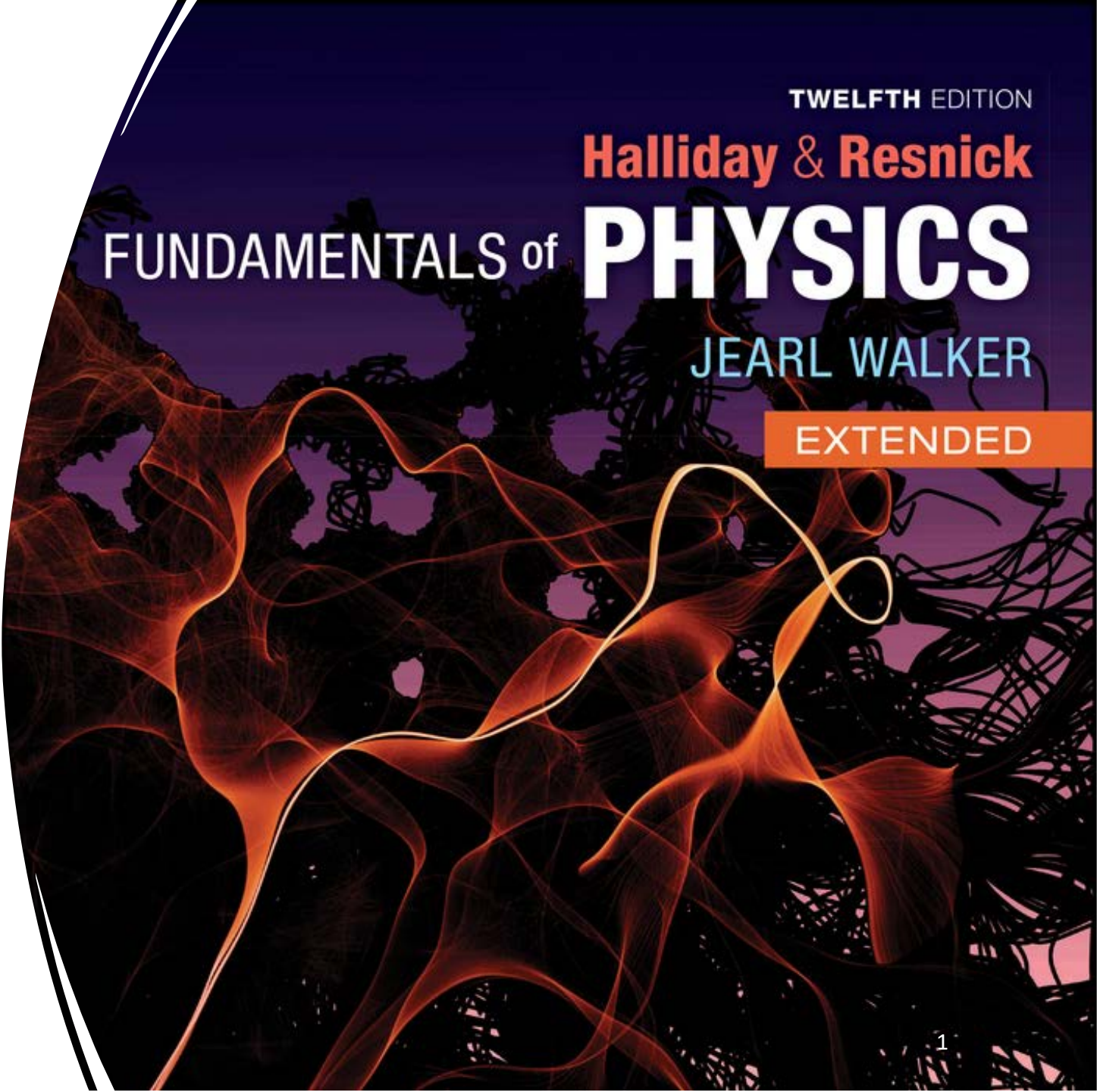


# Chapter 24

# Electric Potential

*Fundamentals of Physics*, Twelfth Edition.  
Halliday & Resnick, Walker



# Chapter 24

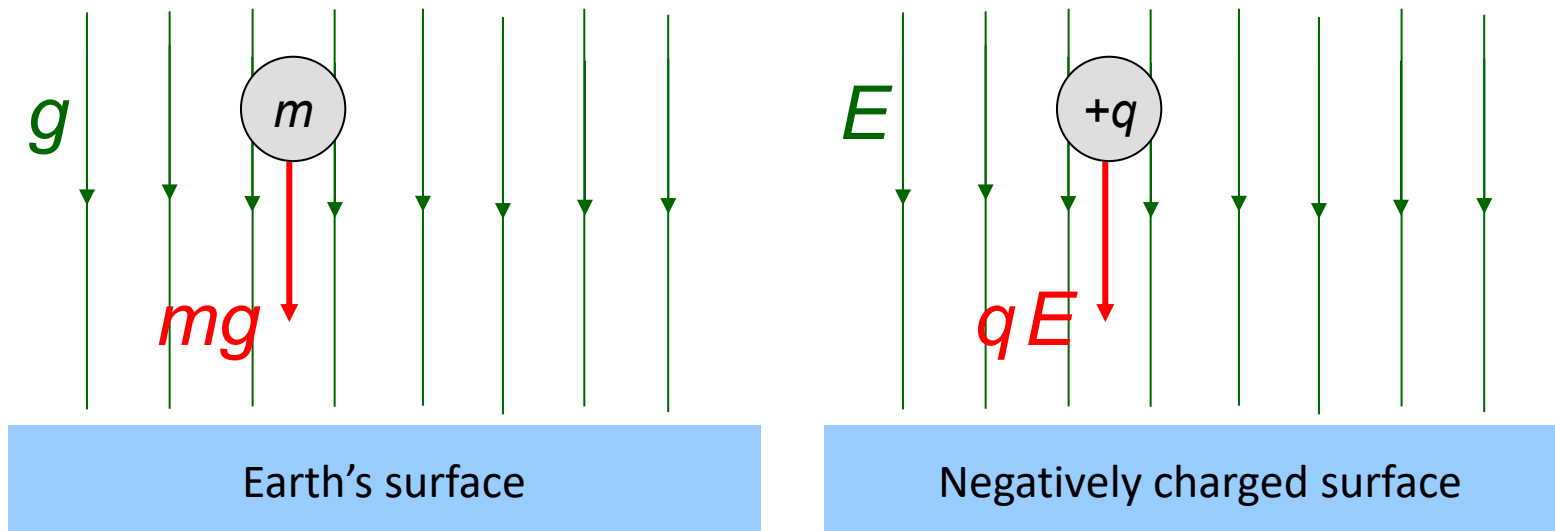
## Electric Potential

- **24.1 Electric Potential**
- **24.2 Equipotential Surfaces and the Electric field**
- **24.3 Potential Due to a Charged Particle**
- **24.6 Calculating the Electric Field from the Potential**
- **24.7 Electric Potential Energy of a System of Charged Particles**
- **24.8 Potential of a Charged Isolated Conductor**

# Recall

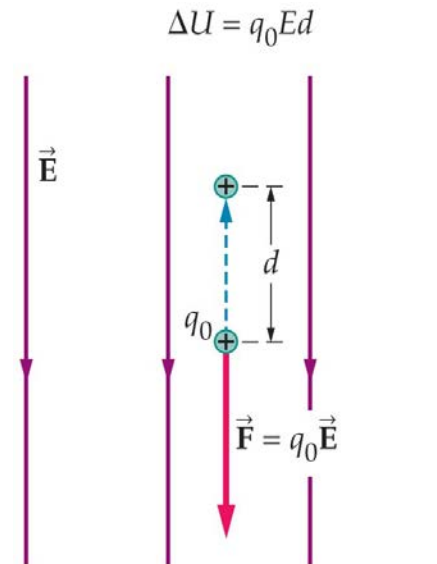
- Electric force is a *conservative force*
  - A force is said to be conservative if the **work** done by the force is independent of the path, that is, it depends only on the start and end points.
- Hence, a potential energy function ( $U$ ) associated with the electric force can be defined such that:

$$W = -\Delta U$$

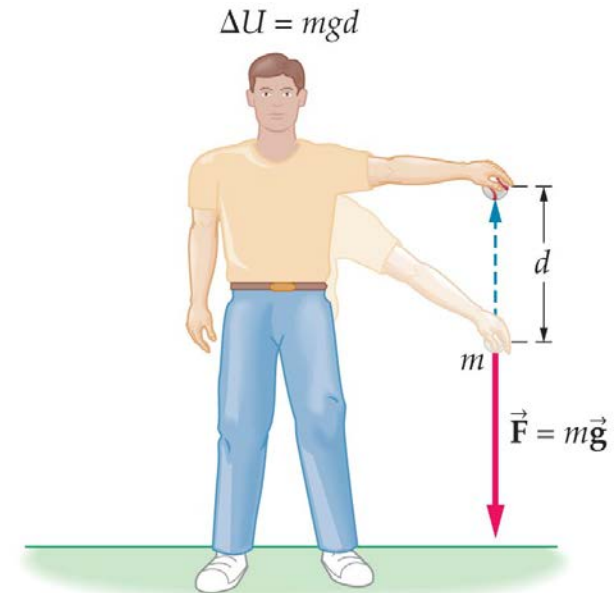


- Moving a mass **up** increases its gravitational potential energy by  $mgd$ 
  - $\Delta U = -W = -mgd \cos 180 = mgd$
  - Recall: work done by a *constant* force is  $W = |F||d| \cos \theta$  where  $\theta$  is the **angle** between the **force** and **displacement**
- Similarly, moving a **positive** charge **opposite** to a **uniform** external field ( $E$ ):
  - $\Delta U = -W = -qEd \cos 180 = qEd$
  - Try calculating  $\Delta U$  when the positive charged is moved in the direction of the field
  - What about a negative charge?
  - What if the charge is moved perpendicular to the field?
  - What if the charged is moved at an angle with respect to the field?

PE ↑ work done +  
 Force and motion same direction



(a) Moving a charge in an electric field  
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(b) Moving a mass in a gravitational field

IMP

- The electric potential energy of a positive charge:
  - decreases when it moves in the direction of the electric field ( $\Delta U -$ )
  - increases when it moves opposite to the electric field ( $\Delta U +$ )
- The electric potential energy of a negative charge:
  - increases when it moves in the direction of the electric field ( $\Delta U +$ )
  - decreases when it moves opposite to the electric field ( $\Delta U -$ )
- Note that in general the potential energy decreases in the direction of the electric force
- Electric potential energy ( $U$ ) a scalar physical quantity, measured in Joules (J)

$$\Delta U = q\Delta V$$

$$U = qV$$

- All-in-one formula when a charge is moved in a **uniform** electric field  $E$

$$\Delta U = -qE \Delta s$$

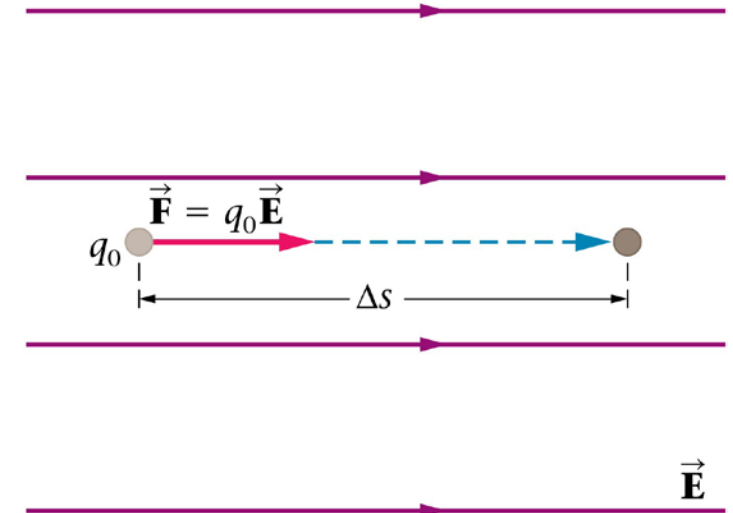
$\Delta s$ : displacement **parallel** to the electric field. Take it positive if it is in the direction of the field and negative if opposite to the direction of the field

$E$ : magnitude of the electric field

$q$ : the charge, with sign (+ or -)

*E field on y-axis  
and there's x & y*

*↳ take y*



Work (+ve)	$\Delta U$ (-ve)	$U \downarrow$
Work (-ve)	$\Delta U$ (+ve)	$U \uparrow$

$$\vec{F} = qE \begin{cases} \rightarrow q(+): F \& E \text{ (same direction)} \\ \rightarrow q(-): F \& E \text{ (opp direction)} \end{cases}$$

**$q(+)$**

1) $\rightarrow$ Motion $\rightarrow \vec{E} \rightarrow \vec{F}_E$	Work (+) $U \downarrow \Delta U (-)$
2) $\rightarrow$ Motion $\leftarrow \vec{E} \leftarrow \vec{F}_E$	Work (-) $U \uparrow \Delta U (+)$

**$q(-)$**

1) $\rightarrow$ Motion $\rightarrow \vec{E} \leftarrow \vec{F}_E$	Work (-) $U \uparrow \Delta U (+)$
2) $\rightarrow$ Motion $\leftarrow \vec{E} \rightarrow \vec{F}_E$	Work (+) $\rightarrow \vec{F}_E$ & motion same direction $U \downarrow \Delta U (-)$

PE increase  
did work to lift it  
which I lost energy  
transforming it

# 24.1 Electric Potential

Forces: charges  
E field: point

Fam1:  
\$100,000 → can be distributed  
between 2 so  
they \$50,000

Fam2:  
\$200,000 → but here 10 members  
so each \$20,000

**Definition** : The electric potential  $V$  at a point  $P$  in the electric field of a charged object is:

$$V = \frac{-W_{\infty}}{q_0} = \frac{U}{q_0}$$

(volts)

Electric field

220 volt plugs

110 volt

if 220 put 110 can detect  
it cause it will be less

where  $W_{\infty}$  is the work that would be done by the electric force on a positive test charge  $q_0$  were it brought from an infinite distance to  $P$ , and  $U$  is the electric potential energy that would then be stored in the test charge-object system.

# Potential Energy of Charged Particle

If a particle with charge  $q$  is placed at a point where the electric potential of a charged object is  $V$ , the electric potential energy  $U$  of the particle–object system is:

$$U = qV.$$

Handwritten annotations in blue ink: an arrow points from 'J' to  $U$ , an arrow points from 'C' to  $q$ , and an arrow points from 'volts (V)' to  $V$ .

The SI unit of the electric potential is  $\text{J/C} = \text{Volt (V)}$

**1 V = 1 J/C** : the energy of 1 C charge changes by 1 J when it moves through a potential difference of 1 V

# Example

- A  $3 \mu\text{C}$  charge is moved through a potential difference of  $640 \text{ V}$ . What is its potential energy change?

$$q = 3 \times 10^{-6} \text{ C}$$

$$V = 640 \text{ V}$$

$$U = qV \rightarrow (3 \times 10^{-6})(640) = 1.92 \times 10^{-3} \text{ J}$$

# Work vs. Change in Electric Potential

**Change in Electric Potential.** If the particle moves through a potential difference  $\Delta V$ , the change in the electric potential energy is:

$$U = qV$$
$$\Delta U = q\Delta V$$

$$\Delta U = q\Delta V = q(V_f - V_i).$$

**Work by the Field.** The work  $W$  done by the electric force as the particle moves from  $i$  to  $f$ :

$$W = -\Delta U = -q \Delta V = -q(V_f - V_i).$$

$$ME = KE + PE$$

conservation of ME

$$ME_i = ME_f$$

$$KE_i + PE_i = KE_f + PE_f$$

$$-U_f + U_i = KE_f - KE_i$$

$$-\Delta U = \Delta KE$$

$$KE = \frac{1}{2}mv^2$$

V: volt  
v: speed

# Work-Energy Conservation

**Conservation of Energy.** If a particle moves through a change  $\Delta V$  in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$\Delta K = -q \Delta V = -q(V_f - V_i).$$

not conservative  
(uncommon)

**Work by an Applied Force.** If some force in addition to the electric force acts on the particle, we account for that work

$$\Delta K = -\Delta U + W_{app} = -q \Delta V + W_{app}.$$

- In general, for a mass moving **from A to B** due to a conservative force:

$$\frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_B^2 + U_B$$

- For the electric force:

$$U = qV$$

- So that

$$\begin{aligned}\frac{1}{2}mv_B^2 &= \frac{1}{2}mv_A^2 + U_A - U_B \\ &= \frac{1}{2}mv_A^2 + q(V_A - V_B)\end{aligned}$$

- Or:

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -q \Delta V = -\Delta U$$

# Example

- A particle of mass  $2 \times 10^{-9}$  kg and charge 6 nC has a speed of 18 m/s at a point where the electric potential is 250 V. It moves through a point where the electric potential is 110 V. What is its speed at this second point? (answer: 34.11 m/s)

$$m = 2 \times 10^{-9} \text{ kg}$$

$$q = 6 \times 10^{-9} \text{ C}$$

$$v_i = 18 \frac{\text{m}}{\text{s}}$$

$$V_i = 250 \text{ V}$$

$$V_f = 110 \text{ V}$$

$$v_f = ?$$

$$ME_i = ME_f$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$

$$\frac{1}{2}(2 \times 10^{-9})(18)^2 + (6 \times 10^{-9})(250) = \frac{1}{2}(2 \times 10^{-9})v_f^2 + (6 \times 10^{-9})(110)$$

KE never -

PE can be -/+

# Example

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

- A proton is accelerated through a potential difference of  $-2 \text{ V}$

a) what is the change in its potential energy?  $\Delta U = ??$

$$\Delta U = q \Delta V$$

$$(1.6 \times 10^{-19}) (-2)$$

$$-3.2 \times 10^{-19} \text{ J}$$

b) how fast will this proton be moving if it started from rest?

$$v_i = 0 \frac{\text{m}}{\text{s}}$$

$$\Delta KE = -\Delta U = +3.2 \times 10^{-19} \text{ J}$$

$$KE_f - \cancel{KE_i} = 3.2 \times 10^{-19} \text{ J}$$

$$\frac{1}{2} m v_f^2 = 3.2 \times 10^{-19} \text{ J}$$

$$J = C \cdot V$$

$$U = qV$$

$$\Delta U = q \Delta V$$

$$ME_i = ME_f$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\Delta KE = -\Delta U$$

# Example

- A proton at rest is released in a **uniform electric field**. What potential difference must it move through in order to acquire a speed of  $6 \times 10^7$  m/s?

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$v_i = 0 \frac{\text{m}}{\text{s}}$$

$$v_f = 6 \times 10^7 \frac{\text{m}}{\text{s}}$$

$$q = +1.6 \times 10^{-19} \text{ C}$$

$$\Delta V = ?$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -q \Delta V$$

$$\frac{1}{2} (1.67 \times 10^{-27}) (6 \times 10^7)^2 - \frac{1}{2} (1.67 \times 10^{-27}) (0)^2 = -(1.6 \times 10^{-19}) \Delta V$$

$$\frac{3.006 \times 10^{-12}}{-1.6 \times 10^{-19}} = \frac{-(1.6 \times 10^{-19}) \Delta V}{-1.6 \times 10^{-19}}$$

$$\Delta V = -1.9 \times 10^7 \text{ V}$$

# Section 24.2

$\vec{E}$  and equipotential surface are perpendicular

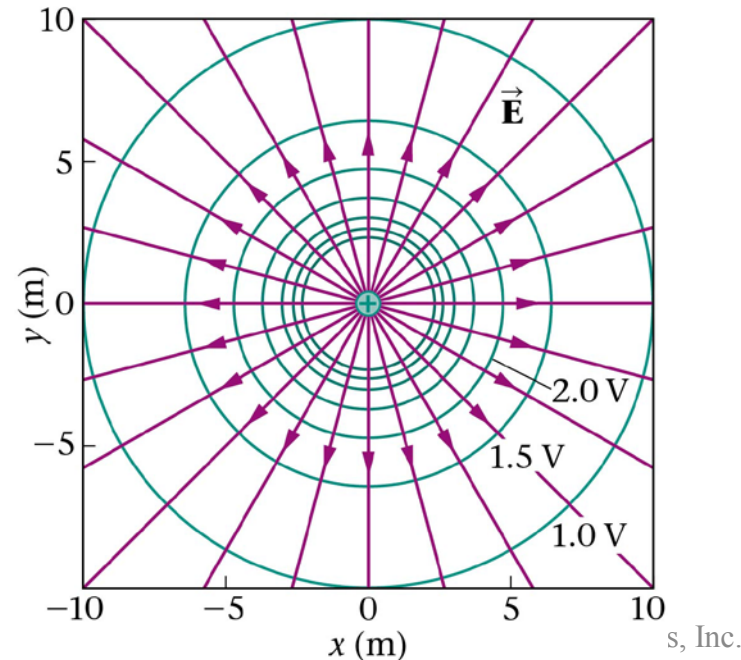
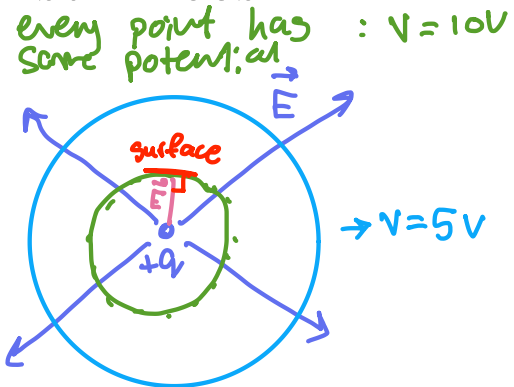
## Equipotential Surfaces and the Electric Field

### Characteristic of Equipotential Surface

surface at every point has same potential/same voltage

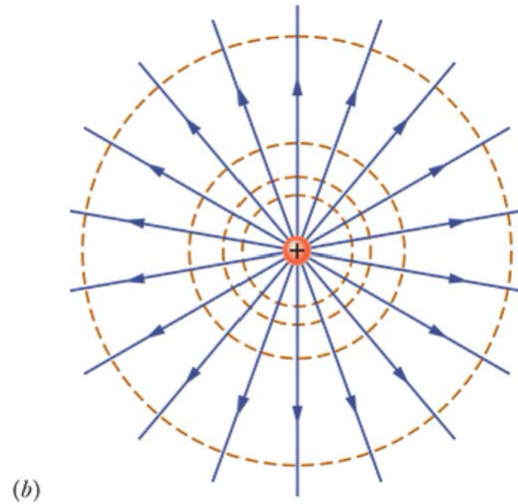
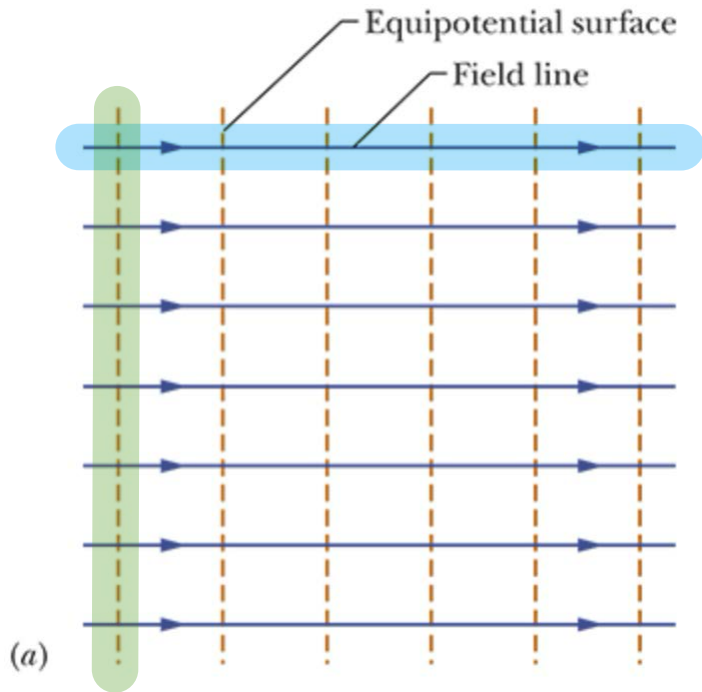
Adjacent points that have the **same electric potential** form an equipotential surface, which can be either an **imaginary surface** or a **real, physical surface**.

For the positive point charge below, the green lines represent equipotential surfaces

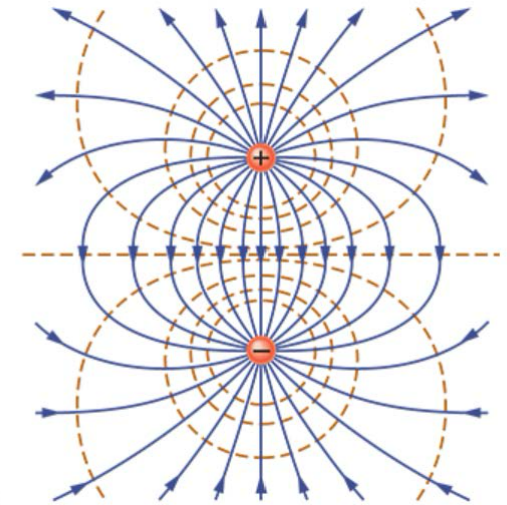


radial since electric potential come out like radiuses

# Examples of Equipotential Surfaces



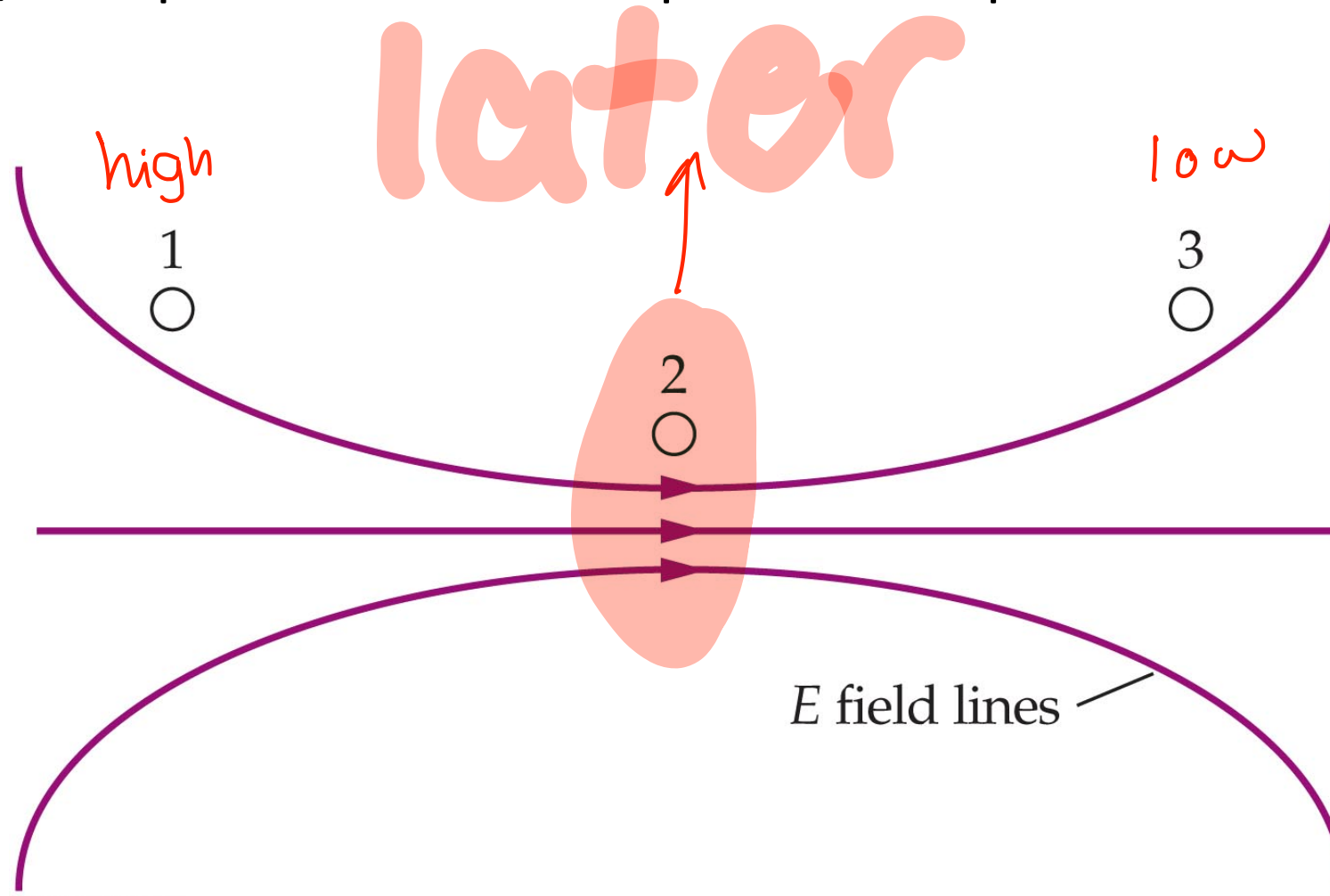
(b)



(c)

Electric field lines (blue) and cross sections of equipotential surfaces (gold) for (a) a uniform electric field, (b) the field due to a charged particle, and (c) the field due to an electric dipole.

- **Question:** Is the electric potential at point 1 in the figure greater than, less than, or equal to the electric potential at point 3?




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# Equipotential Surfaces and The Electric Field

- The work done to move a charge from any point on the equipotential surface to any other point on the equipotential surface is zero since they are at the same potential. Furthermore, equipotential surfaces are always perpendicular to the electric field lines

Work =  $-\Delta U$   
work =  $-q\Delta V$   
or  
Work =  $Fd \cos\theta$       $\cos(90) = 0$  [perpendicular]

*up & v; same so diff = 0*

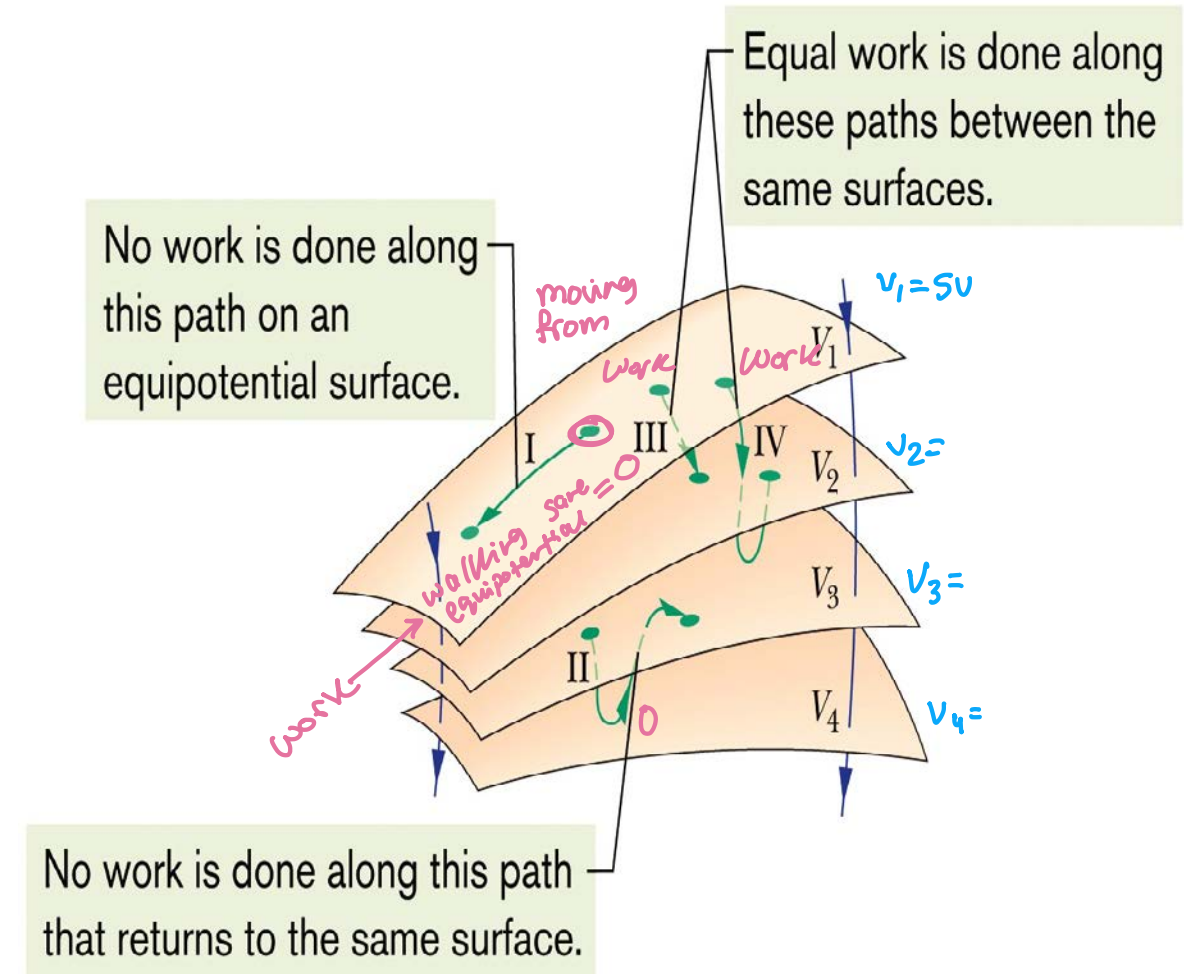


- The electric field must be perpendicular to equipotential lines. Why?
  - Otherwise, work would be required to move a charge along an equipotential surface, and it would not be equipotential.
- In the static case (charges not moving) the surface of a conductor is an equipotential surface. Why?
  - Otherwise, charges would flow, and then it wouldn't be a static case.

# Energy Changes for Paths Within and Between Equipotential Surfaces

The adjacent figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges.

- The work done by the electric field on a charged particle as the particle moves from one end to the other of paths **I** and **II** is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential.
- The work done as the charged particle moves from one end to the other of paths **III** and **IV** is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths **III** and **IV** connect the same pair of equipotential surfaces.

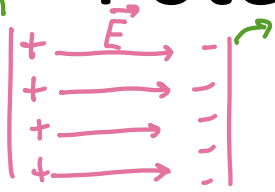


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# Potential Difference vs. Travel Along Path in Electric Field

high potential



+ high potential  
- low potential

Travel from High potential to low potential

➤ The electric potential difference between two points  $i$  and  $f$  is :

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s},$$

Ed      Lebanese dialect

where the integral is taken over any path connecting the points.

➤ In a **uniform field** of magnitude  $E$ , the change in potential from a higher equipotential surface to a lower one, separated by distance  $\Delta x$ , is

$$\Delta V = -E \Delta x.$$

same exactly as

- From this relation, electric field has units of V/m, which is the same as N/C
- The electric potential decreases in the direction of the electric field ( $\Delta V - ve$ ) and increases opposite to the direction of the electric field ( $\Delta V + ve$ )

Work =  $-\Delta U$   
 Work =  $-q \Delta V$   
 $\int \vec{F} \cdot d\vec{s} = -q \Delta V$   
 $\int q \vec{E} \cdot d\vec{s} = -q \Delta V$   
 $-\int \vec{E} \cdot d\vec{s} = \Delta V = V_f - V_i$

Now  $E$  has 2 units

$\Delta V = -E d$        $F = qE$

$\downarrow$  V     $\downarrow$  V/m     $\downarrow$  m       $\downarrow$  N     $\downarrow$  C     $\downarrow$  N/C

# Example

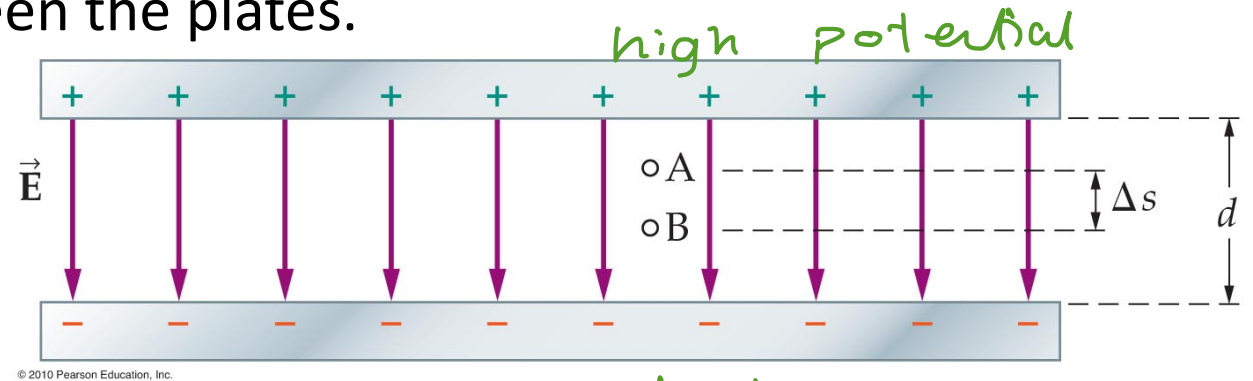
# IMP

The electric potential at point B in the parallel plate capacitor shown is less than the electric potential at point A by 4.5 V. The separation between the points A and B is 0.12 cm, and the separation between the plates is 2.55 cm.

- Find the electric field within the capacitor
- Find the potential difference between the plates.

①  $V_A - V_B = 4.5 \text{ V}$   
 $V_B - V_A = -4.5 \text{ V}$   
 $\Delta V = -Ed$       $d = 2.55 \times 10^{-2} \text{ m}$   
 $\frac{-4.5}{0.12 \times 10^{-2}} = \frac{+ E \times (0.12 \times 10^{-2})}{0.12 \times 10^{-2}}$   
 $E = 3750 \frac{\text{V}}{\text{m}}$

*d parallel to  $\vec{E}$*



*low*  
uniform:  $\vec{E}$  at A is not diff to  $\vec{E}$  at B

②  $\Delta V = -Ed \rightarrow (-3750)(2.55 \times 10^{-2})$   
 $-95.625 \text{ V}$

# Section 24.3

## Potential due to a Charged Particle

Potential Change for a Radial Path through a Particle-Generated Electric Field (part one)

The electric potential difference between two points  $i$  and  $f$  is:

$$V_f - V_i = -\int_i^f E \, d\vec{s},$$

For radial path



$$V_f - V_i = -\int_R^\infty E \, dr.$$

The magnitude of the electric field at the site of the test charge



$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

We set  $V_f = 0$  (at  $\infty$ )

and  $V_i = V$  (at  $R$ )



$$0 - V = -\frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} \, dr = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} \right]_R^\infty$$

Missed

## Potential Change for a Radial Path through a Particle-Generated Electric Field (part two)

Solving for  $V$  and switching  $R$  to  $r$ , we get the electric potential for a **point charge**:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

The potential due to a **collection of charged** particles is:

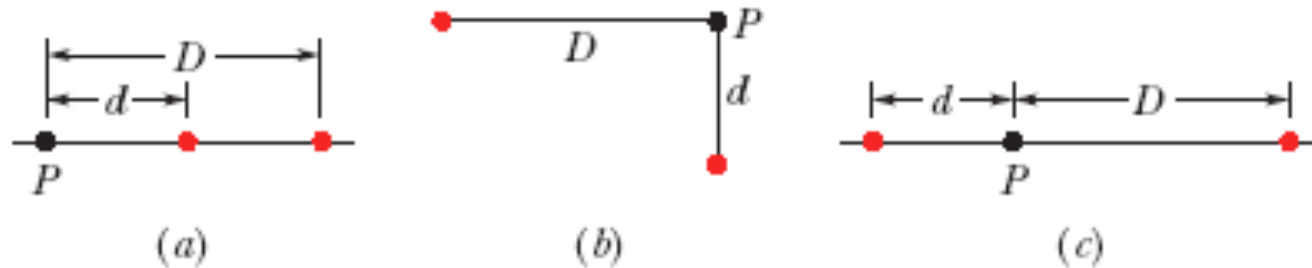
$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Thus, the potential is the **algebraic sum** of the individual potentials, with **no consideration of directions**.

A **positively** charged particle produces a **positive electric potential**. A **negatively** charged particle produces a **negative electric potential**.

# Potential due to a Charged Particle: Checkpoint

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point  $P$  by the protons, greatest first.



**Answer:**

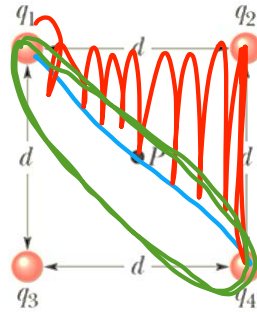
Same net potential (a) = (b) = (c)

# Home Practice

What is the electric potential at point  $P$ , located at the center of the square of charged particles shown in [Fig. 24.3.3a](#)? The distance  $d$  is 1.3 m, and the charges are

$$q_1 = +12 \text{ nC}, \quad q_3 = +31 \text{ nC}, \\ q_2 = -24 \text{ nC}, \quad q_4 = +17 \text{ nC}.$$

calculate Electric PE of the system:



(a)

Figure 24.3.3 (a) Four charged particles.

$$U = \frac{k q_1 q_3}{d} + \frac{k q_1 q_4}{d\sqrt{2}} + \frac{k q_3 q_4}{d} \\ = 2.58 \times 10^{-6} + 9.98 \times 10^{-7} + 3.65 \times 10^{-6}$$

## Solution:

$$V = \sum_{i=1}^4 V_i = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r} \right).$$

The distance  $r$  is,  $d/\sqrt{2}$ , which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9} \text{ C} \\ = 36 \times 10^{-9} \text{ C}.$$

Thus,

$$V = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}}$$

$$\approx 350 \text{ V}.$$

(Answer)

# Section 24.6 Calculating the Field from the Potential

## Work to Move a Charge Through an Electric Field

Suppose that a positive test charge  $q_0$  moves through a displacement  $d\vec{s}$  from one equipotential surface to the adjacent surface. The work the electric field does on the test charge during the move is  $-q_0 dV$ . On the other hand the work done by the electric field may also be written as the scalar product  $(q_0 \vec{E}) \cdot d\vec{s}$ .

Equating these two expressions for the work yields:

$$-q_0 dV = q_0 E (\cos \theta) ds,$$

or

$$E \cos \theta = -\frac{dV}{ds}.$$

Since  $E \cos \theta$  is the component of  $\vec{E}$  in the direction of  $d\vec{s}$ , we get:

$$E_s = -\frac{\partial V}{\partial s}.$$

Partial derivative ←

$$\text{Work}_{F_e} = -\Delta V$$

$$U = qV$$

$$\text{Work} = -q\Delta V$$

$$F_e \Delta s \cos\theta = -q\Delta V$$

~~$$qE \Delta s \cos\theta = -q\Delta V$$~~

$$E_x \Delta s \cos\theta = -\Delta V$$

$$E_x \Delta x = -\Delta V$$

$$E_x dx = -\partial V$$

$$E_x = \frac{-\partial V}{\partial x}$$

$$E_y = \frac{-\partial V}{\partial y}$$

If we take the  $s$  axis to be, in turn, the  $x$ ,  $y$ , and  ~~$z$~~ , we find that the  $x$ ,  $y$ , and  $z$  components of  $\vec{E}$  at any point are:

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad \cancel{E_z = -\frac{\partial V}{\partial z}}$$

not anymore with us

Thus, if we know for all points in the region around a charge distribution—that is, if we know the function  $V(x,y,z)$  we can find the components of  $\vec{E}$ , and thus  $\vec{E}$  itself, at any point by taking partial derivatives.

For the simple situation in which the electric field  $\vec{E}$  is uniform

$$E = -\frac{\Delta V}{\Delta s}$$

$$\Delta V = -Ed$$

↓ volt     ↓  $\frac{V}{m}$       $\rightarrow m$

## Extra Practice 1

**35 E CALC** The electric potential at points in an  $xy$  plane is given by  $V = (2.0 \text{ V/m}^2)x^2 - (3.0 \text{ V/m}^2)y^2$ . In unit-vector notation, what is the electric field at the point (3.0 m, 2.0 m)?  $(3 \text{ m}, 2 \text{ m})$

**Solution:**

$$V = 2x^2 - 3y^2$$

$$E_x = -\frac{\partial V}{\partial x} = (-4)(3) = -12 \frac{\text{V}}{\text{m}} (-\hat{x})$$

$$-(-6y) = +6y \text{ at } y=2\text{m}$$

$$E_y = -\frac{\partial V}{\partial y} = -(-6)(2) = 12 \frac{\text{V}}{\text{m}} (+\hat{y})$$

35. We use Eq. 24.6.4:

$$E_x(x, y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}((2.0 \text{ V/m}^2)x^2 - 3.0 \text{ V/m}^2)y^2) = -2(2.0 \text{ V/m}^2)x;$$

$$E_y(x, y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}((2.0 \text{ V/m}^2)x^2 - 3.0 \text{ V/m}^2)y^2) = 2(3.0 \text{ V/m}^2)y.$$

Not in slide but will be needed:

$$E = \sqrt{(-12)^2 + (12)^2} = 16.9 \frac{\text{V}}{\text{m}}$$

We evaluate at  $x = 3.0 \text{ m}$  and  $y = 2.0 \text{ m}$  to obtain

$$\vec{E} = (-12 \text{ V/m})\hat{i} + (12 \text{ V/m})\hat{j}.$$

$$\theta = \tan^{-1}\left(\frac{12}{-12}\right) = 135^\circ$$

add 180

## Extra Practice 2

**36 E CALC** The electric potential  $V$  in the space between two flat parallel plates 1 and 2 is given (in volts) by  $V = 1500x^2$ , where  $x$  (in meters) is the perpendicular distance from plate 1. At  $x = 1.3$  cm, (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?

$$x = 0.013 \text{ m}$$

**Solution:**  $E_x = -\frac{\partial V}{\partial x} \rightarrow -(2 \times 1500x) \text{ at } x = 0.013 \text{ m}$   $E_x = -39 \frac{\text{V}}{\text{m}} (-\hat{x})$

36. We use Eq. 24.6.4. This is an ordinary derivative since the potential is a function of only one variable.

$$\begin{aligned}\vec{E} &= -\left(\frac{dV}{dx}\right)\hat{i} = -\frac{d}{dx}(1500x^2)\hat{i} = (-3000x)\hat{i} = (-3000 \text{ V/m}^2)(0.0130 \text{ m})\hat{i} \\ &= (-39 \text{ V/m})\hat{i}.\end{aligned}$$

(a) Thus, the magnitude of the electric field is  $E = 39 \text{ V/m}$ .

(b) The direction of  $\vec{E}$  is  $-\hat{i}$ , or toward plate 1.

# Extra practice

## Extra Practice 3

not required

**37** **M** **CALC** **SSM** What is the magnitude of the electric field at the point  $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})$  m if the electric potential in the region is given by  $V = 2.00xyz^2$ , where  $V$  is in volts and coordinates  $x$ ,  $y$ , and  $z$  are in meters?

$$E_x = -\frac{\partial V}{\partial x} \rightarrow -2yz^2$$

**Solution:**

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2,$$

$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2,$$

$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz,$$

$$E_y = -\frac{\partial V}{\partial y} \rightarrow -2xz^2$$

which, at  $(x, y, z) = (3.00 \text{ m}, -2.00 \text{ m}, 4.00 \text{ m})$ , gives

$$(E_x, E_y, E_z) = (64.0 \text{ V/m}, -96.0 \text{ V/m}, -64.0 \text{ V/m}).$$

**ANALYZE** The magnitude of the field is therefore

$$\begin{aligned} |\vec{E}| &= \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0 \text{ V/m})^2 + (-96.0 \text{ V/m})^2 + (-64.0 \text{ V/m})^2} \\ &= 150 \text{ V/m} = 150 \text{ N/C}. \end{aligned}$$

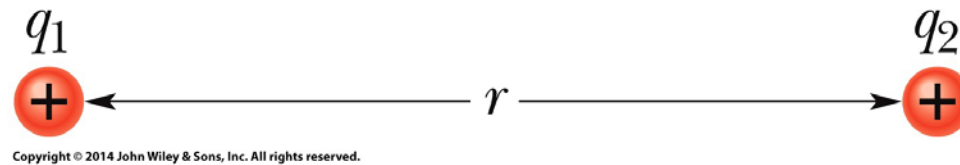
## Section 24.7 Electric Potential Energy of a System of Charged Particles

### Electric Potential Energy Between a Pair of Charged Particles

- The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.
- The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other.

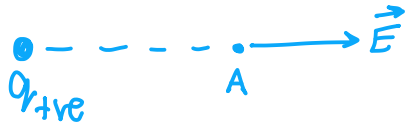
For two particles at separation  $r$ ,

*Boi*



$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (\text{two-particle system}).$$

Electric : enough to have 1 charge



$$E_A = k \frac{|q|}{r^2}$$

$$V_A = \frac{kq}{r}$$

no direction  
so no components...

no way  
negative  
(obv)

the  
high  
potential

-ve  
low  
potential



$$F = k \frac{|q_1 q_2|}{r^2}$$

$$E_A = \frac{k|q|}{r^2}$$

$$U = \frac{k q_1 q_2}{r} \quad \text{--- } C$$

$$V_A = \frac{kq}{r}$$

forgetting  $r^2$   
makes it that  
you are calculating  
something else

# Electric Potential Energy Amongst Three Charged Particles (part one)

## Sample Problem

The figure shows three charges fixed at the vertices of an equilateral triangle. . What is the electric potential energy  $U$  of this system of charges?

Assume that  $d = 12$  cm and that:

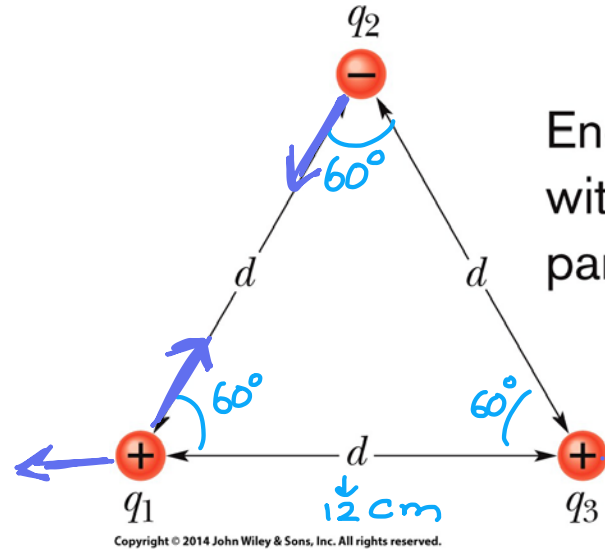
$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which  $q = 150$  nC.

$$q_1 q_2 \quad U = \frac{k q_1 q_2}{r} = \frac{9 \times 10^9 (150 \times 10^{-9}) (-4 (150 \times 10^{-9}))}{0.12} = -6.75 \times 10^{-3}$$

$$q_2 q_3$$

$$q_1 q_3$$



Energy is associated with each pair of particles.

What is the electric potential energy of the system?  $U$

$$U = \frac{k q_1 q_2}{r} + \frac{k q_1 q_3}{r} + \frac{k q_2 q_3}{r}$$

$$-6.75 \times 10^{-3} + 3.375 \times 10^{-3} + -0.0135$$

$$-0.016875 \text{ J}$$

# Answer to Sample Problem

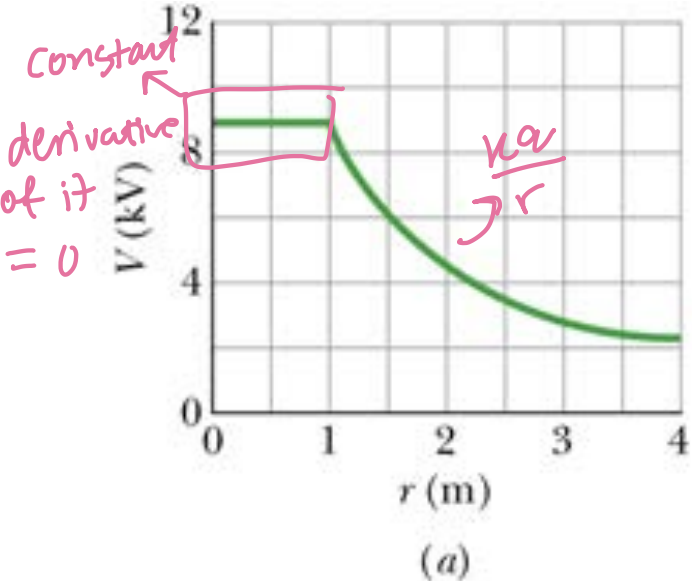
$$\begin{aligned}U &= U_{12} + U_{13} + U_{23} \\&= \frac{1}{4\pi\epsilon_0} \left( \frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d} \right) \\&= -\frac{10q^2}{4\pi\epsilon_0 d} \\&= -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10)(150 \times 10^{-9} \text{ C})^2}{0.12 \text{ m}} \\&= -1.7 \times 10^{-2} \text{ J} = -17 \text{ mJ.} \quad \text{(Answer)}\end{aligned}$$

**The negative potential energy means that negative work would be done in to create the three-charge structure.**

# Section 24.8 Potential of Charged Isolated Conductor

## Potentials and Electrical Field Strength of a Charged Isolated Conductor

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor whether on the surface or inside come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge. Our proof follows directly from:

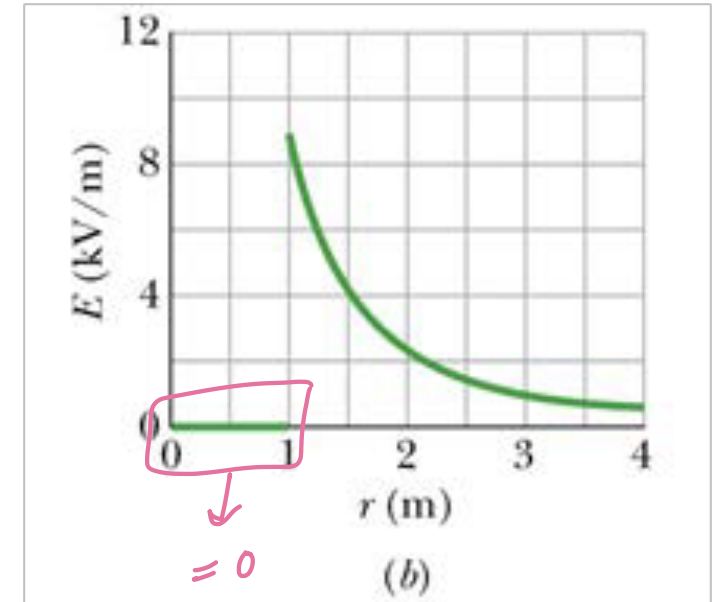


can get them from each other

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$

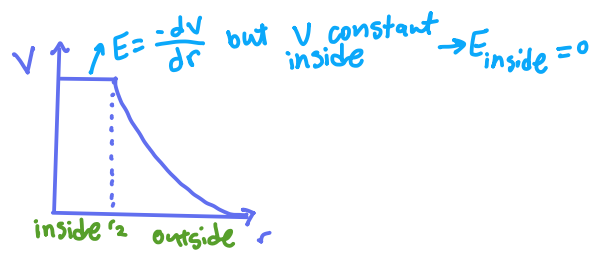
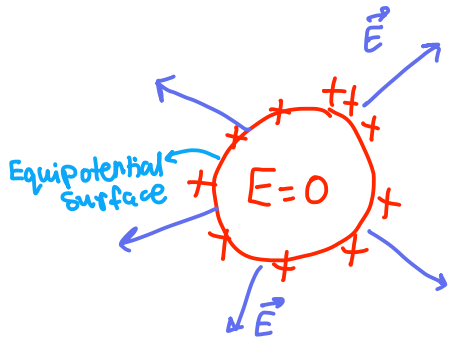
$$E_s = -\frac{\partial V}{\partial s}$$

$$E_r = -\frac{\partial V}{\partial r}$$



a) A plot of  $V(r)$  both inside and outside a charged spherical shell of radius 1.0 m.

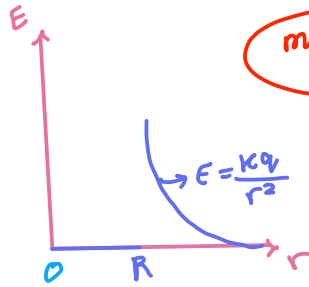
b) A plot of  $E(r)$  for the same shell.



$$V = kq r^{-1}$$

$$E = -\frac{dV}{dr} = -kq(-r^{-2})$$

$$E = +\frac{kq}{r^2}$$



most probably MCQ

# Practical Application

It is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero?

A car (unless it is a convertible or made with a plastic body) is almost ideal!



Courtesy Westinghouse Electric Corporation