Tue 13 Jan

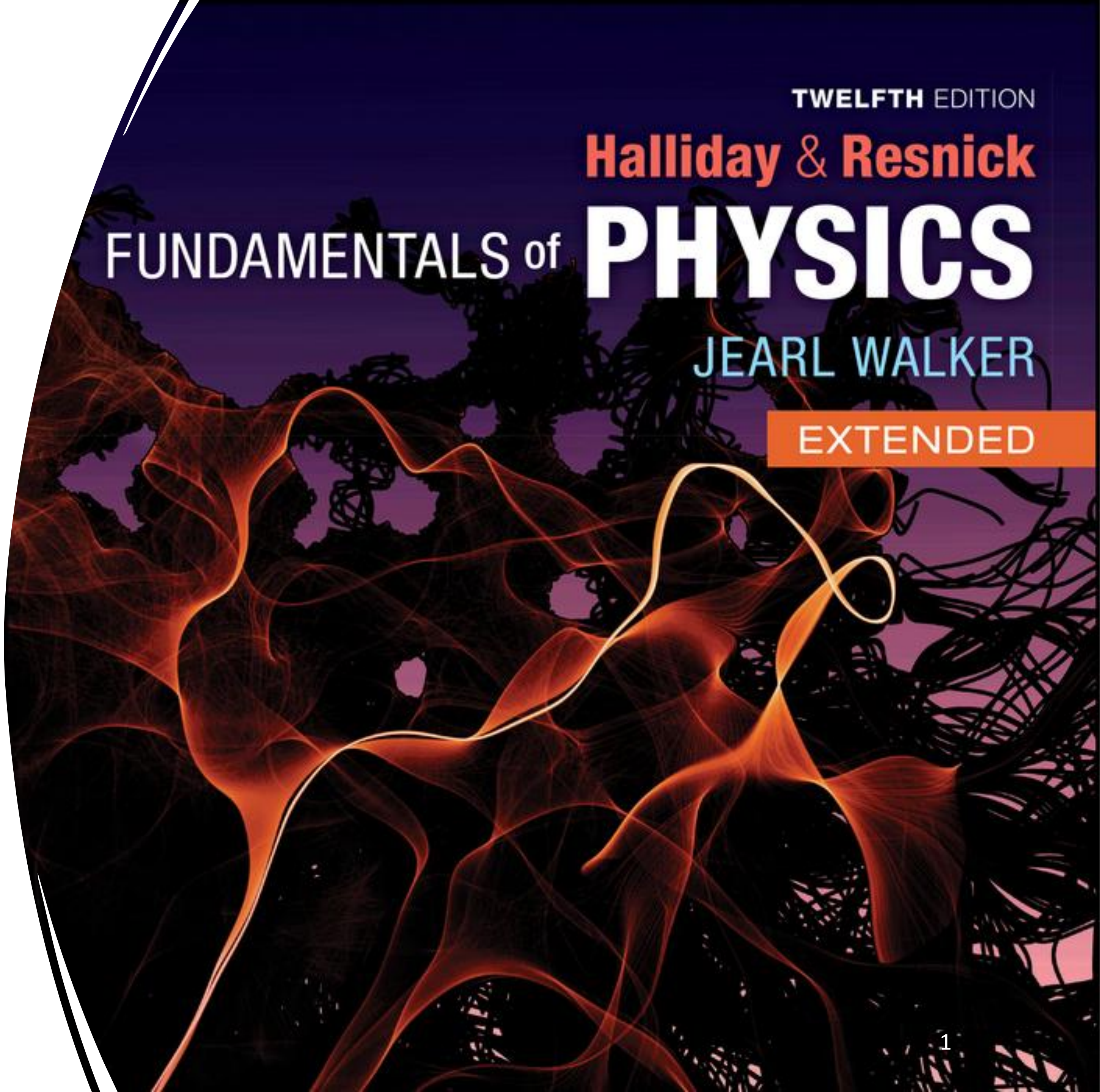
Chapter 23
Gauss' Law

23.1 Electric Flux → Flow

23.2 Gauss' Law

23.3 A Charged Isolated Conductor

***Fundamentals of Physics*, Twelfth Edition.**
Halliday & Resnick, Walker



Recap:

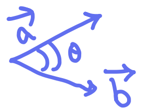
Dot product/scalar product:

$|\vec{a}|\cdot|\vec{b}|\cos\theta$

$\theta = 90^\circ$ → 0

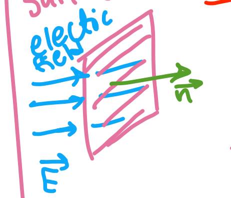
$\theta = 0^\circ$ → max

$\theta = 180^\circ$ → −ve



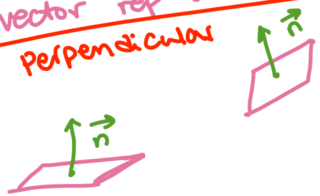
$\cos 90 = 0$	$\cos 0 = 1$	$\cos 180 = 1$
$\sin 90 = 1$	$\sin 0 = 0$	$\sin 180 = 0$

Flux = Flow



vector rep area = normal

perpendicular



$$\Phi = \int \vec{E}\cdot d\vec{A}$$

$$\Phi = \int E \times dA \times \cos\theta$$

angle between $\vec{E}$ & $\vec{n}$ (normal to Area)

if E is constant & uniform & flat surface

$$\boxed{\Phi = E\cos\theta \times A}$$

$$\boxed{K = \frac{1}{4\pi\varepsilon_0}}$$

$$F = \frac{K|q_1 q_2|}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Section 23.1 **Electric Flux**

Charge Enclosed by a Surface vs. Electric Field at the Surface

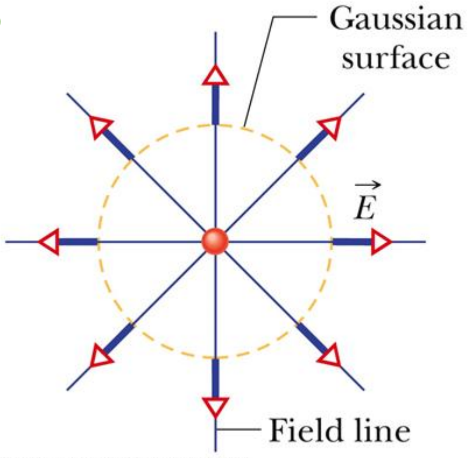
Gauss’ law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

q+: away

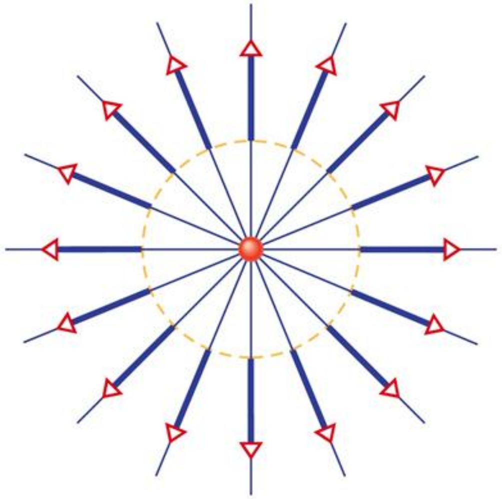
yellow is imaginary

sphere chosen because

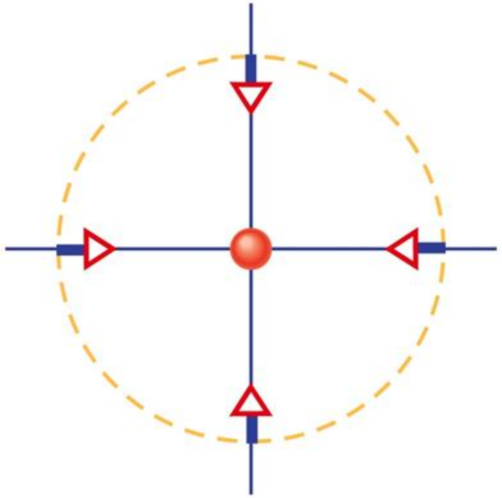
if rectangle some points would have different charge



Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$.



Now the enclosed particle has charge $+2Q$.



Can you tell what the enclosed charge is now?

Answer: $-0.5Q$

Halfed and towards

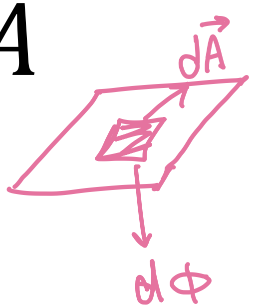
Area Vector and Electric Flux

The area vector $d\vec{A}$ for an area element (patch element) on a surface is a vector that is perpendicular to the area element and has a magnitude equal to the area dA of the element.

The **electric flux $d\Phi$** through a patch element with area vector $\boldsymbol{d\vec{A}}$ is given by a dot product:

flux

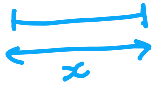
$$d\Phi = \vec{E} \cdot d\vec{A}$$



stronger field, more flux
bigger area, " "

$\int dx = x$

dx means I have x and I cut it into infinitismal parts those are called

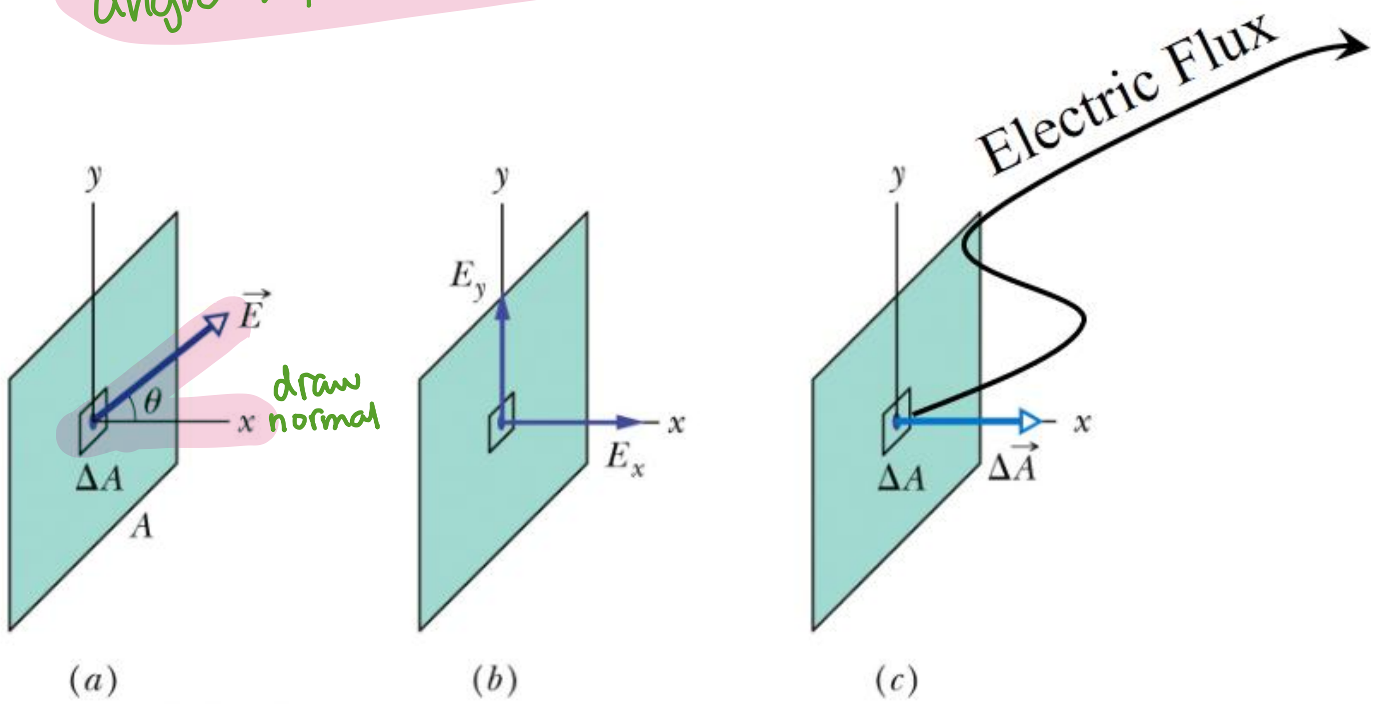


which gives the whole x

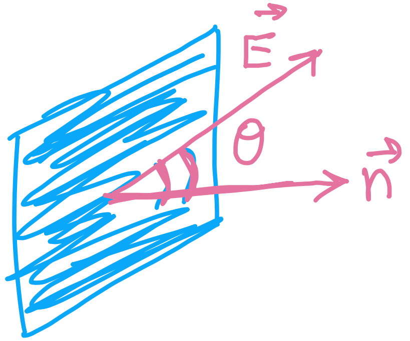
Σ: adding discrete amount
$\int$: adding continuous part

Geometric Definition of Electric Flux

angle with $\vec{E}$ and $\vec{n}$



$$\Delta\Phi = \left(E\cos\theta\right)\Delta A.$$



a) An electric field vector pierces a small square patch on a flat surface.

b) Only the *x* component actually pierces the patch; the *y* component skims across it.

c) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.

Calculating Total and Net Electric Flux

Now we can find the total flux by integrating the dot product over the full surface.

The **total flux** through a **surface** is given by

we integrate to get whole

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \text{(total flux)}.$$

$\cancel{\int d\Phi} = \int \vec{E} \cdot dA$

The **net flux** through a **closed surface** (which is used in Gauss' law) is given by

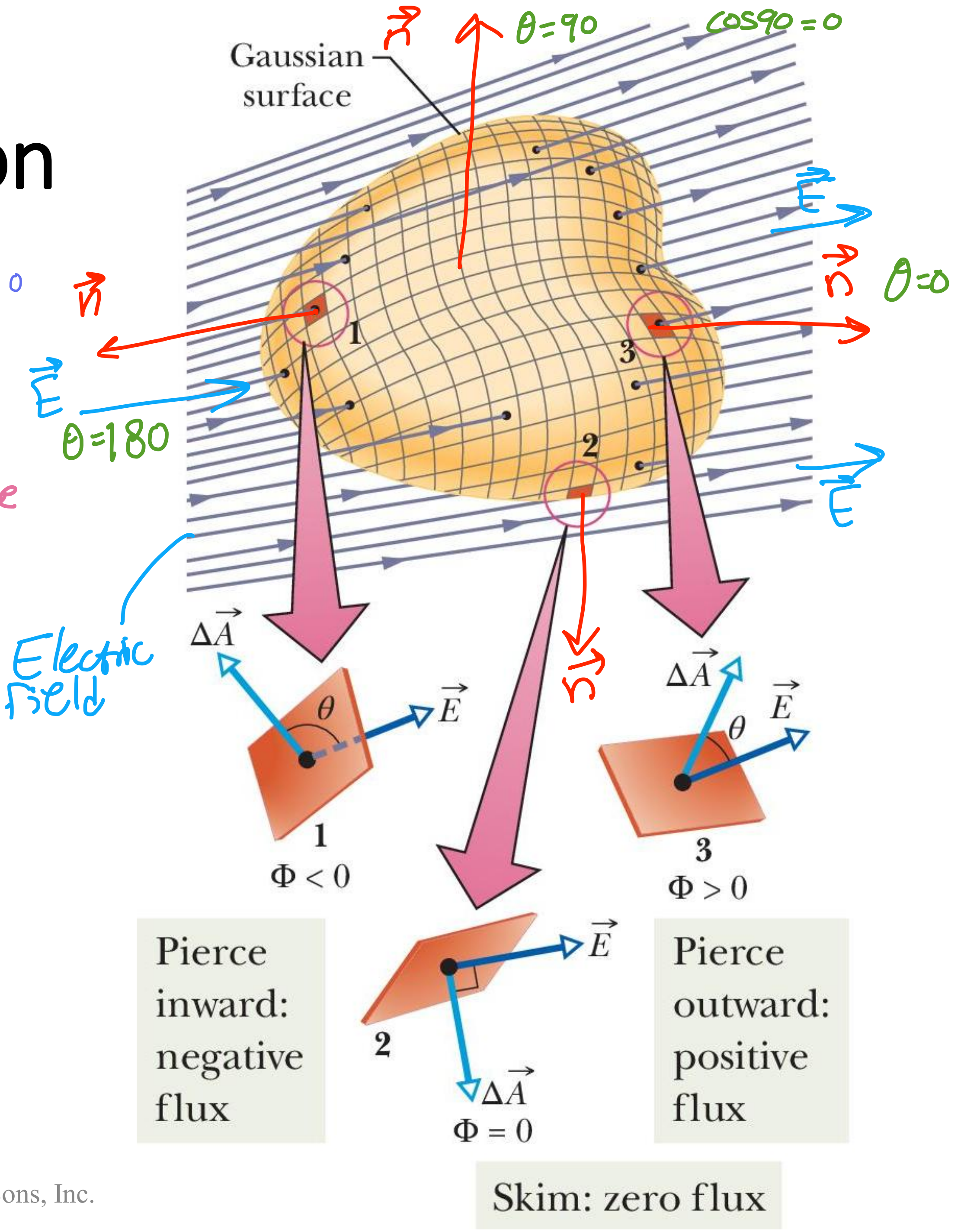
$$\boxed{\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux)}}$$

where the integration is carried out over the entire closed surface.

Graphic Example of Flux Direction

1. - An **inward** piercing electric field is **negative flux.**
2. - An **outward** piercing electric field is **positive flux.**
3. - A skimming field is zero flux.

Always: normal take away



Calculating Flux Through a Closed Cylinder: part one

given to us ig…

The figure below shows a Gaussian surface in the form of a closed cylinder (a Gaussian cylinder) of radius R. It lies in a uniform electric field $\vec{E}$ with the cylinder's central axis (along the length of the cylinder) parallel to the field.

What is the net flux Φ of the electric field through the cylinder?

$\Phi = \int \vec{E} \cdot dA$

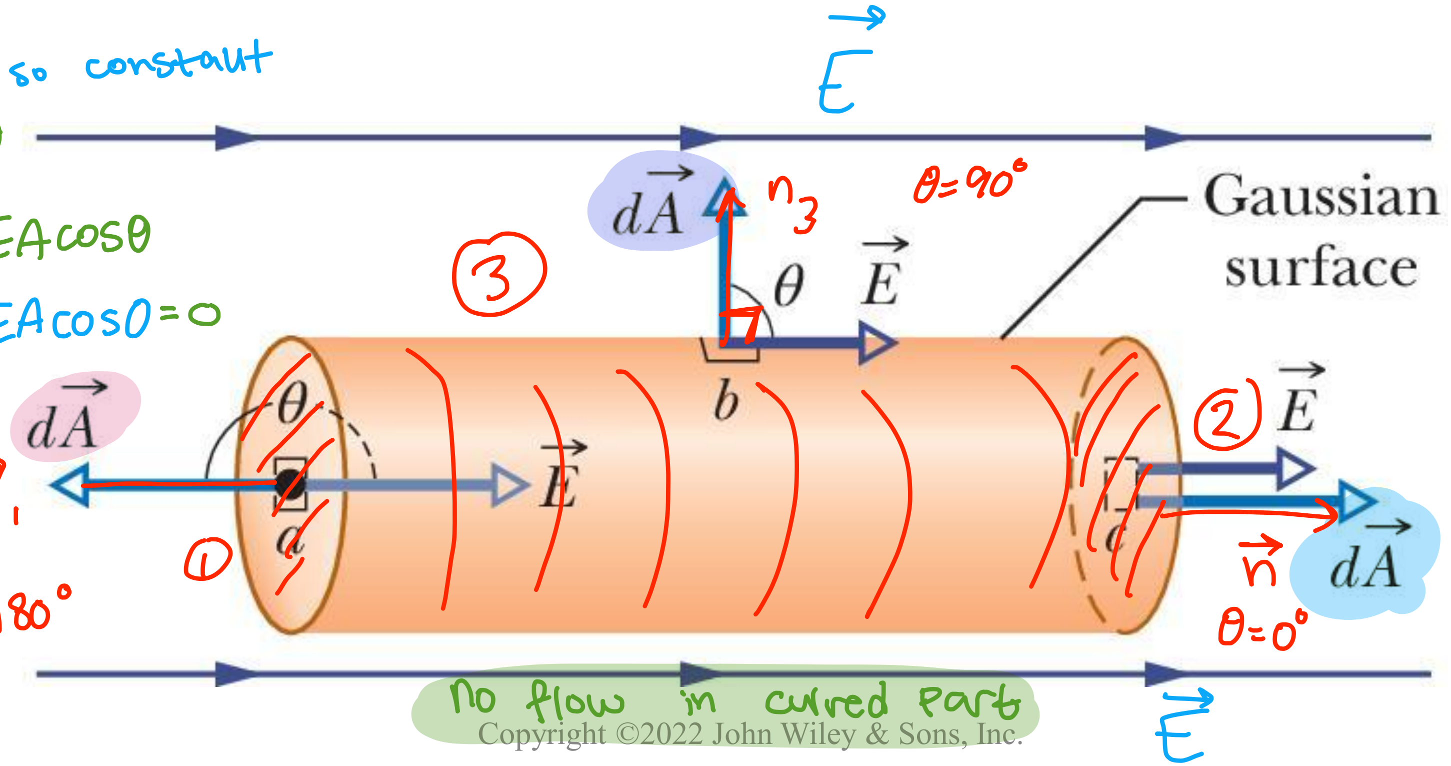
uniform so constant

$a = \int E \times dA \times \cos\theta$

$E\cos\theta \int \cancel{d}A = EA\cos\theta$

$-EA\cos 180 + \cancel{EA\cos 90} + EA\cos 0 = 0$

0

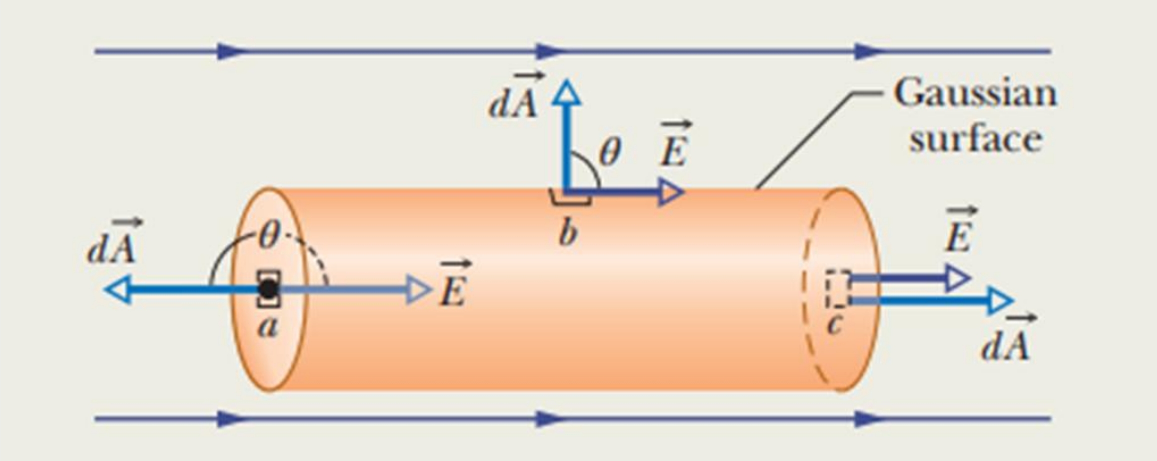


Calculating Flux Through a Closed Cylinder: part two

We can find the net flux Φ by integrating the dot product $\vec{E} \cdot d\vec{A}$ over the cylinder's surface. However, we cannot write out functions so that we can do that with one integral. Instead, we need to be a bit clever:

We break up the surface into sections with which we can actually evaluate an integral.

Calculations: We break the integral into **three** terms: integrals over the left cylinder cap *a*, the curved cylindrical surface *b*, and the right cap *c*:



$$\Phi = \oint \vec{E} \cdot d\vec{A}$$
$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}.$$

Calculating Flux Through a Closed Cylinder: part three

1. Pick a patch element on the left cap:
   - Its area vector $d\vec{A}$ must be perpendicular to the patch and pointing away from the interior of the cylinder $\Rightarrow \theta = 180°$.
   - The electric field through the end cap is uniform and thus E can be pulled out of the integration.

So, we can write the flux through the left cap as:

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180°)\, dA = -E\int dA = -EA,$$

Where $\int dA$ gives the cap's area $A\ (= \pi R^2)$

Calculating Flux Through a Closed Cylinder: part four

2. Similarly, for the right cap where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

3. Finally, for the cylindrical surface, where the angle $\theta = 90°$ at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

Substituting adding these results leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The **net flux is zero** because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

A special case: a constant electric field and a flat surface

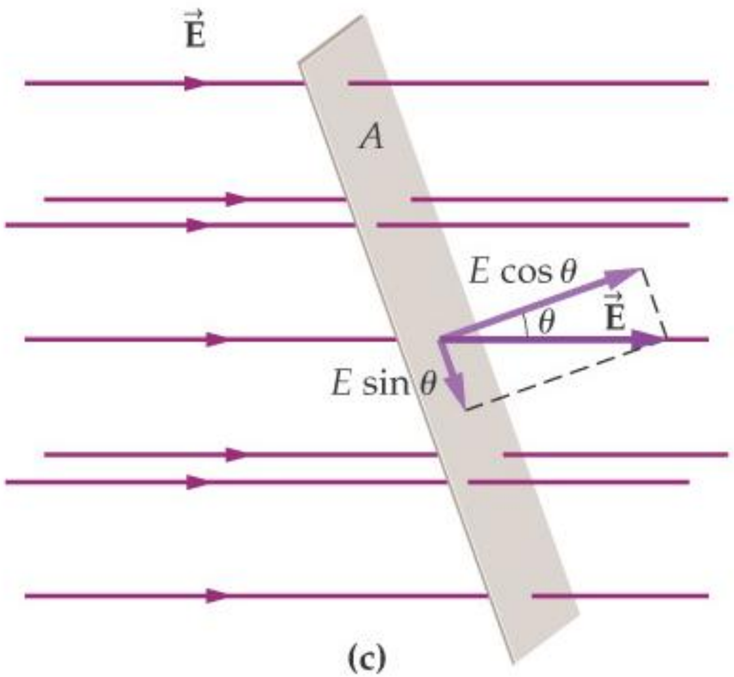
Electric Flux:

$$\Phi = \int \vec{E}.d\vec{A} = \int E(\cos\theta)dA = E\cos\theta \int dA = EA\cos\theta$$

That is,

$$\Phi = \overset{\frac{N}{C}}{E}\,\overset{m^2}{A}\cos\theta$$

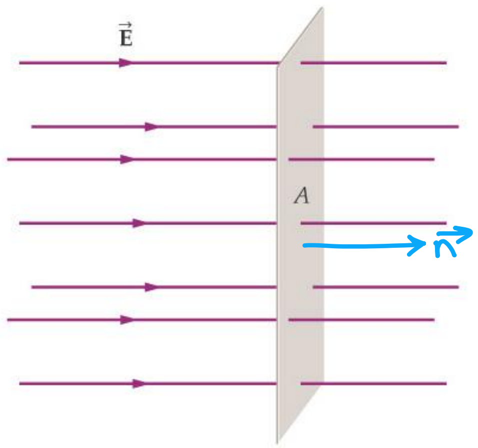
(SI unit of Φ: $\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$)



(c)

Example

$1\,cm = 10^{-2}\,m$
$1\,cm^2 = 10^{-4}\,m^2$



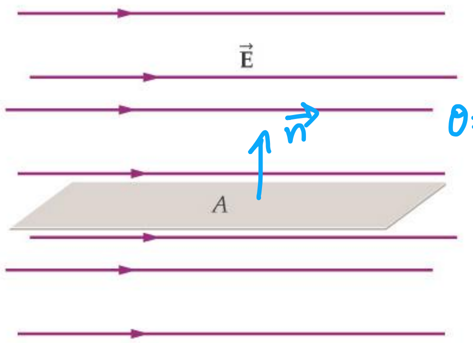
$\theta = 0°$

(a)

Calculate the flux of the electric field E = 200 N/C, through the surface $A = 350\ \mathrm{cm}^2$, in each of the three cases shown. (In part (c) θ=15°)

$1 cm^2 = 350 \times 10^{-4}\ m^2$

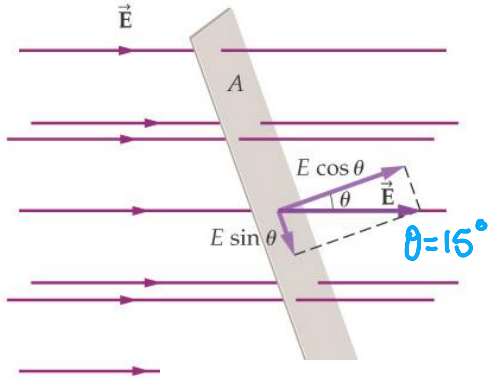
a) $\Phi = EA\cos 0 = (350\times10^{-4})(200)\cos 0 = 7\ \frac{N}{C}m^2$



$\theta = 90°$

(b)

b) $\Phi = EA\cos\theta = 0$



$\theta = 15°$

c) $\Phi = E_{net} A \cos\theta = (200)(350\times10^{-4})\cos 15 = 6.76\ \frac{N}{C}m^2$

Wed 14 Jan

Section 23.2 Gauss' Law

Gauss' Law Definition

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the net charge $\boldsymbol{q_{enc}}$ **that is enclosed by that surface**. It tells us that

continiuous distribution of charges

$$\boxed{\varepsilon_0 \Phi = q_{enc} \qquad (\text{Gauss' law}).}$$

where ε_0 is the permittivity constant.

inside surface

Gaussian (given)

$$\Phi = \frac{q_{enclosed}}{\varepsilon_0}$$

now we include the sign, but questions before usually was charge abs value

given constant

We can also write Gauss' law as

$$\boxed{\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}}$$

$$\boxed{\int \vec{E} \cdot \vec{dA} = \frac{q_{enclosed}}{\varepsilon_0}}$$

no charge no flux

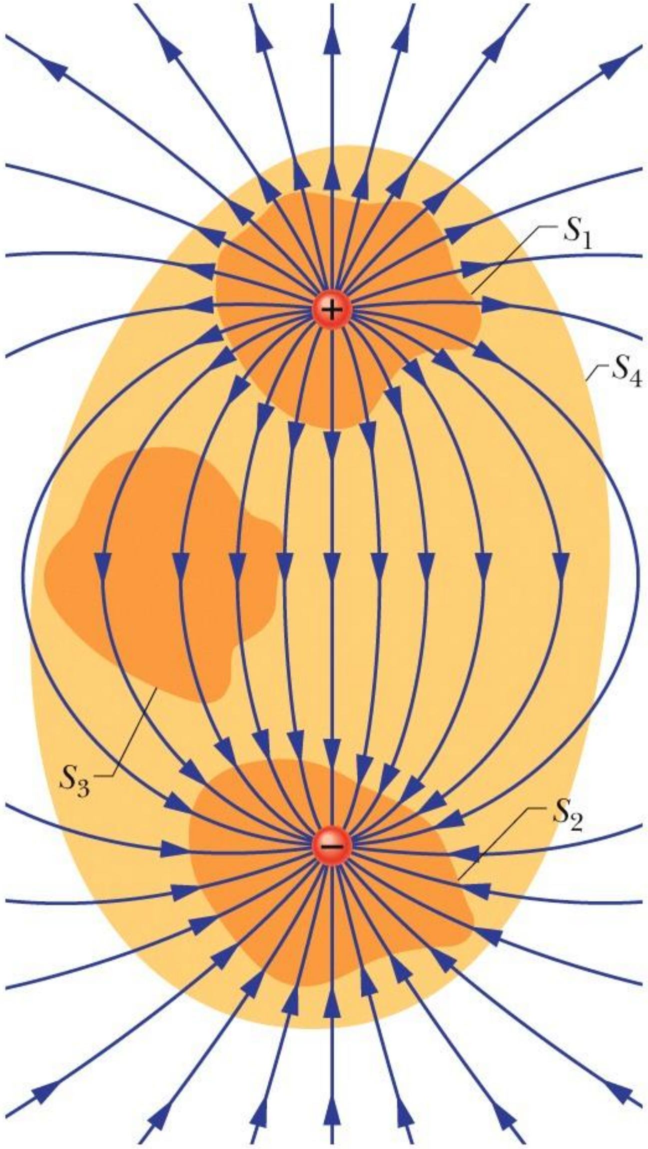
net flux = 0

That is, the **electric flux** through a **closed surface is proportional to the charge enclosed by the surface**

Flux Direction at Gaussian Surfaces

- The figure shows two charges, equal in magnitude but opposite in sign, and their net electric field lines.
- Electric Flux Through the Closed Gaussian Surfaces:
  - **Surface S_1**: E points outward everywhere
    → Positive electric flux
    → Positive enclosed charge (Gauss' law)
  - **Surface S_2**: E points inward everywhere
    → Negative electric flux
    → Negative enclosed charge (Gauss' law)
  - **Surface S_3**:
    → No enclosed charge ($q_{\text{enc}} = 0$)
    → Zero net flux (Field lines enter and leave the surface)
  - **Surface S_4**: Equal positive and negative charges enclosed
    → Net enclosed charge = 0
    → Zero net flux (Field lines leaving = field lines entering)

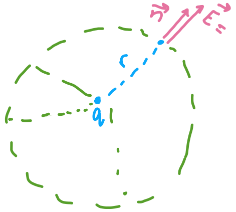
S_4 is skimming field



Gauss' Law and Coulomb's Law

Sphere best
Same distance
So same E

$E = \frac{K|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$



$\int \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

$\int E \times dA \times \cos\theta = \frac{q}{\varepsilon_0}$

$EA = \frac{q}{\varepsilon_0}$

$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$

$E = \frac{q}{4\pi r^2 \cdot \varepsilon_0}$

- Gauss' law can be used to find the electric field of a point charge.
- The field has spherical symmetry: Depends only on distance r
- Choose a Gaussian sphere centered on a positive charge.
  1. For a surface element dA, the area vector is radially outward. The electric field $\vec{E}$ is also radial, that is $\theta = 0$

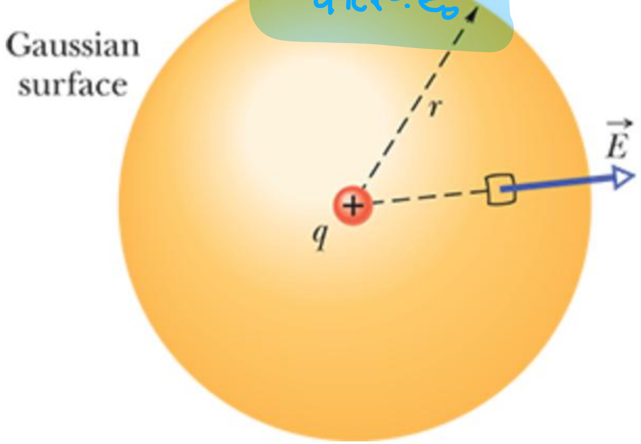
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

  2. Since E is the same at every area element, can be pulled outside the integral:

$$\varepsilon_0 E \oint dA = q.$$

  3. The remaining integral is just an instruction to sum all the areas of the patch elements on the sphere, we know this gives $A = 4\pi r^2$ (sphere). Solving for E:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$



- This is exactly what we find using **Coulomb's law**.

Example

Consider the surface S shown in the sketch. Is the electric flux through this surface **(a)** negative, **(b)** positive, or **(c)** zero?

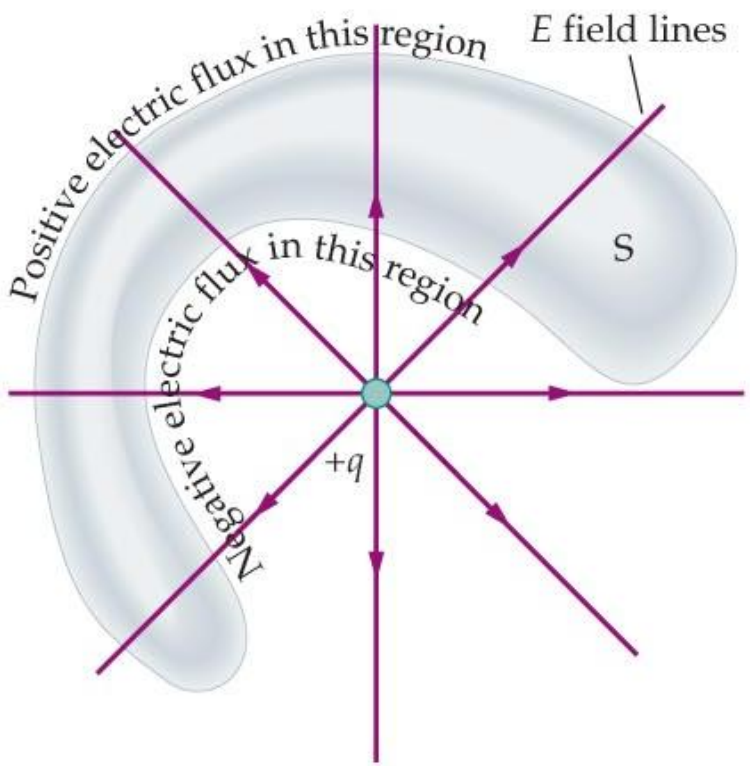
REASONING AND DISCUSSION

Because the surface S encloses no charge, the net electric flux through it must be zero, by Gauss's law. The fact that a charge $+q$ is nearby is irrelevant, because it is outside the volume enclosed by the surface.

We can explain why the flux vanishes in another way. Notice that the flux on portions of S near the charge is negative, since field lines enter the enclosed volume there. On the other hand, the flux is positive on the outer portions of S where field lines exit the volume. The combination of these positive and negative contributions is a net flux of zero.

ANSWER

(c) The electric flux through the surface S is zero.



Because no $q_{enclosed}$ so $= 0$

Example

Calculate the net electric flux (Φ) of the electric field for each of the closed surfaces a, b, c, and d, given that $q = 26.55$ nC

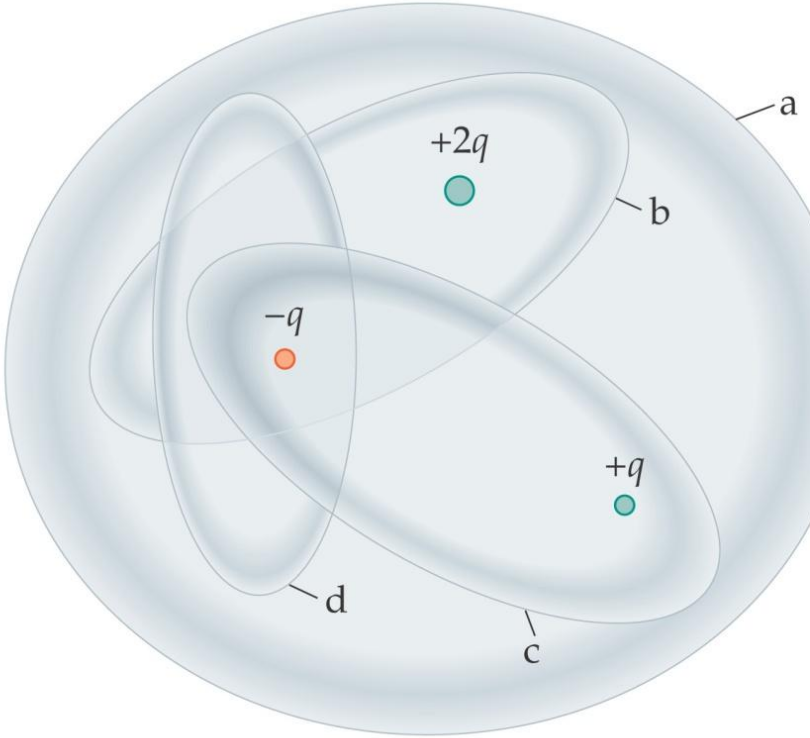
$\epsilon_0 = 8.854 \times 10^{-12}$

Surface a, Φ_a = $\frac{+q-q+2q}{\epsilon_0} = \frac{+2(26.55 \times 10^{-9})}{8.854 \times 10^{-12}} = 5997.28 \frac{N}{C} \times m^2$

Surface b, Φ_b = $\frac{+2q-q}{\epsilon_0} = \frac{(26.55 \times 10^{-9})}{8.854 \times 10^{-12}} = 2998.64 \frac{N}{C} m^2$

Surface c, Φ_c = $\frac{-q+q}{\epsilon_0} = 0$

Surface d, Φ_d = $\frac{-q}{\epsilon_0} = -2998.64 \frac{N}{C} m^2$



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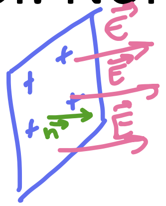
Section 23.3 A Charged Isolated Conductor

Distribution of Excess Charge on Conductors

- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.



$\Phi = \int \vec{E} \cdot d\vec{A}$

$\Phi_{total} = \frac{q_{enclosed}}{\epsilon_0}$

$\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$

$\oint E \times dA \times \cos 0$

$EA = \frac{q}{\epsilon_0}$

$\sigma = \frac{q}{A}$

$q = \sigma A$

$E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$

$\boxed{E = \frac{\sigma}{\epsilon_0}}$

- Applying Gauss's law, it can be shown that the electric field near the surface of a charged conductor equals:

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface)}$$

charge conductor
it's charge goes on the surface

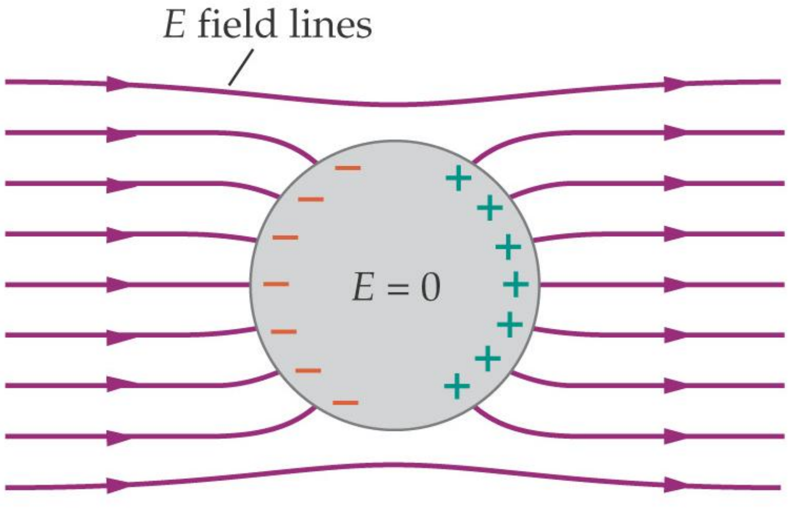
Where σ is the surface charge density (charge per unit area)

Additional remarks:

1. **Electrostatic shielding:** **A conductor shields it's interior from external electric fields.** When electric charges are at rest (electrostatic), the electric field within a conductor is zero.

$$E = 0 \quad \text{(Inside the conductor)}$$

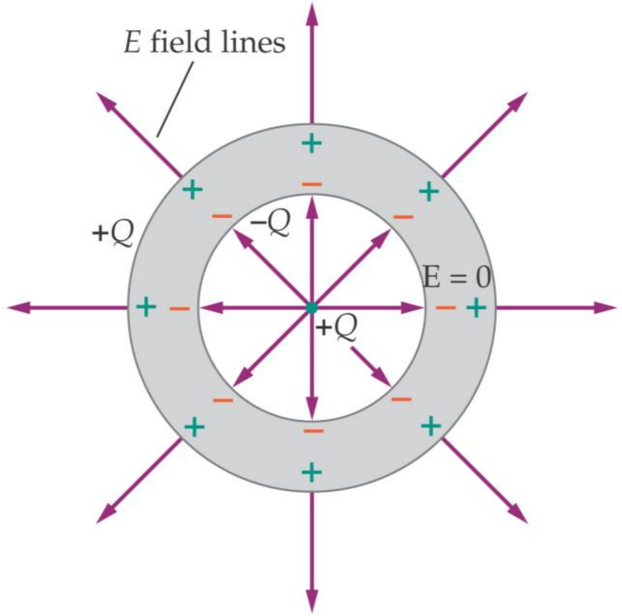
surface : charge
inside : free of charge / so Electric field inside is 0



Conductor electric charge = 0

Shielding works in one direction only.

- In the figure on the right, the charge $+Q$ in the cavity induces a charge $-Q$ on the interior surface and $+Q$ on the exterior surface. That is, the field outside the sphere will not be zero.



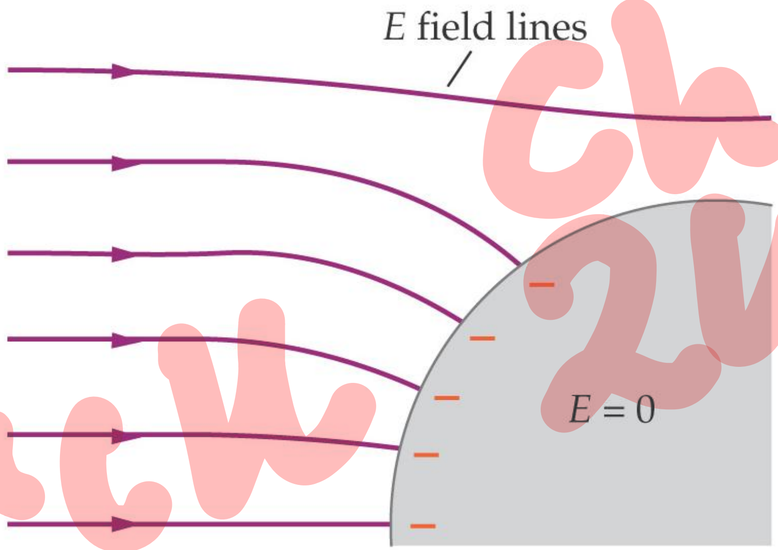
$\int E \times dA \times \cos\theta = \cancel{\frac{q}{\epsilon_0}}$ 0

$\theta = 90°$

but area exists so can't be 0

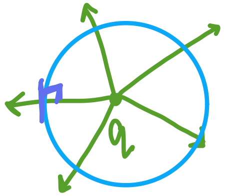
$\oint \vec{E} \cdot d\vec{A} = \cancel{\frac{q}{\epsilon_0}}$ 0 0

2. The electric **field** is always **perpendicular to the surface** of a conductor. If it weren't, the charges would move along the surface.

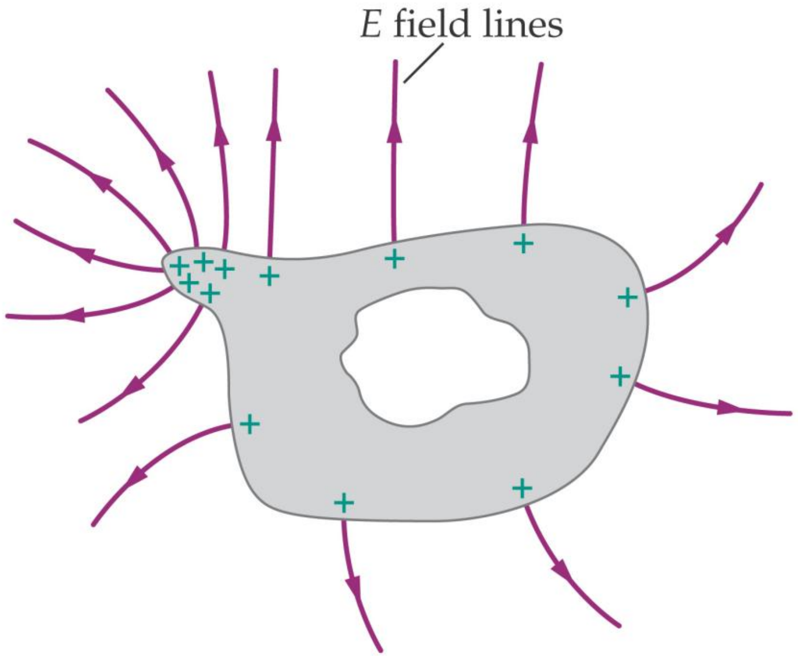


come back ch 24

3. The electric **field** is **stronger** where the surface is more sharply curved (pointy bit).



radius & tan are ⊥ to each other



lightning: stay inside car because charge would stay on metal surface

Faraday Cage

- Faraday stated that the charge on a charged conductor resided only on its exterior
- To demonstrate this fact he built a room coated with metal foil, and allowed high-voltage discharges from an electrostatic generator to strike the outside of the room

- He used an electroscope to show that there was no excess electric charge on the inside of the room's walls.

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Forces

conservative vs. dissaptive

① PE: gravitational PE
② PE: tension (elastic PE)
③ PE: electric PE

dot product: $\cos\theta$
cross product: $\sin\theta$

F & W same direction = + work
F & W opp " = − wor

independent of path
only care about h_i & h_f

↓ Motion Work by weighted = +ve (Motive) GPE ↓ ΔGPE = −ve GPE = mgh

↑ Motion work by weight = −ve (Resistive) GPE ↑ ΔGPE = +ve

$PE = U$
U_g: gravitational
U_E: elastic
U_i

$Work = -\Delta U$

$W = m \times g$ (W: gravitational force; m: mass; g: gravitational field)

$U_g = mgh$ so qEd

$F = q \times E$ (F: electric force; q: charge; E: electric field)