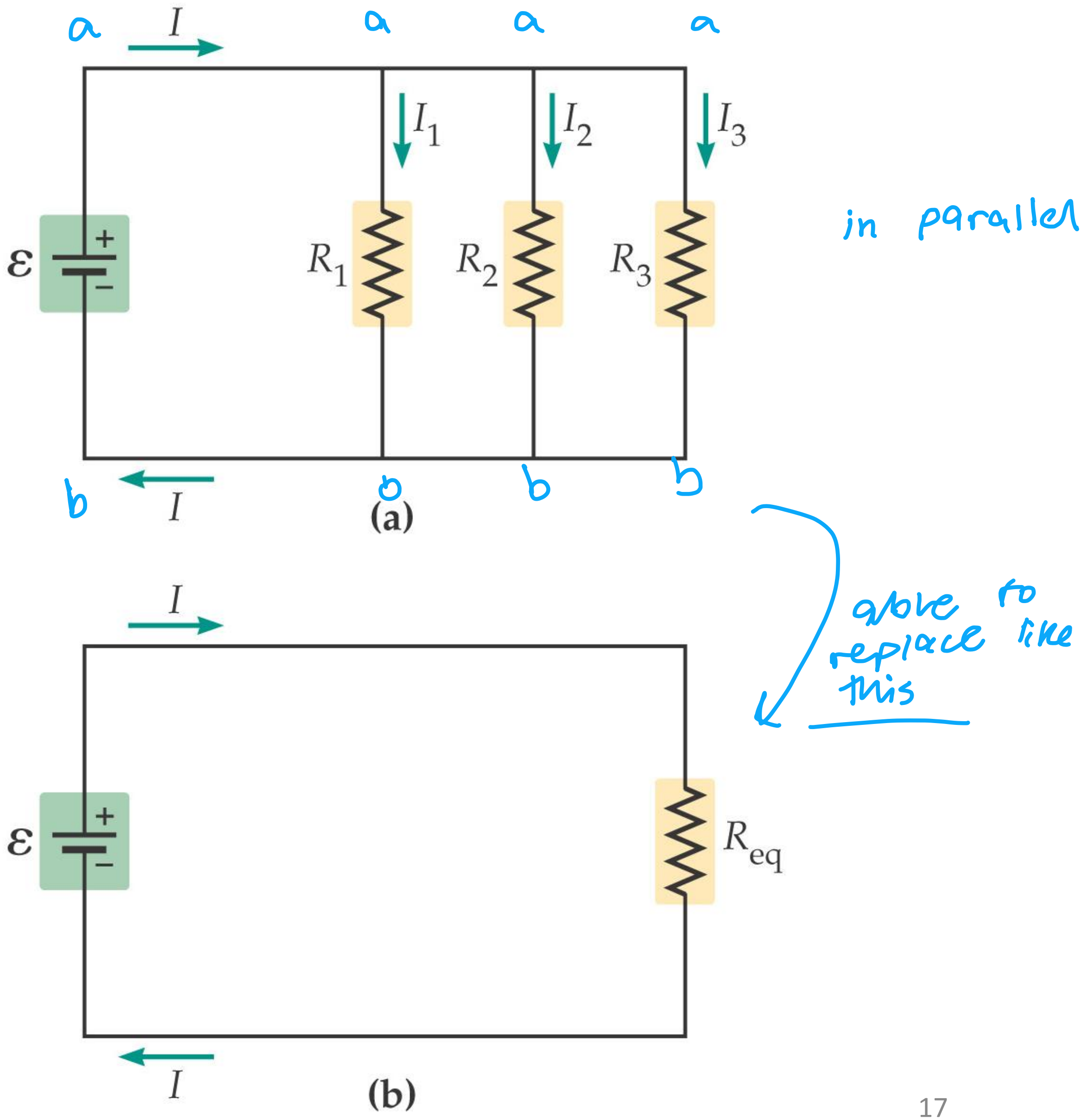



Series resistors and their equivalent have the same current ("ser-i").



# Resistors in Parallel

Resistors are in parallel when they are across the same potential difference; they can again be replaced by a single equivalent resistance:



Using the fact that the potential difference across each resistor is the same, and the total current is the sum of the currents in each resistor, we find:

**Equivalent Resistance for Resistors in Parallel**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R}$$

SI unit: ohm, $\Omega$

must do reciprocal

Note that this equation gives you the inverse of the resistance, not the resistance itself!

# Resistors in Parallel

- Total current in the circuit is the sum of the current in all its paths (branches)

$$I_{eq} = I_1 + I_2 + I_3 + \ldots$$

- Voltage is the same in each path.



$$V_{eq} = V_1 = V_2 = V_3 = \ldots$$

- The equivalent resistance decreases with more parallel resistors such that.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

# Summary
# Resistors: Series vs. Parallel Combo's

## Series and Parallel Resistors

| Series | Parallel |
|---|---|
| Resistors | Resistors |
| $R_{\text{eq}} = \sum_{j=1}^{n} R_j$ **Equation (27.1.7)** | $\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}$ **Equation (27.2.7)** |
| Same current through all resistors | Same potential difference across all resistors |

imp difference

# Capacitors: Series vs. Parallel Combo's

## Series and Parallel Capacitors

| Series | Parallel |
|---|---|
| Capacitors | Capacitors |
| $\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}$ **Equation (25.3.2)** | $C_{\text{eq}} = \sum_{j=1}^{n} C_j$ **Equation (25.3.1)** |
| Same charge on all capacitors | Same potential difference across all capacitors |

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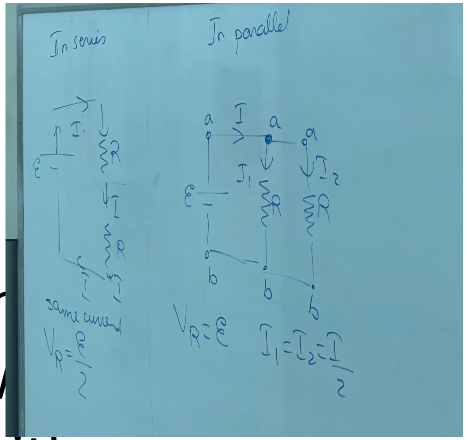
# Checkpoint #2




A battery, with potential $V$ across it, is connected to a combi
of two identical resistors and then has current $i$ through it. W
are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

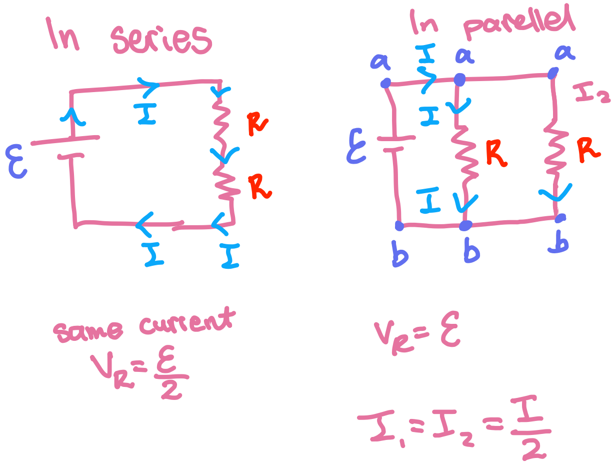
**Answer:**

(a) Potential difference across each resistor: $\dfrac{V}{2}$

Current through each resistor: $i$

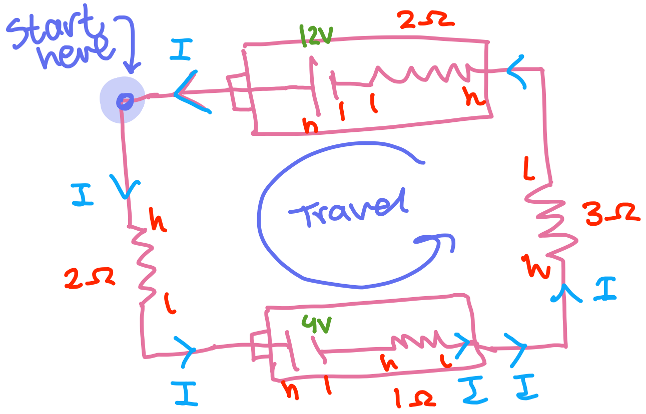
(b) Potential difference across each resistor: $V$

Current through each resistor: $\dfrac{i}{2}$



# Dr. question

IMP test question




battery $-\varepsilon$
resistor $-iR$

$-2I - 4V - 1I - 3I - 2I + 12V = 0$

$8 - 2I - 2I - 1I - 3I = 0$

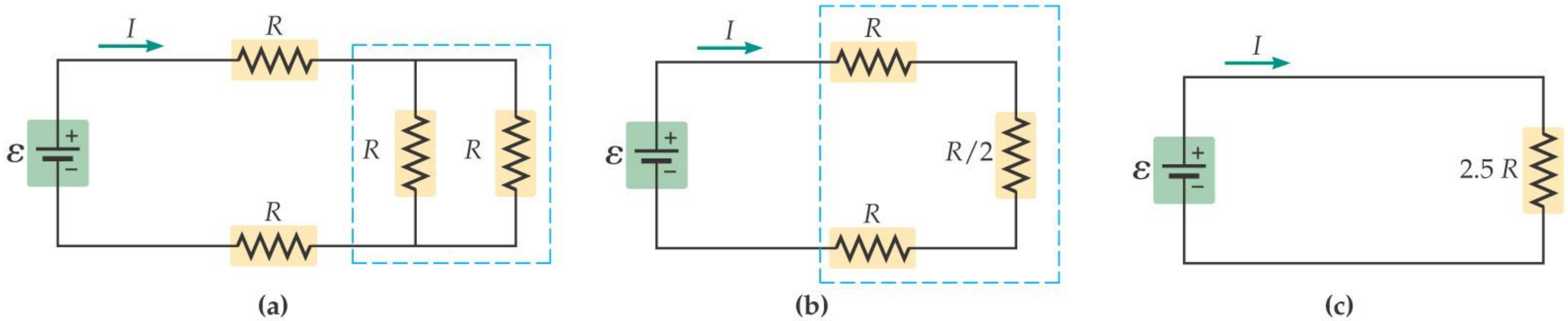
$8 - 8i = 0$

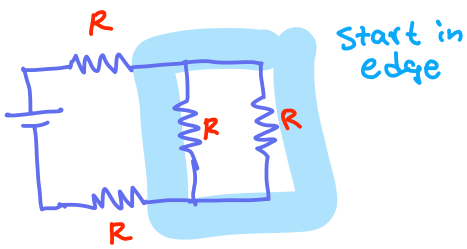
$\frac{8}{8} = \frac{\cancel{8}i}{\cancel{8}}$

$\boxed{i = 1\,A}$

# Resistors in Series and Parallel

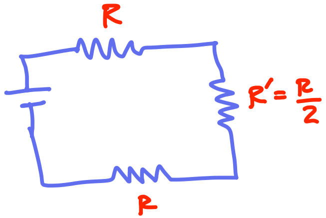
If a circuit is more complex, start with combinations of resistors that are either purely in series or in parallel. Replace these with their equivalent resistances; as you go on you will be able to replace more and more of them.






$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

$$R' = \frac{R}{2}$$




$$R_{eq} = R + \frac{R}{2} + R$$

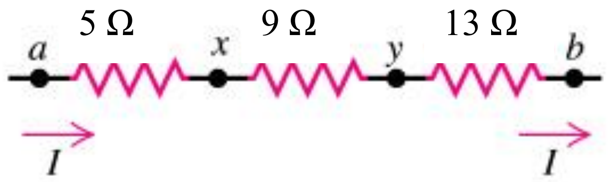
$$R_{eq} = \frac{5}{2}R = 2.5R$$

# Examples

What is the equivalent resistance of each of the following cases.

series

$R_{eq} = 27$

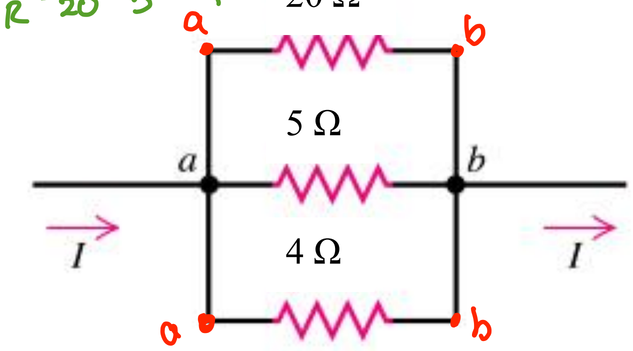



(a)

Parallel

$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{1}{20} + \frac{1}{5} + \frac{1}{4}$

$R_{eq} = 2$

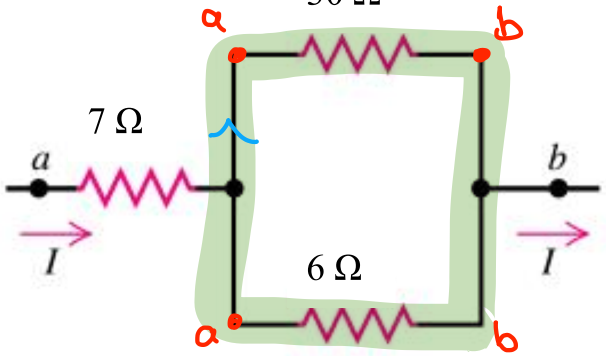



(b)

$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} = \frac{1}{30} + \frac{1}{6} = \frac{1}{5}$

$R' = 5$

$R_{eq} = R' + R_a = 5 + 7 = 12$




(c)

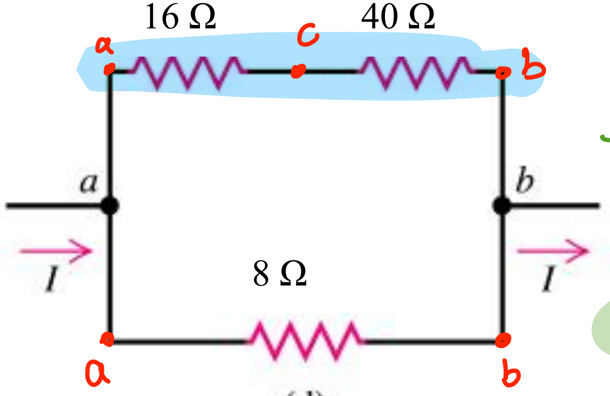
$\frac{1}{R'} = \frac{1}{16} + \frac{1}{8}$

$R_{eq} = 16 + 40 = 56$

$\frac{1}{R'} = \frac{1}{R_{eq}} + \frac{1}{R}$

$= \frac{1}{56} + \frac{1}{8} = \frac{1}{7}$

$R' = 7$




(d)

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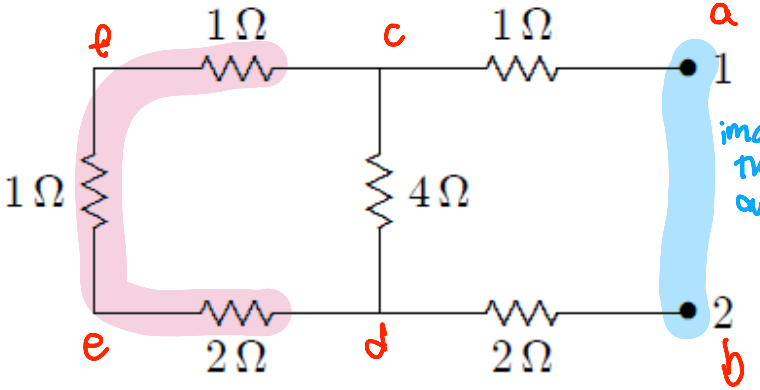
# Examples

① series : $R_{eq} = 4\,\Omega$



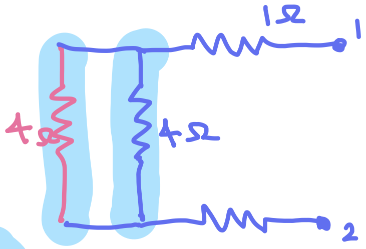

(e)

② Parallel

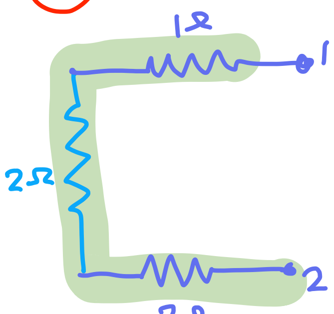
$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R}$

$= \frac{1}{4} + \frac{1}{4}$

$R' = 2\,\Omega$




③

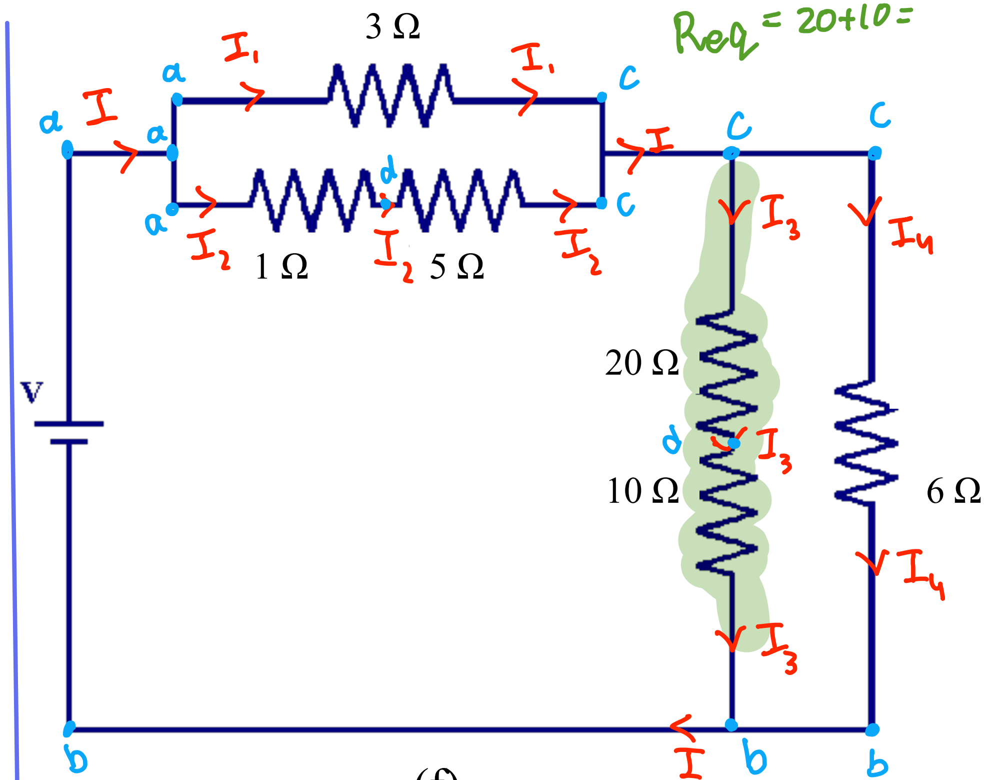



series

$R_{eq} = 5\,\Omega$

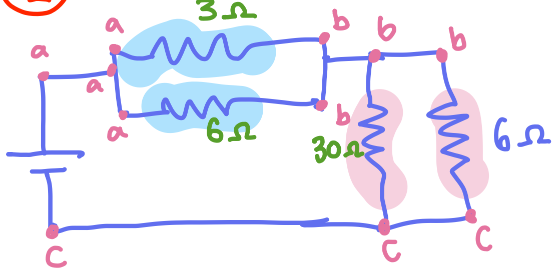
① Series

$R_{eq} = 20 + 10 =$




(f)

②



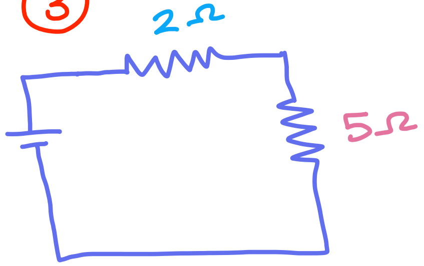

$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$R_{eq} = 2\,\Omega$

$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{6}$

$R_{eq} = 5\,\Omega$

③

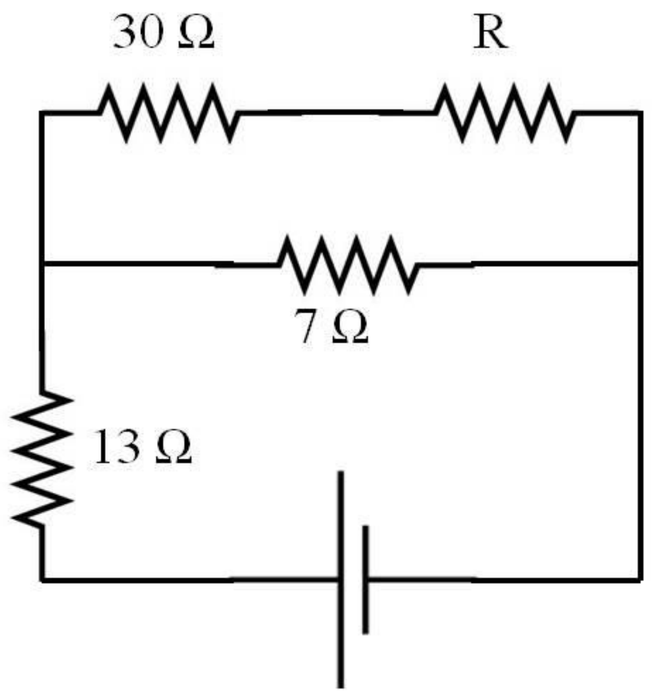


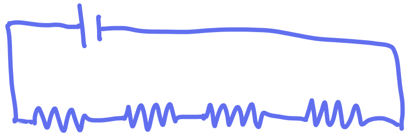

$R_{eq} = 7\,\Omega$

# Example

If the equivalent resistance of the shown network is $R_{eq} = 19\ \Omega$, what is the value of the unknown resistance R?

Self study





$R_{eq} \uparrow$

$I \uparrow$ $I = \frac{V}{R}$

# SERIES CIRCUITS

conceptual question: can draw & put numbers from me

Explain what happens to the current in a series circuit when there is a break in the circuit.

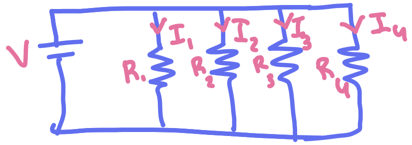
current = 0

The circuit is no longer complete, therefore current can not flow

Explain what happens to the voltage across each bulb as more bulbs are added to the circuit.

Decrease

The voltage decreases because the current is decreased and the resistance increases.




$I_1 = \frac{V}{R_1}$

At home → parallel since more practical

# PARALLEL CIRCUITS

Explain what happens to the current in each bulb as more bulbs are added to the circuit.

The current remains the same. The total resistance drops in a parallel circuit as more bulbs are added

**Series**

$R_1 = 5\Omega$

$R_2 = 2\Omega$

$R_3 = 1\Omega$

$\boxed{R_{eq} = 8\Omega}$

Greater than the greatest

**Parallel**

$R_1 = 5\Omega$

$R_2 = 2\Omega$

$R_3 = 1\Omega$

$\boxed{R_{eq} = 0.59\Omega}$

less than the least

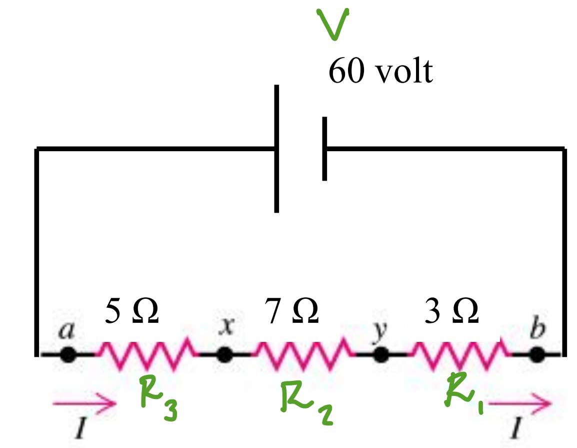
Explain what happens to the total current provided by the battery as more bulbs are added to the circuit.

The current increases.

# Examples

Find the voltage and current for each resistance in the figures shown.

(a)




(a)

$I = \frac{V}{R}$

$R_{eq} = 5 + 7 + 3 = 15$

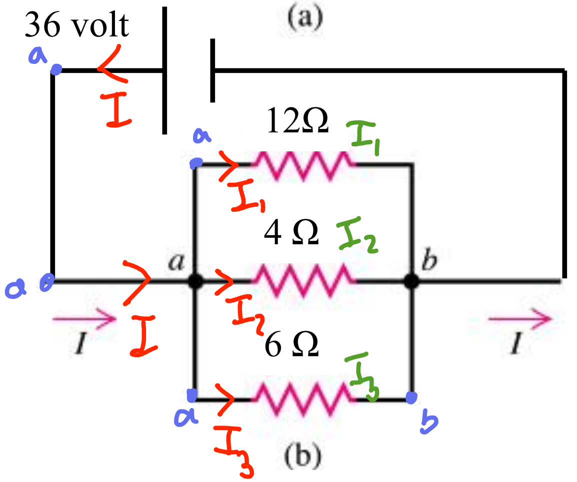
$I = \frac{60}{15} = 4A$

$V_{ax} = IR = 5 \times 4 = 20V$

$V_{xy} = 4 \times 7 = 28V$

$V_{yb} = 3 \times 4 = 12V$

(b)




(b)

$I_1 = \frac{V}{R_1} \rightarrow I_1 = \frac{36}{12} = 3A$

$I_2 = \frac{V}{R_2} \rightarrow I_2 = \frac{36}{4} = 9A$

$I_3 = \frac{V}{R_3} \rightarrow I_3 = \frac{36}{6} = 6A$

$I_{eq} = 18A$

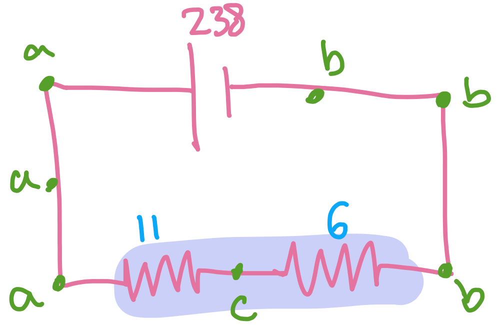
Tue 14 Apr

# Examples

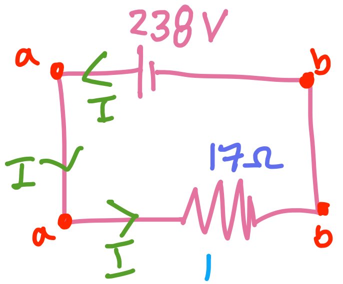
Find the voltage and current for each resistance in the figures shown.

(a)

$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \longrightarrow R_{23} = 6\,\Omega$




series:
$R_{eq} = 11 + 6 = 17\,\Omega$




(1) $I = \frac{V}{R_{eq}} \longrightarrow I = \frac{238}{17}$

$\boxed{I = 14A}$

↳ work backward

(2) $V_{ac} = I R_{ac} \rightarrow 154V$

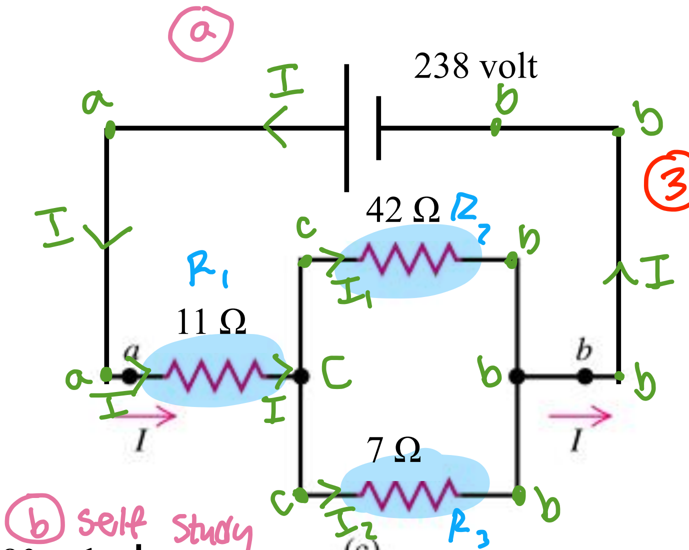
$V_{cb} = I R_{cb} \rightarrow 84V$

$\boxed{OR}$

$238 - 154 = 84V$

same thing

(a)



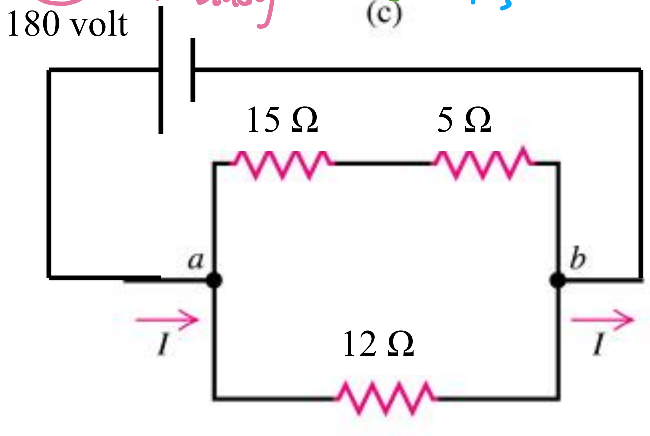

(c)

$I_1 = \frac{V_{cb}}{R_1} = \frac{84}{42} = 2A$

$I_2 = \frac{V_{cb}}{R_2} = \frac{84}{7} = 12A$

(3) $\boxed{I_{eq} = 14A}$ ✓

(b) self study




(d)

# Examples

Find the voltage and current for each resistance in the figures shown.

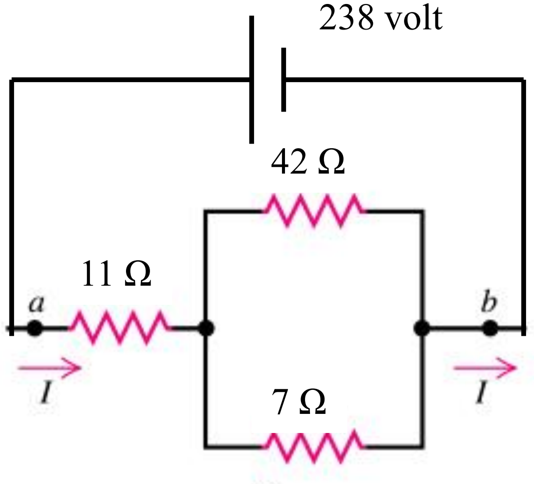
$R_{eq_{inner}} = 6 \text{ Ohm}, R_{eq_{tot}} = 17 \text{ Ohm}$

$I_{tot} = \frac{238}{17} = 14\ A,$

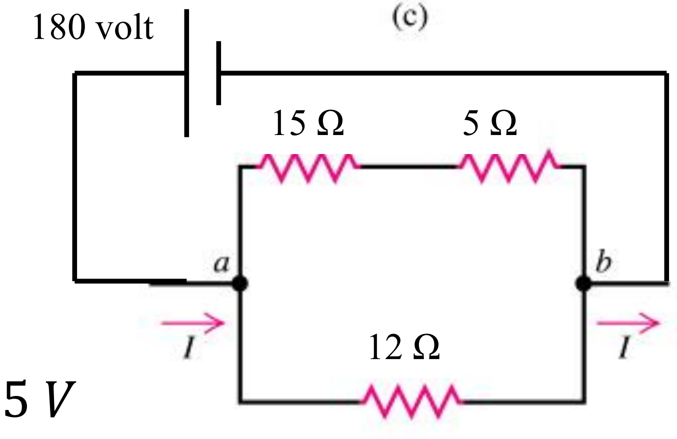
$V_1 = 14 \times 11 = 154\ V,$

$V_{2,3} = 14 \times 6 = 84 = 238 - 154 = 84\ V,$

$I_1 = 14\ A, I_2 = \frac{84}{42} = 2\ A, I_3 = \frac{84}{7} = 12\ A$



(c)



(d)

$V_3 = 180\ V, I_3 = \frac{180}{12} = 15\ A,$

$V_{1+2} = 180, I_{1,2} = \frac{180}{20} = 9\ A$

$V_1 = 9 \times 15 = 135\ V, V_2 = 9 \times 5 = 45\ V$

# Kirchhoff's Rules

$I_{enter} = I_{leave}$

More complex circuits cannot be broken down into series and parallel pieces.

For these circuits, Kirchhoff's rules are useful.

→ Junction

→ Loop rule $\Sigma V = 0$

The junction rule is a consequence of charge conservation; the loop rule is a consequence of energy conservation.

neither series nor parallel

Junction rule:

$I = I_1 + I_2$

Loop 1:

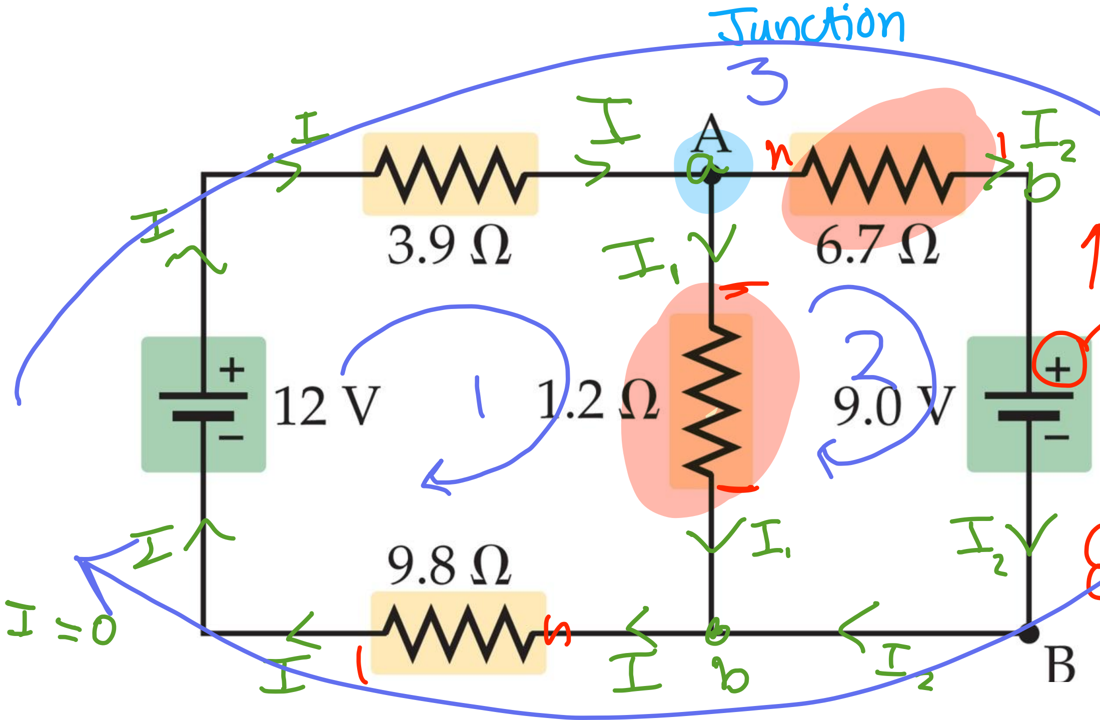
$+12 - 3.9I - 1.2I_1 - 9.8I = 0$

Loop 2:

$-9 + 1.2I_1 - 6.7I_2 = 0$

Loop 3:

$+12 - 3.9I - 6.7I_2 - 9 - 9.8I = 0$




current go start this way as well

if − means wrong direction

Junction rule $I = I_1 + I_2 \rightarrow I - I_1 - I_2 = 0$ ①

$+13.7I + 1.2I_1 + 0I_2 = +12$ ②

Loop 1:

$+12 - 3.9I - 1.2I_1 - 9.8I = 0$

Loop 2:

$-9 + 1.2I_1 - 6.7I_2 = 0$

Loop 3:

$+12 - 3.9I - 6.7I_2 - 9 - 9.8I = 0$

$0I + 1.2I_1$

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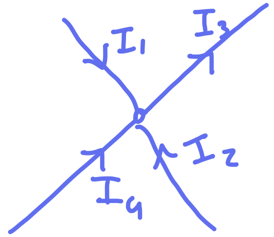
n→1 minus (w/ current)
1→n plus (against current)

# Kirchhoff's Rules

NOTE:



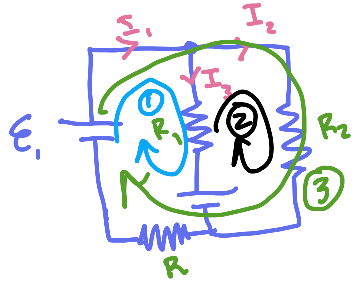

sum enter = sum leave

so

$I_1+I_2+I_4=I_3$

The junction rule: At any junction, the current entering the junction must equal the current leaving it.

here $I_3$ must be leaving

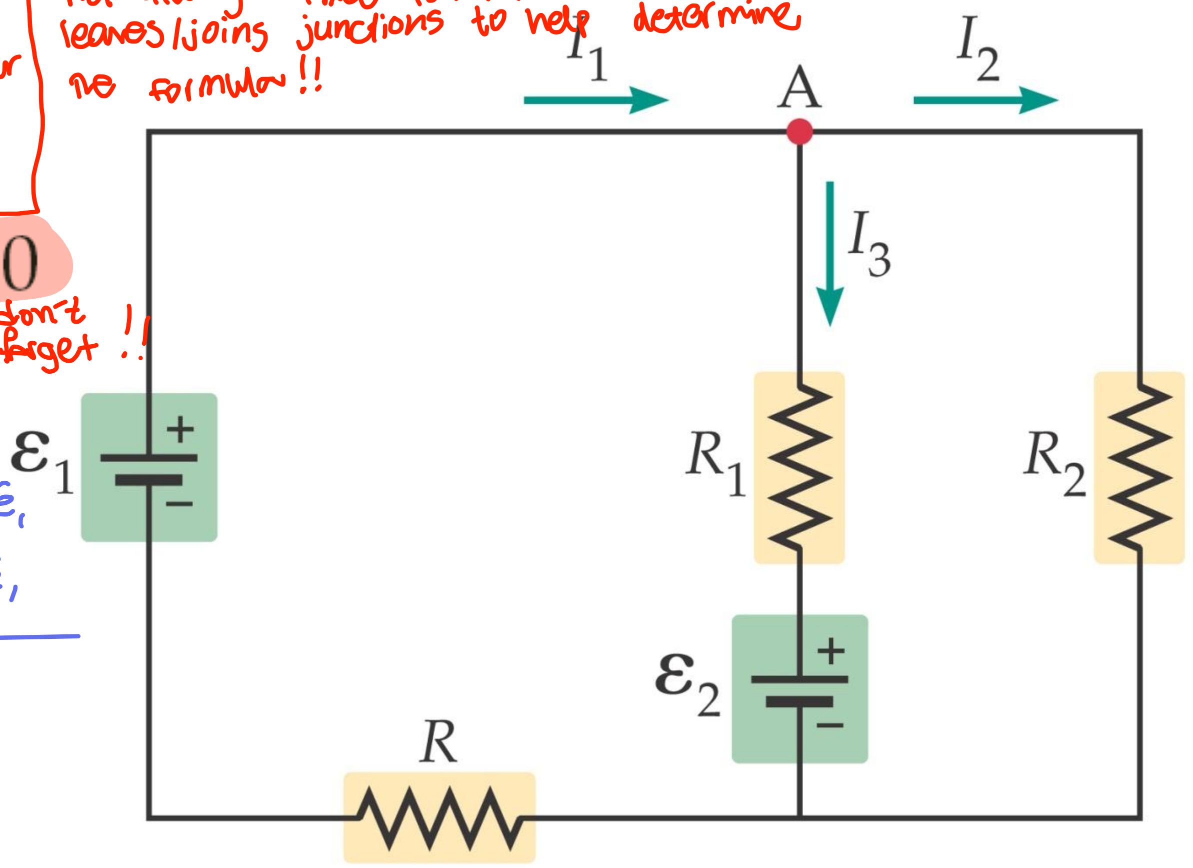



if answer – just get absolute value cause this means your direction is in the opposite

not always fixed formula look at who leaves/joins junctions to help determine no formula!!

$$I_1 - I_2 - I_3 = 0$$

!! don't forget !!




① Junction Rule

$I_1 = I_2 + I_3$

Loop 1: $+\varepsilon_1 - I_3R_1 - \varepsilon_2 - I_1R = 0$

Loop 2: $+\varepsilon_1 - I_2R_2 + \varepsilon_2 + I_3R_1 = 0$

Loop 3: $+\varepsilon_1 - I_2R_2 - I_1R = 0$
extra

Rearrange

$I_1 - I_2 - I_3 = 0$

$I_1R + 0I_2 + I_3R_1 = \varepsilon_2 - \varepsilon_1$

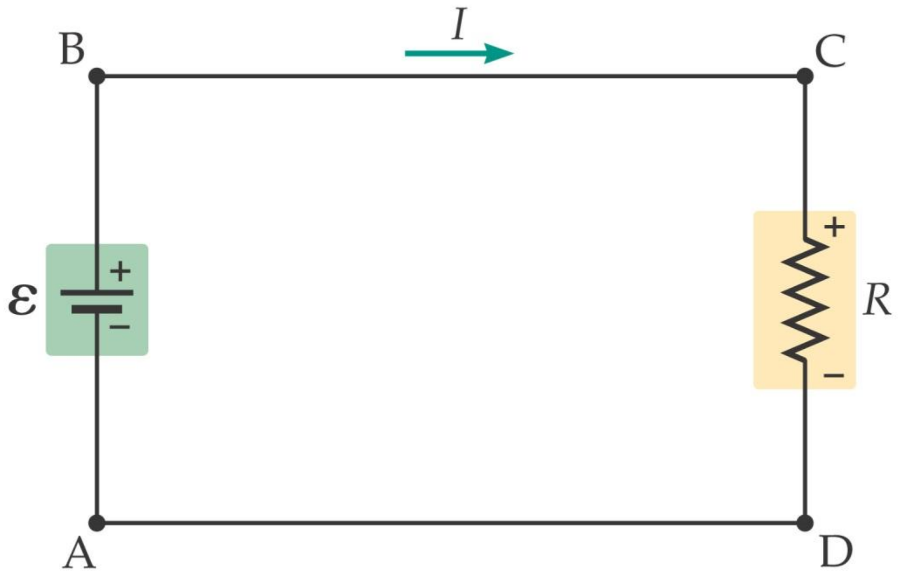
$0I_1 - I_2R_2 + I_3R_1 = -\varepsilon_1$

Put numbers

# Kirchhoff's Rules

The loop rule: The algebraic sum of the potential differences around a closed loop must be zero (it must return to its original value at the original point).

$$\mathcal{E} + \Delta V_{CD} = 0$$



Using Kirchhoff's rules:

• The variables for which you are solving are the currents through the resistors.

• You need as many independent equations as you have variables to solve for.

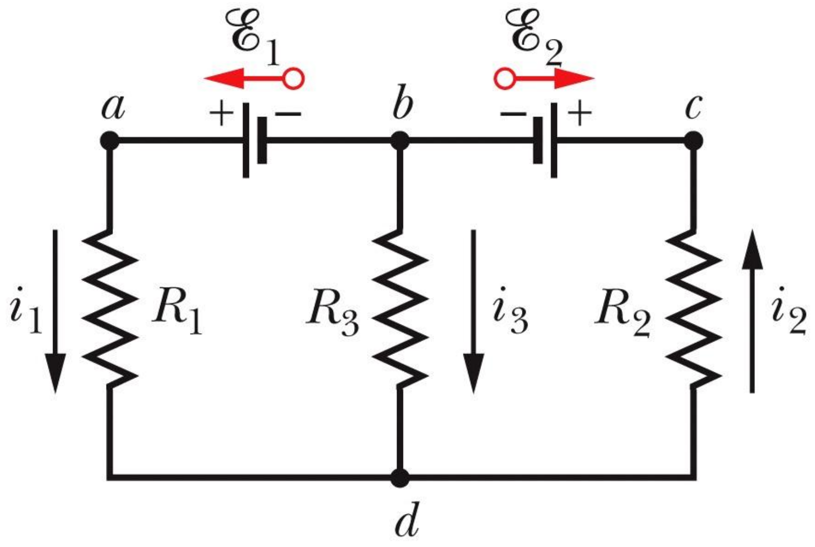
• You will need both loop and junction rules.

**Junction Rule:** The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. $i_{2=}\ i_1\ +\ i_3$

The current into the junction must equal the current out (charge is conserved).

Figure shows a circuit containing more than one loop. If we traverse the left-hand loop in a counterclockwise direction from point *b*, the loop rule gives us

$$\varepsilon - i_1 R_1 + i_3 R_3 = 0$$



If we traverse the right-hand loop in a counterclockwise direction from point $b$, the loop rule gives us:

$$-i_3R_3 - i_2R_2 - E_2 = 0$$

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from $b$) the equation

$$E_1 - i_1R_1 - i_2 R_2 - E_2 = 0$$

which is the sum of two small loops equations.

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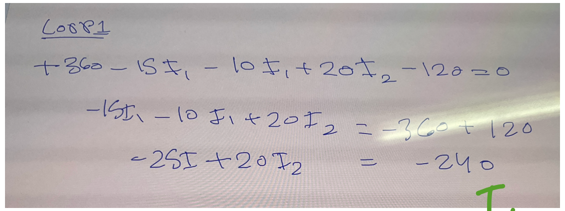
# Example



Loop 1

$$+360 - 15I_1 - 10I_1 + 20I_2 - 120 = 0$$

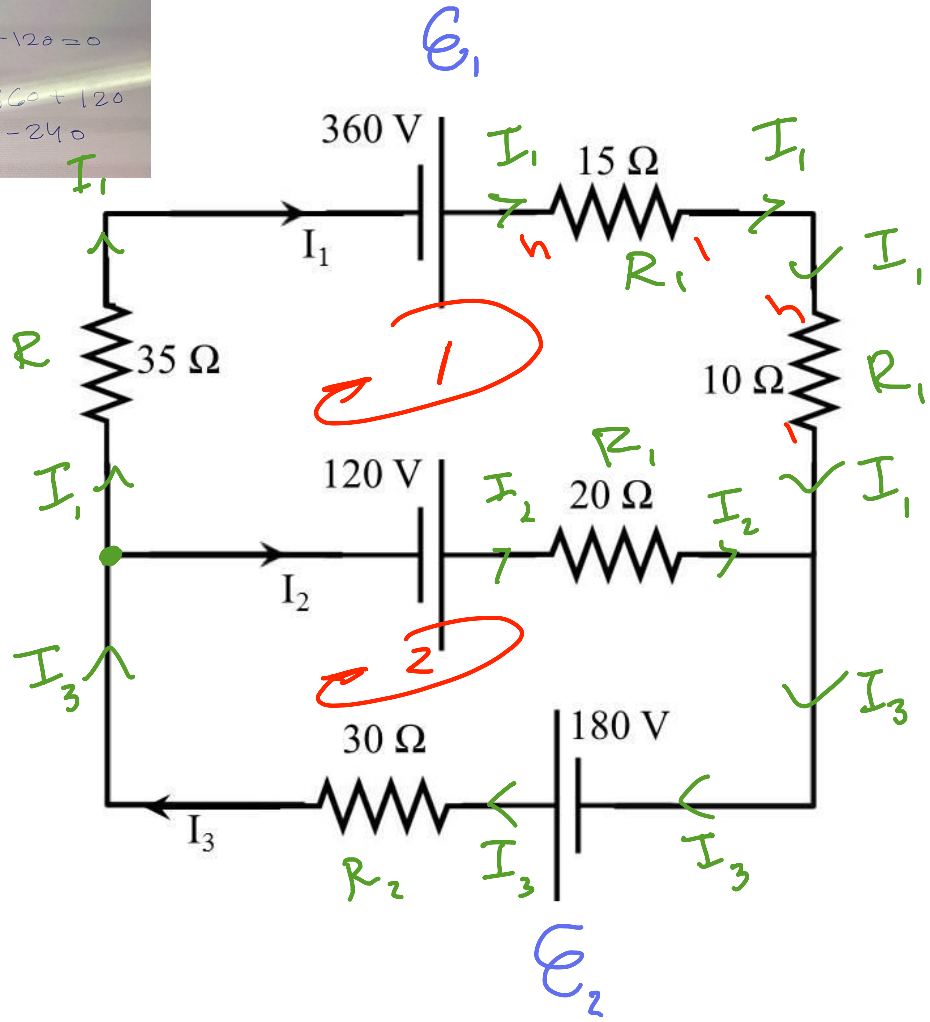
$$-15I_1 - 10I_1 + 20I_2 = -360 + 120$$

$$-25I + 20I_2 = -240$$

Use Kirchhoff's rules to evaluate the currents $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ in the shown circuit.

Loop 1 $+\mathcal{E}_1 - I_1R_1 - I_1R_1 + I_1R_1$

Picture




Ans. $I_1 = 5$ A, $I_2 = 3$ A, $I_3 = 8$ A

On Tue 21 Apr

# Example

Rare & difficult

In the circuit shown, use Kirchhoff's rules to find the values of the unknowns $R_2$, $I_1$, $I_2$

$I_3 = 1.8A$
$I_1 = ?$
$I_2 = ?$
$R_2 = ?$

$I_1 + I_3 = I_2$

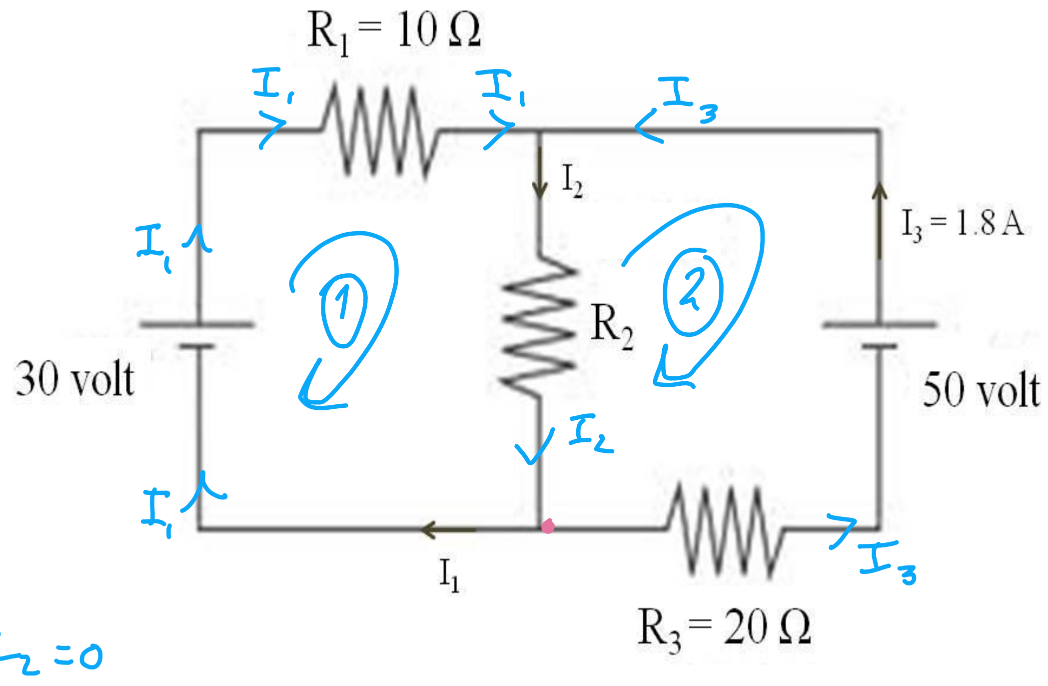
$I_1 + 1.8 = I_2$ (1)

loop 1: $30V - 10I_1 - I_2R_2 = 0$ (2)

loop 2: $-20I_3 + 50 - I_2R_2 = 0$ (3)

$-36 + 50 - I_2R_2 = 0$

$14 - I_2R_2 = 0 \rightarrow 14 = I_2R_2$




$\boxed{2}$ $30 - 10I_1 - 14 = 0$

$-10I_1 + 16 = 0$

$\frac{16}{10} = \frac{10I_1}{10}$

$I_1 = 1.6A$

$\boxed{3}$ $1.6 + 1.8 = I_2 = 3.4$

$30 - 10(1.6) - (3.4)R_2 = 0$

$14 - 3.4R_2 = 0$

$\frac{14}{3.4} = \frac{3.4R_2}{3.4}$

$R_2 = 4.11$

# Additional Practice

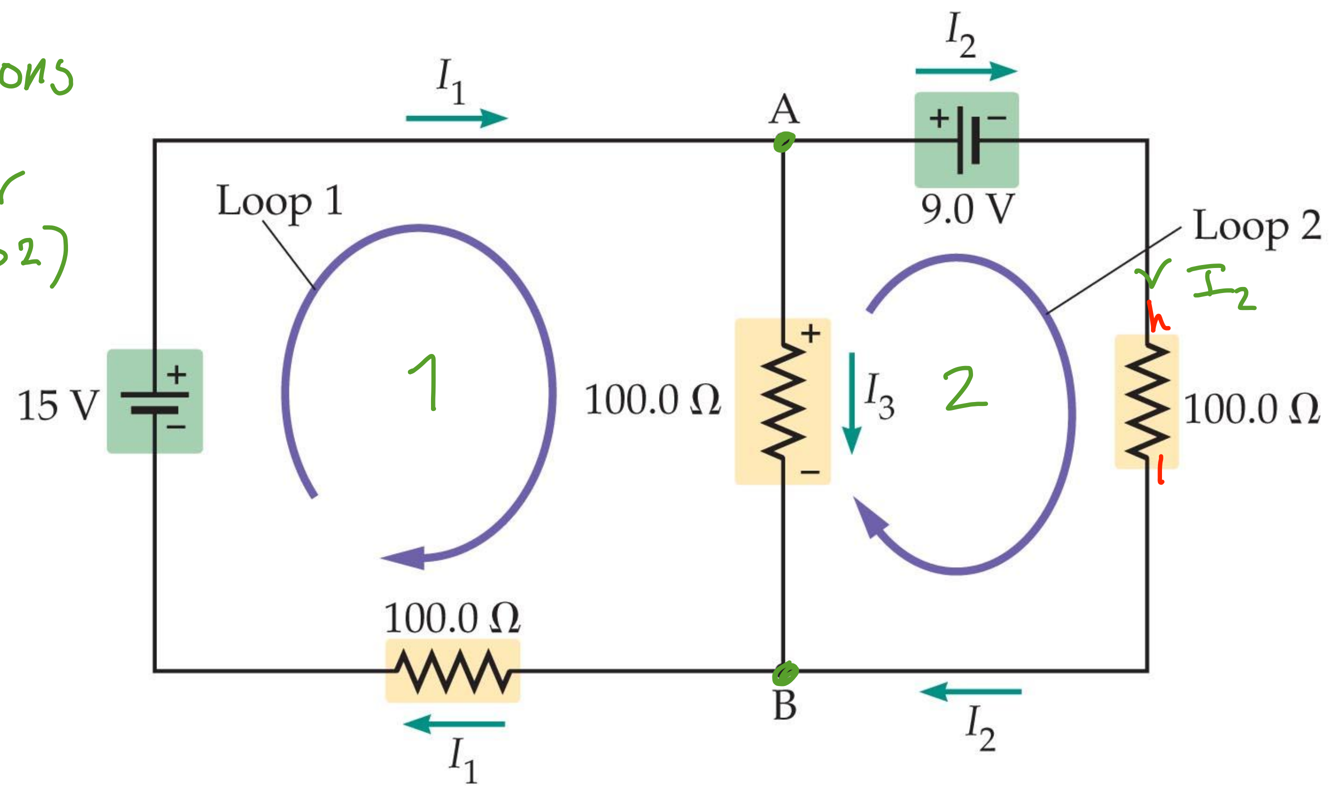
In the circuit shown, use Kirchhoff's rules to find the values of the unknown currents

Junction rule: $I_1 = I_2 + I_3$

loop 1: $15 - 100I_1 - 100I_3 = 0$

loop 2: $-9 - 100I_2 + 100I_3 = 0$

3 equations (put in calculator mode → 5 → 2)






Ans.

$I_1 = 0.07A, \quad I_2 = -0.01A, \quad I_3 = 0.08A$

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# Additional Practice

- Find the magnitude and direction (clockwise or counterclockwise ) of the current in the circuit shown. $+15V-8.5I+11.5V-6.22I-15.1I=0$

$26.5V-29.82I=0$

$I=0.89A$ clockwise

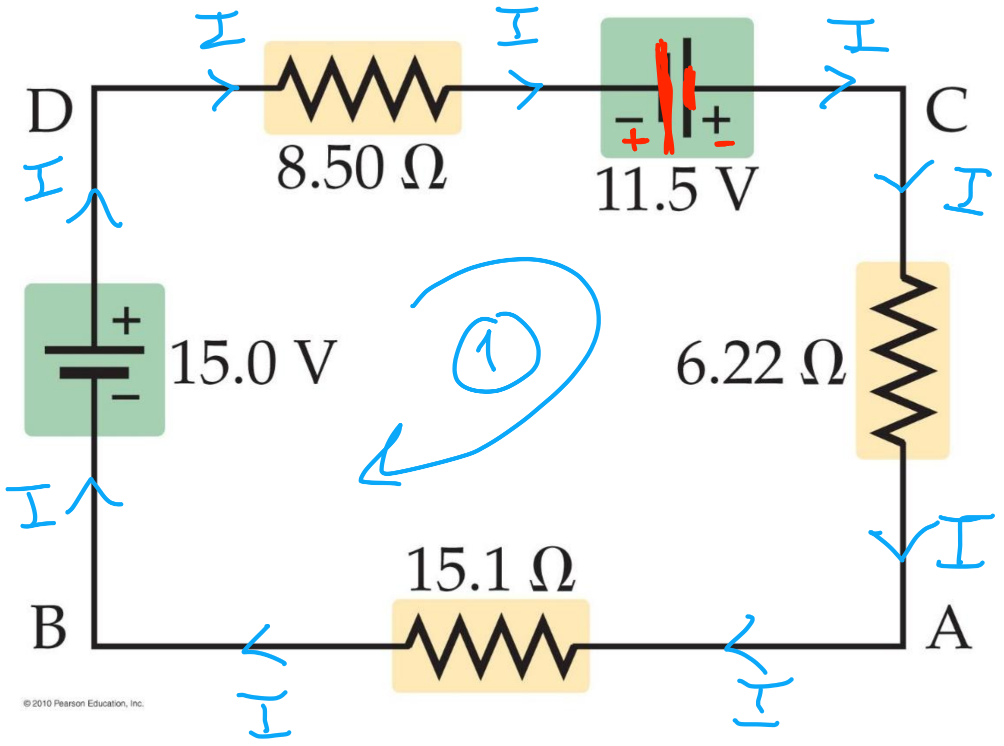
They are helping each other

- For the same circuit, if the polarity of the (11.5 v) battery is reversed, do you expect this to increase or decrease the amount of current flowing in the circuit? Calculate the magnitude and direction (clockwise or counterclockwise) of the current in this case.

$+15V-8.5I-11.5V-6.22I-15.1I=0$

$3.5V-29.82I=0$

$I=\frac{3.5}{29.82}=0.117A$ clockwise




important to start from bigger

# Home Practice

**Sample Problem 27.1.1**

**Single-loop circuit with two real batteries**

The emfs and resistances in the circuit of Fig. 27.1.8a have the following values:

$\mathscr{E}_1 = 4.4\ \text{V},\quad \mathscr{E}_2 = 2.1\ \text{V},$

$r_1 = 2.3\ \Omega,\quad r_2 = 1.8\ \Omega,\quad R = 5.5\ \Omega.$

(a) What is the current $i$ in the circuit?

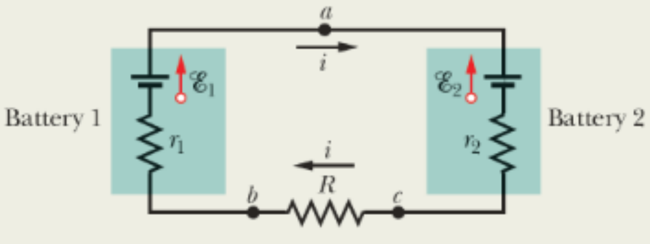
**KEY IDEA**

We can get an expression involving the current $i$ in this single-loop circuit by applying the loop rule, in which we sum the potential changes around the full loop.

*Calculations:* Although knowing the direction of $i$ is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_1$ is greater than $\mathscr{E}_2$, battery 1 controls the direction of $i$, so the direction is clockwise. Let us then apply the loop rule by going counterclockwise—against the current—and starting at point $a$. (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) We find

$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0.$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than $a$. Also, take the time to compare this equation term by term with Fig. 27.1.8b, which shows the potential changes graphically (with the potential at point $a$ arbitrarily taken to be zero).




(a)

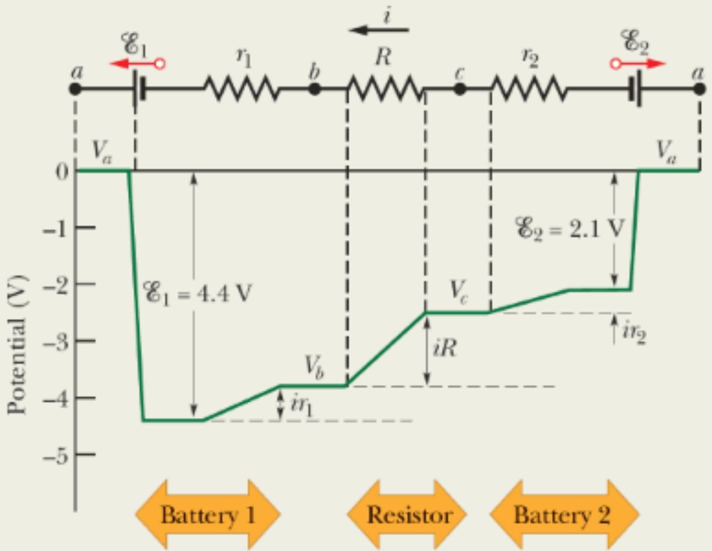


Figure 27.1.8 (*a*) A single-loop circuit containing two real batteries and a resistor. The batteries oppose each other; that is, they tend to send current in opposite directions through the resistor. (*b*) A graph of the potentials, counterclockwise from point *a*, with the potential at *a* arbitrarily taken to be zero. (To better link the circuit with the graph, mentally cut the circuit at *a* and then unfold the left side of the circuit toward the left and the right side of the circuit toward the right.)

Solving the above loop equation for the current $i$, we obtain

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} = \frac{4.4\ \text{V} - 2.1\ \text{V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega}$$

$$= 0.2396\ \text{A} \approx 240\ \text{mA}. \qquad \text{(Answer)}$$

(b) What is the potential difference between the terminals of battery 1 in Fig. 27.1.8*a*?

**KEY IDEA**

We need to sum the potential differences between points $a$ and $b$.

*Calculations:* Let us start at point $b$ (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point $a$ (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathscr{E}_1 = V_a,$$

which gives us

$$V_a - V_b = -ir_1 + \mathscr{E}_1$$

$$= -(0.2396\ \text{A})(2.3\ \Omega) + 4.4\ \text{V}$$

$$= +3.84\ \text{V} \approx 3.8\ \text{V}, \qquad \text{(Answer)}$$

which is less than the emf of the battery. You can verify this result by starting at point $b$ in Fig. 27.1.8*a* and traversing the circuit counterclockwise to point $a$. We learn two points here. (1) The potential difference between two points in a circuit is independent of the path we choose to go from one to the other. (2) When the current in the battery is in the "proper" direction, the terminal-to-terminal potential difference is low, that is, lower than the stated emf for the battery that you might find printed on the battery.

**WileyPLUS** Additional examples, video, and practice available at *WileyPLUS*

Please refer to the Wiley plus e-textbook for extra practice : 27.2.2, 27.2.2 and 27.2.3

27.2 MULTILOOP CIRCUITS | Fundamentals of Physics, Extended - Wiley Reader

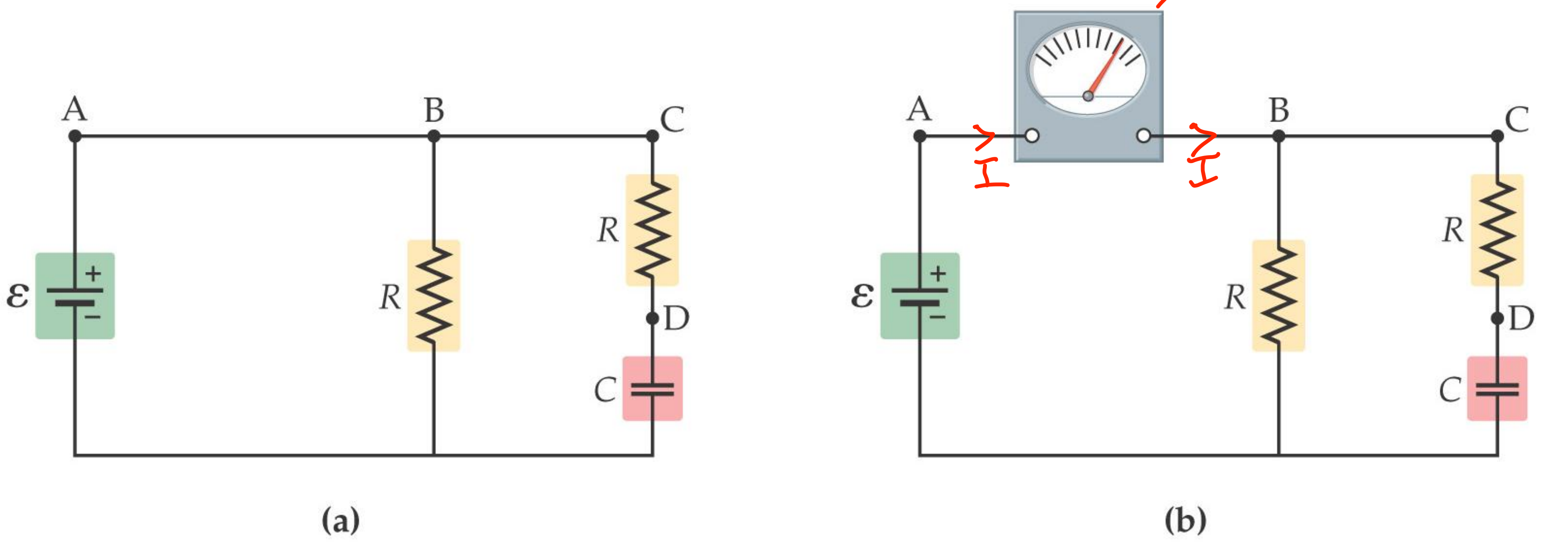
# Section 27.3 The Ammeter and The Voltmeter

# Ammeters

most imp points

An ammeter is a device for measuring current, and a voltmeter measures voltages. In series *

The current in the circuit must flow through the ammeter; therefore the ammeter should have as low a resistance as possible, for the least disturbance. Resistance should be so small *



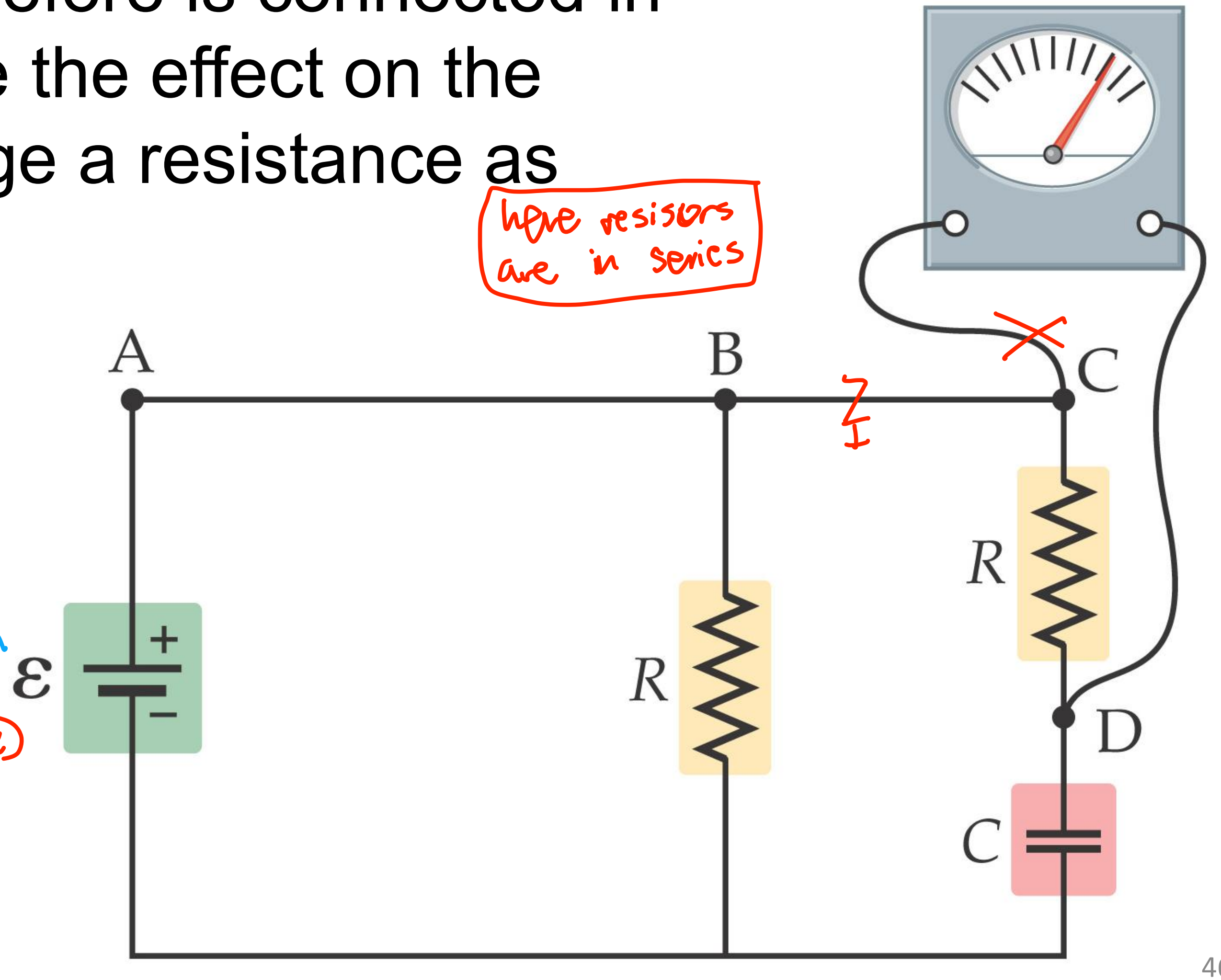

resistor in Ammeter is parallel

# Voltmeters

A voltmeter measures the potential drop between two points in a circuit. It therefore is connected in parallel; in order to minimize the effect on the circuit, it should have as large a resistance as possible.

* parallel connection
* resistance so big

current choose the easiest (less resistance)



# Section 27.4 RC Circuits

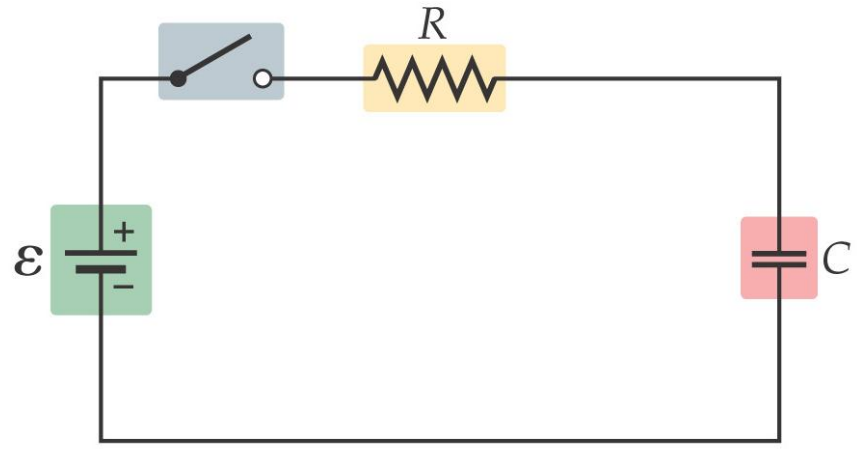
# RC Circuits

resistor $\longleftrightarrow$ capacitor = control charging/discharging

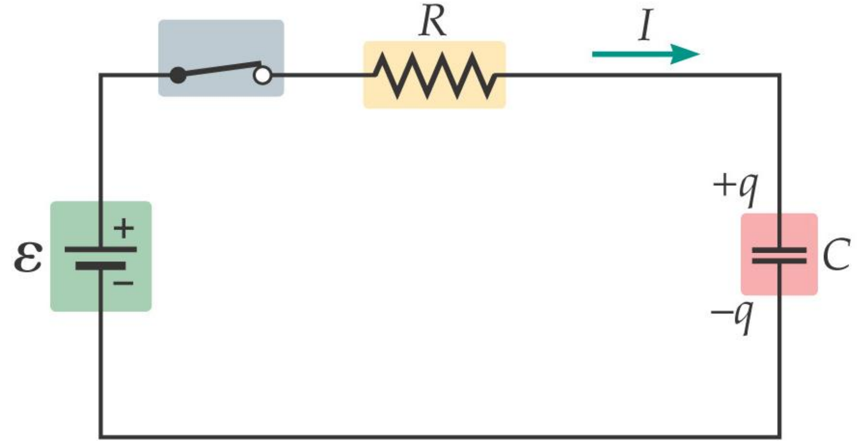
When the circuit contains only batteries and capacitors, charge appears almost instantaneously on the capacitors when the circuit is connected.

If the circuit contains a resistance , battery and a capacitor, this is NOT the case. This circuit is called an RC circuit.

Resistors limit the rate at which charge can flow, and an amount of time may be required for the capacitor to become charged.




**(a)** $t < 0$

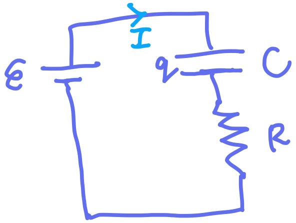



**(b)** $t > 0$

## RC

$V = IR$ : resistor
$Q = CV$ : capacitor

### ① Charging




$V_{battery} = V_R + V_C$

$\varepsilon = iR + \frac{q}{C}$

① At $t=0$ — initially starting point

$q = 0 \quad V_C = 0$

$\varepsilon = I_0 R$

$I_0 = I_{max} = \frac{\varepsilon}{R}$

③ At the end, when capacitor fully charged

$V_{Battery} = V_C$

$V_R = 0$

$q_{max} = C\varepsilon$

② At any instant $t$

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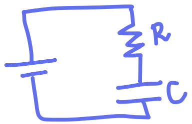
Trick : get $R_{eq}$ then $C_{eq}$

emergency light → slow discharging

flash → fast charging

### ① Charging




$V_{battery} = V_R + V_C$

$V_R = IR \quad q = CV_C$

①
$t=0 \quad q=0 \quad V_C = 0$ — capacitor act as closed switch

$V_{battery} = V_R$

$\varepsilon = I_{max} R$

$I_{max} = \frac{\varepsilon}{R}$

③ $t = \infty$ when capacitor fully charged — capacitor act as open switch

$V_C = \varepsilon$

$q_{max} = C\varepsilon$

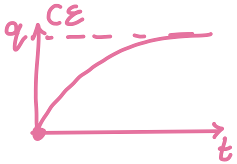
$V_R = 0$

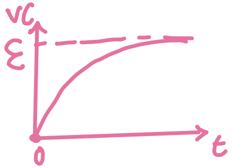
$i = 0$

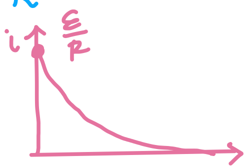
② At any instant charging

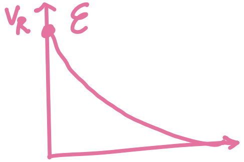
$q = \underbrace{C\varepsilon}_{Q_{max}}(1 - e^{-\frac{t}{RC}})$

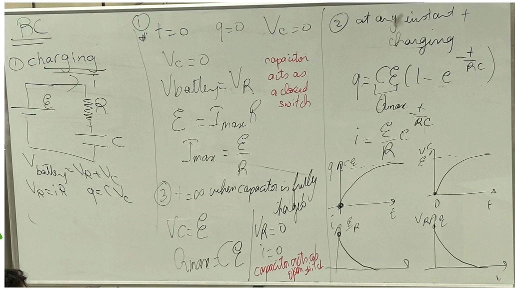
$i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$



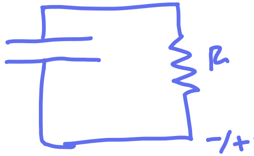



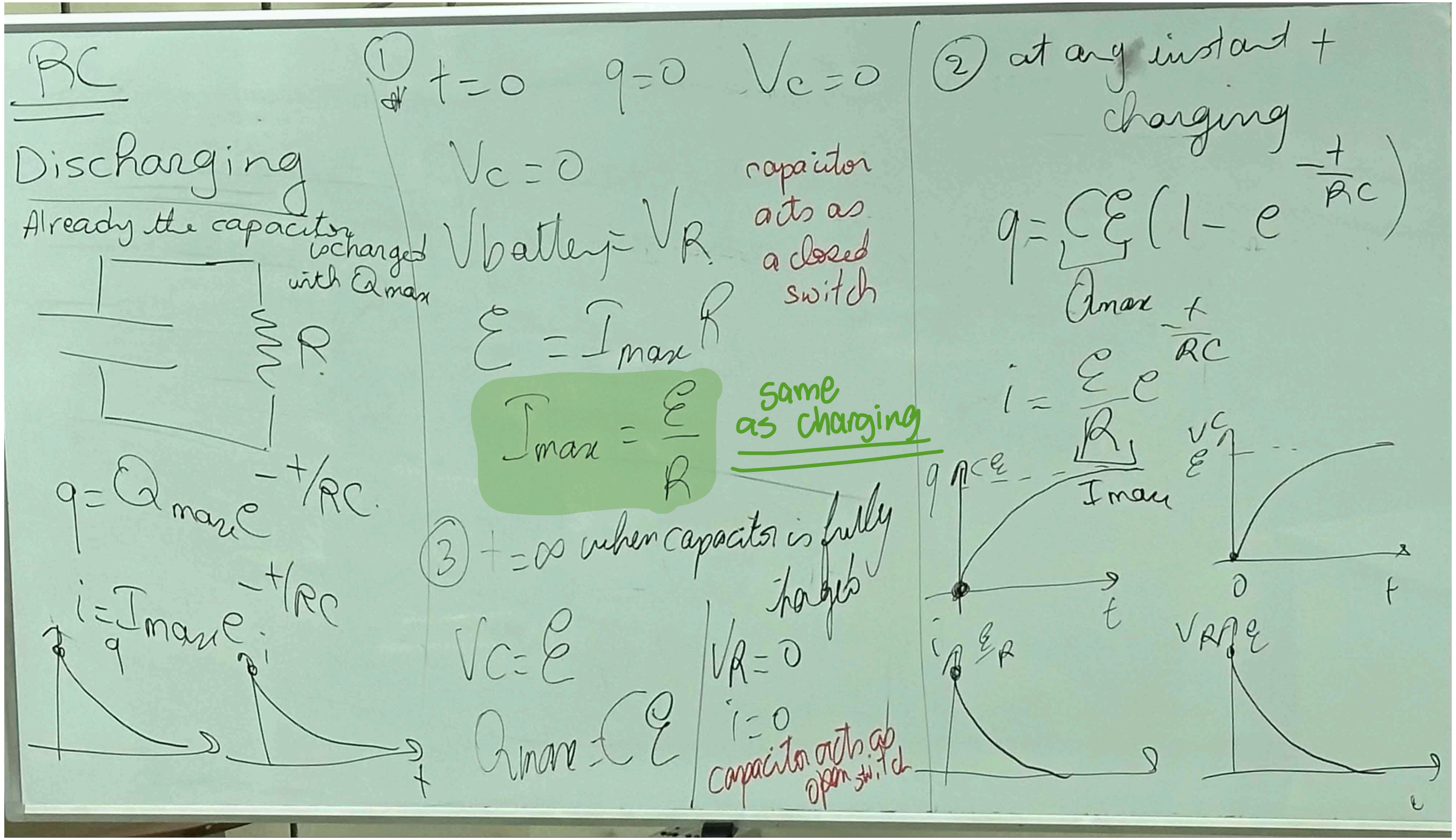



**Discharging** : Already capacitor charged w/ $Q_{max}$




$-t/RC$

$q = Q_{max} e^{-t/RC}$



# RC Circuits - Charging

Charge does not flow from the positive plate to the negative plate through the capacitor. Charge is transferred from one plate to the other plate through the resistor, switch, and battery until the capacitor is fully charged.

Consider a series circuit containing a resistance and a capacitor that is initially uncharged.

- With switch is open, there is no current in the circuit.
- When switch is closed at t = 0 s, charges begin to flow, and a current is present in the circuit and the capacitor begins to charge.
- As the capacitor charges, the current varies over time.
- The value of the maximum charge depends on the voltage of the battery.
- Once the maximum charge is reached, the current in the circuit is zero.

# RC Circuits - Charging

- At t = 0 s, when the switch is closed, the charge on the capacitor is 0 and the initial current is:

$$I_{\text{max}} = V_{\text{battery}}/R$$

  - Note $V_{\text{battery}} \equiv \mathcal{E} \equiv emf$

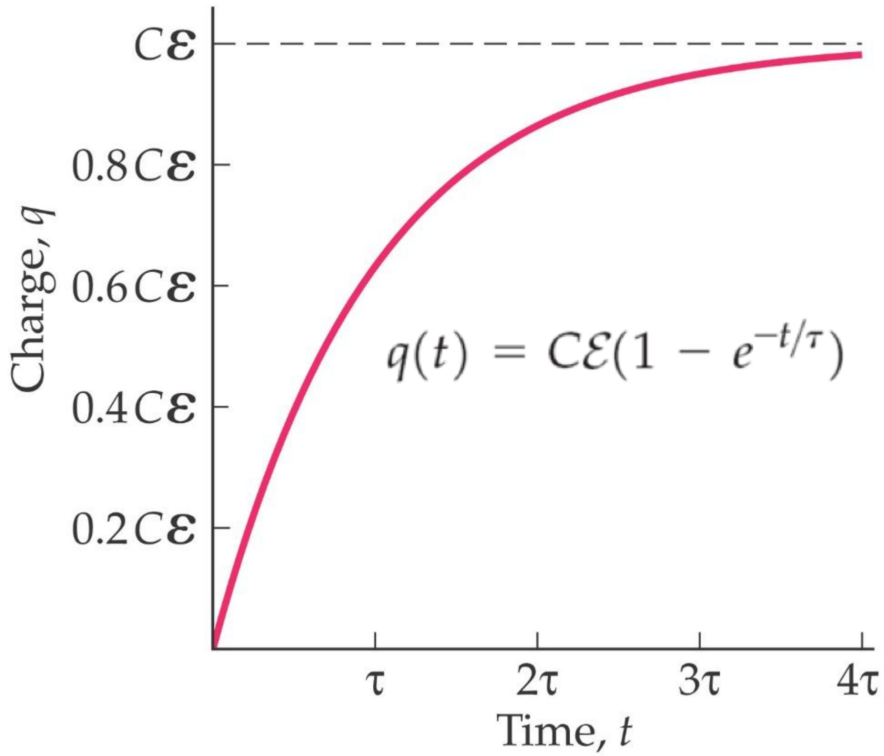
- At t = 0 s, the potential drop is entirely across the resistor.
- As the capacitor is charged to its maximum value Q, the charges stop flowing and the current in the circuit is 0 A and the potential drop is entirely across the capacitor.
- After a long time. the maximum charge on the capacitor is reached

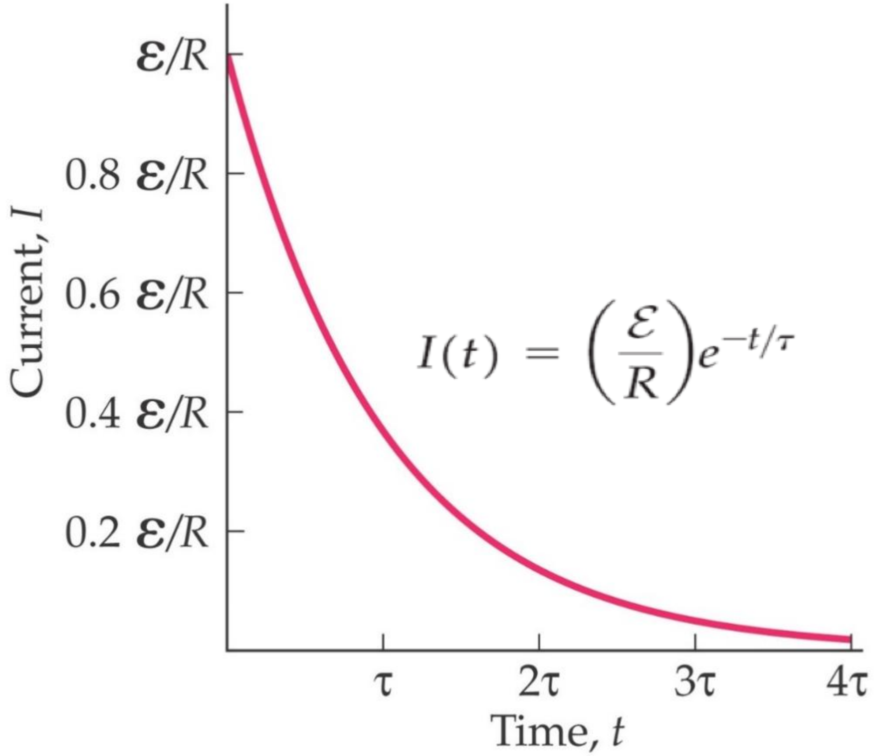
$$Q_{\text{max}} = CV_{\text{battery}}$$

- From t = 0 s until the capacitor is fully charged and the current stops, the amount of current in the circuit decreases over time and the amount of charge on the capacitor increases over time.

# RC Circuits - Charging

Charge and current in a charging RC circuit as a function of time: ($\mathcal{E} = V_{\text{battery}}$ )




$q(t) = Q_{\max}(1 - e^{-t/RC})$
$q(t = 0) = 0$
$q(t = \infty) = Q_{\max} = CV_{\text{battery}}$

$I(t) = I_{\max}\, e^{-t/RC}$
$I(t = 0) = I_{\max} = V_{\text{battery}}/R$
$I(t = \infty) = 0$

# RC Circuits – the time constant ($\boldsymbol{\tau}$)

- The quantity $RC$, which appears in the exponential component of the charge and current equations is called the time constant $\boldsymbol{\tau}$ of the circuit.

$$\tau = RC \text{ (the time constant)}$$

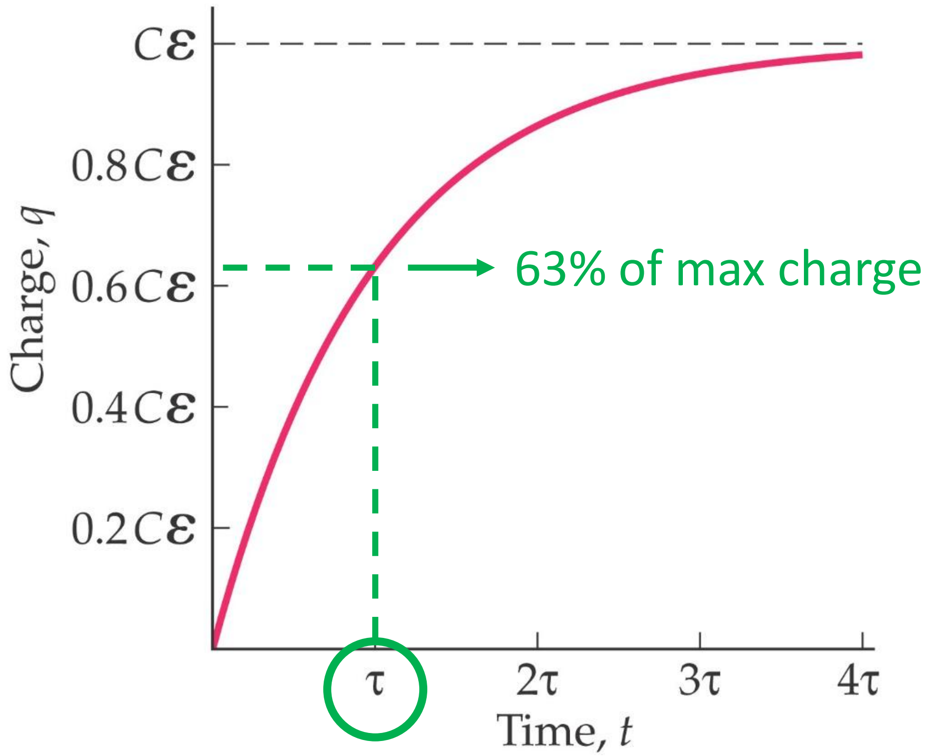
- The unit for the time constant is seconds.

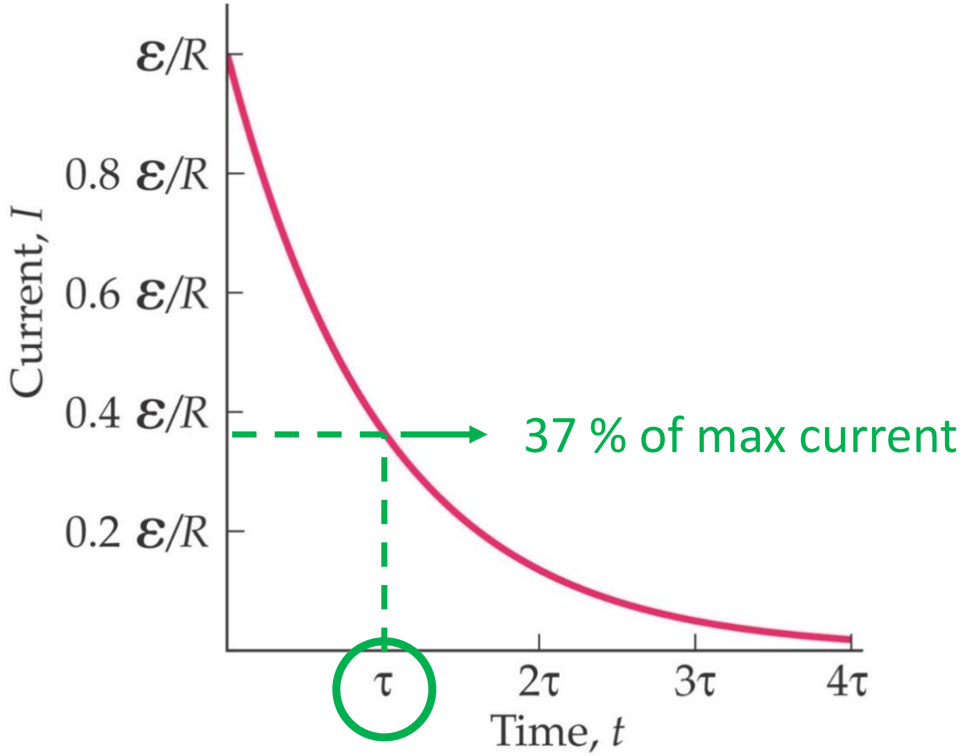
Ω · F = (V/A)(C/V) = C/(C/s) = s

- The time constant is a measure of how quickly the capacitor becomes charged.
- The time constant represents the time it takes the:
    - current to decrease to $1/e = 0.37$ of its initial (max) value ($0.37\ I_{\max}$).
    - charge to increase from 0 to $(1 - e^{-1}) = 0.63$ its final (max) value $(0.63\ Q_{\max})$

# RC Circuits – the time constant ($\boldsymbol{\tau}$)

After 1 time constant ($t = 1\tau$):

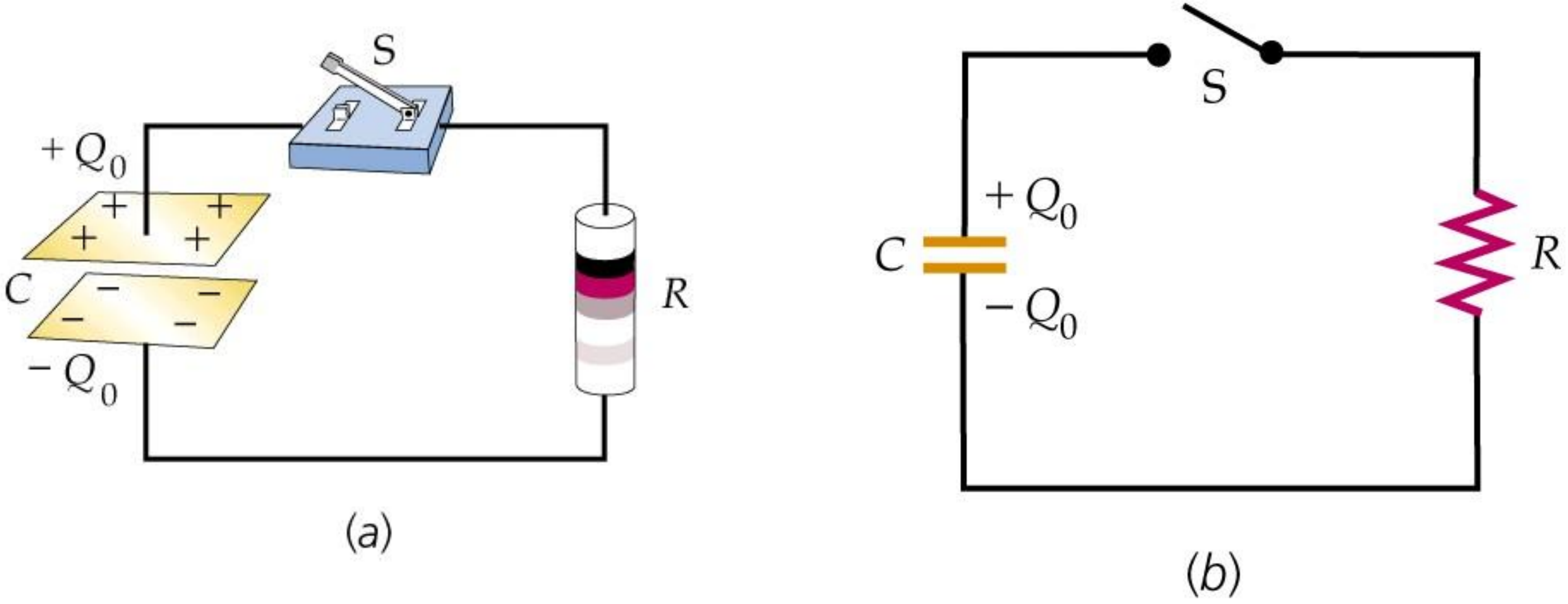


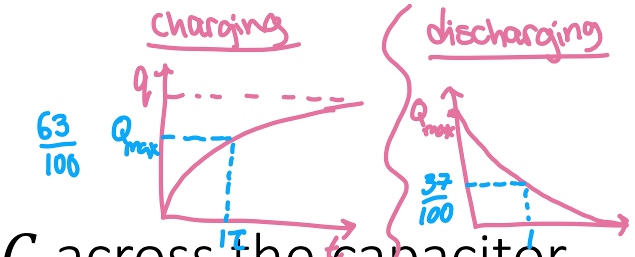

In a time $t$ of one time constant $\tau$, the charge q rises to 63% of its maximum, while the current $I$ decays to 37% of its maximum value

# RC Circuits – Discharging

Removing the battery from the circuit while keeping the switch open leaves us with a circuit containing only a charged capacitor and a resistor.



# RC Circuits – Discharging




- When the switch is open, there is a potential difference of $Q/C$ across the capacitor and $0$ V across the resistor since $I = 0$ A.
- If the switch is closed at time $t = 0$, the capacitor begins to discharge through the resistor and a current flows through the circuit.
- Both the current and the charge are maximum at $t = 0$, and zero after discharging is complete
- At some time during the discharge, current in the circuit is $I$ and the charge on the capacitor is $q$.

$V = IR$
$Q = CV$
$I = \frac{Q}{t}$

$\tau = RC$
$\frac{\cancel{V}}{I}\frac{Q}{\cancel{V}}$
$\frac{Q}{I} = t$

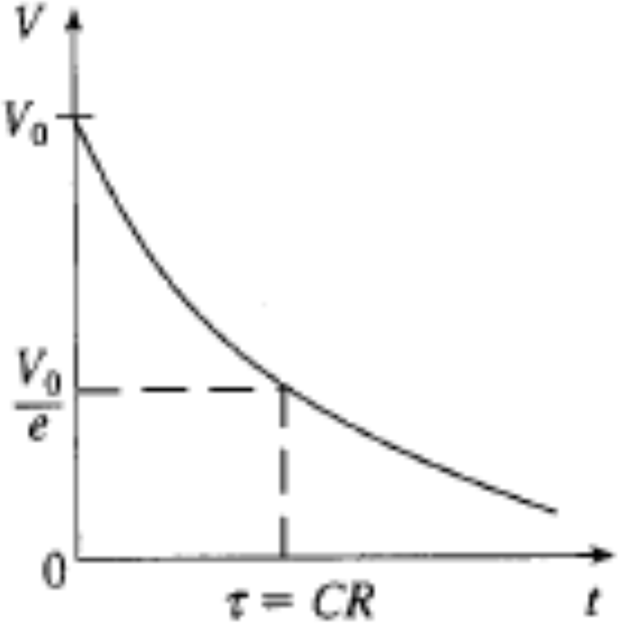
$\boxed{\tau = t}$

$$q(t) = \mathrm{Q}_{\max}\, e^{-t/RC}$$

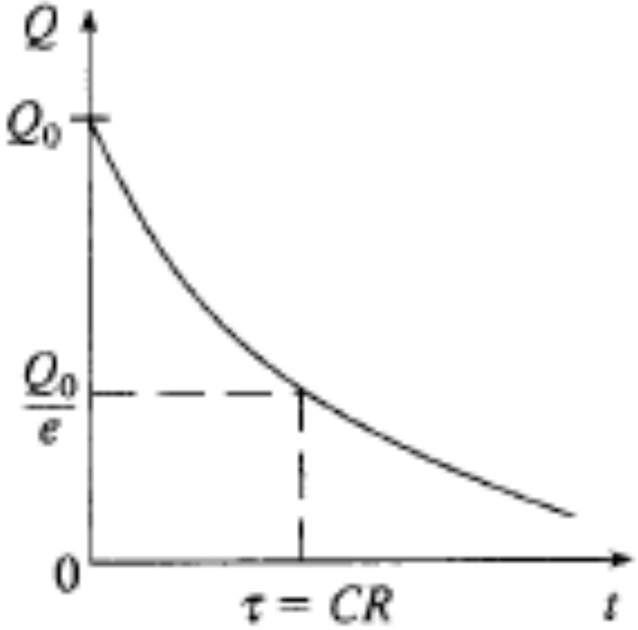
$$I(t) = I_{\max}\, e^{-t/RC}$$

- Note: $I_{\max} = I_0 = V_0/R$ and $Q_{\max} = Q_0 = CV_0$ where $V_0$ is the initial potential on the capacitor

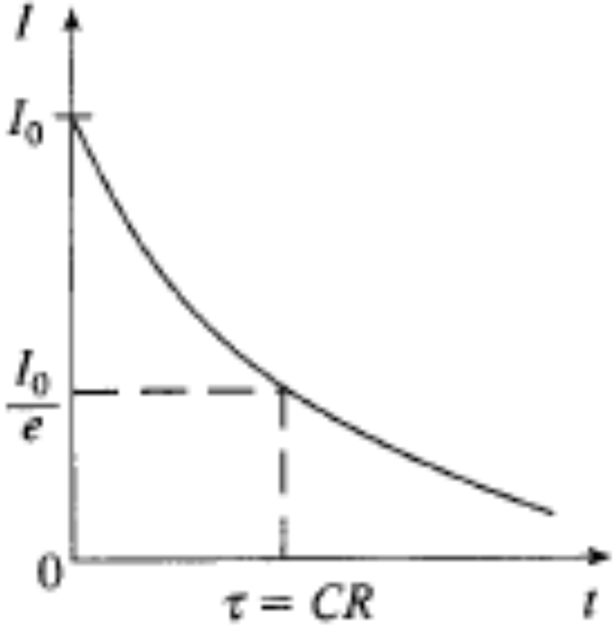
# RC Circuits – Discharging




voltage




charge




current

# Example

$C = 10\times10^{-6}F = 10^{-5}F$

$R = 1\times10^{6}\Omega$

A 10 µf capacitor is connected through a 1 MΩ resistance to a constant potential difference of 100 volt $\mathcal{E} = 100V$

a) What is the maximum current in the circuit? $I_{max} = \frac{\mathcal{E}}{R} = \frac{100}{10^6} = 10^{-4}A$

b) What is the maximum charge on the capacitor? $Q_{max} = C\mathcal{E} = 10^{-5} \times 100 = 10^{-3}C$

c) What is the charging current at t=5 s? $i = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} = 6.06\times10^{-5}A$

d) What is the charge on the capacitor at t= 20 s? $8.65\times10^{-4}\ C$

# Example

Consider the RC circuit shown in the figure. Find:

a) The time constant

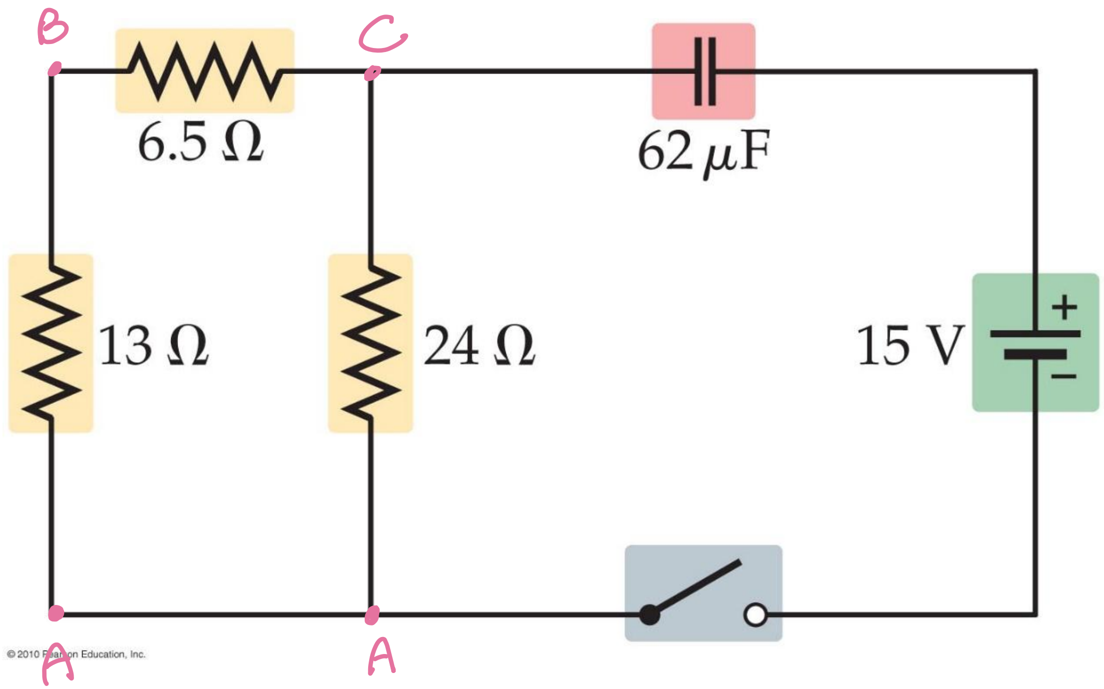
b) The initial current

c) It is desired to increase the time constant of this circuit by adjusting the value of the 6.5 Ω resistor. Should the resistance of this resistor be increased or decreased to have the desired effect?

Find R first:

$R_{series} = 13 + 6.5 = 19.5\,\Omega$

$R_{parallel} = \left(\frac{1}{19.5} + \frac{1}{24}\right)^{-1} = 10.8\,\Omega$



# Example

Start as charging

A capacitor is charged to a potential of 12 volts and then disconnected and reconnected to a voltmeter of internal resistance 3 MΩ. After 7 seconds, the voltmeter reads 5 volts.

a) What is the capacitance of the capacitor?

b) How long it takes the capacitor voltage to be decreased to 2 volts?

# Example

The time constant for an RC circuit is 14.4325s. What is the time required for the capacitor to lose one half its maximum charge?

Screenshot

# Example

How many time constants are required for a capacitor in an RC circuit to lose 70% of its charge?

$q = Q_{max} e^{-\frac{t}{\tau}}$

$\frac{30}{100} \cancel{Q_{max}} = \cancel{Q_{max}} e^{-\frac{t}{\tau}}$

$\frac{30}{100} = e^{-\frac{t}{\tau}}$

$\ln\left(\frac{30}{100}\right) = -\frac{t}{\tau}$

$1.2 = \frac{-t}{\tau}$

$t = 1.2\tau$

ï ?

$\frac{70}{100} \rightarrow \frac{7}{10}$

$\frac{3}{10}$

# Biomedical Applications: Problem 27

*Side Flash*. Figure 27.21 indicates one reason no one should stand under a tree during a lightening storm. If lightening comes down the side of the tree, a portion can jump over to the person, especially if the current on the tree reaches a dry region on the bark and thereafter must travel through air to reach the ground. In the figure, part of the lightening jumps through distance *d* in air and then travels through the person (who has negligible resistance relative to that of air because of the highly conductive salty fluids within the body). The rest of the current travels through air alongside the tree, for a distance *h*. If $d/h = 0.400$ and the total current is I= 5000 A, what is the current through the person?

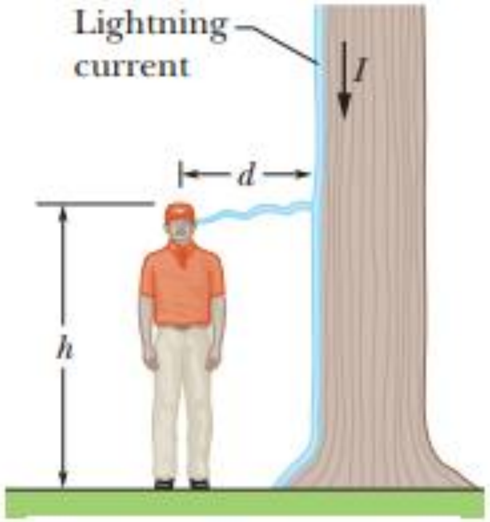



**Figure 27.21** Problem 27.

Answer: 3.6 kA

# Biomedical Applications: Problem 75

Suppose that, while you are sitting in a chair, charge separation between your clothing and the chair puts you at a potential of 200 v, with the capacitance between you and the chair at 150 pF. When you stand up, the increased separation between your body and the chair decreases the capacitance to 10 pF. (a) What then is the potential of your body? That potential is reduced over time, as the charge on you drains through your body and shoes (you are a capacitor discharging through a resistance). Assume that the resistance along that route is 300 GΩ. If you touch an electrical component while your potential is greater than 100 V, you could ruin the component. (b) How long must your wait until your potential reaches the safe level of 100 V?

If you wear a conducting wrist strap (Figure 27.55) that is connected to ground, your potential does not increase as much when you stand up; you also discharge more rapidly because the resistance through the grounding connection is much less than through your body and shoes. (c) Suppose that when you stand up, your potential is 1400 V and the chair-to-you capacitance is 10 pF. What resistance in that wrist-strap grounding connection will allow you to discharge to 100 V in 0.30 s, which is less time than you would need to reach for, say, your computer?


Photo Cornwall/Alamy Stock Photo

**Figure 27.55** Problem 75. Wrist strap to discharge static electric charge.

Answers: (a) $3.0\times10^{3}$ V, (b) 10 s, (c) $1.1\times10^{10}$ Ω