http://algs4.cs.princeton.edu

4.2 Directed Graphs → edges → معرفة بجهه معينه

- *introduction*
- *digraph API*
- *digraph search*
- *topological sort*
- *strong components*

Algorithms

Robert Sedgewick | Kevin Wayne

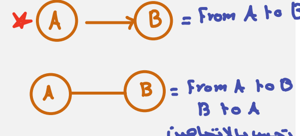
http://algs4.cs.princeton.edu

4.2 Directed Graphs

- *introduction*
- *digraph API*
- *digraph search*
- *topological sort*
- *strong components*

Directed graphs

Digraph. Set of vertices connected pairwise by directed edges.



outdegree = 4

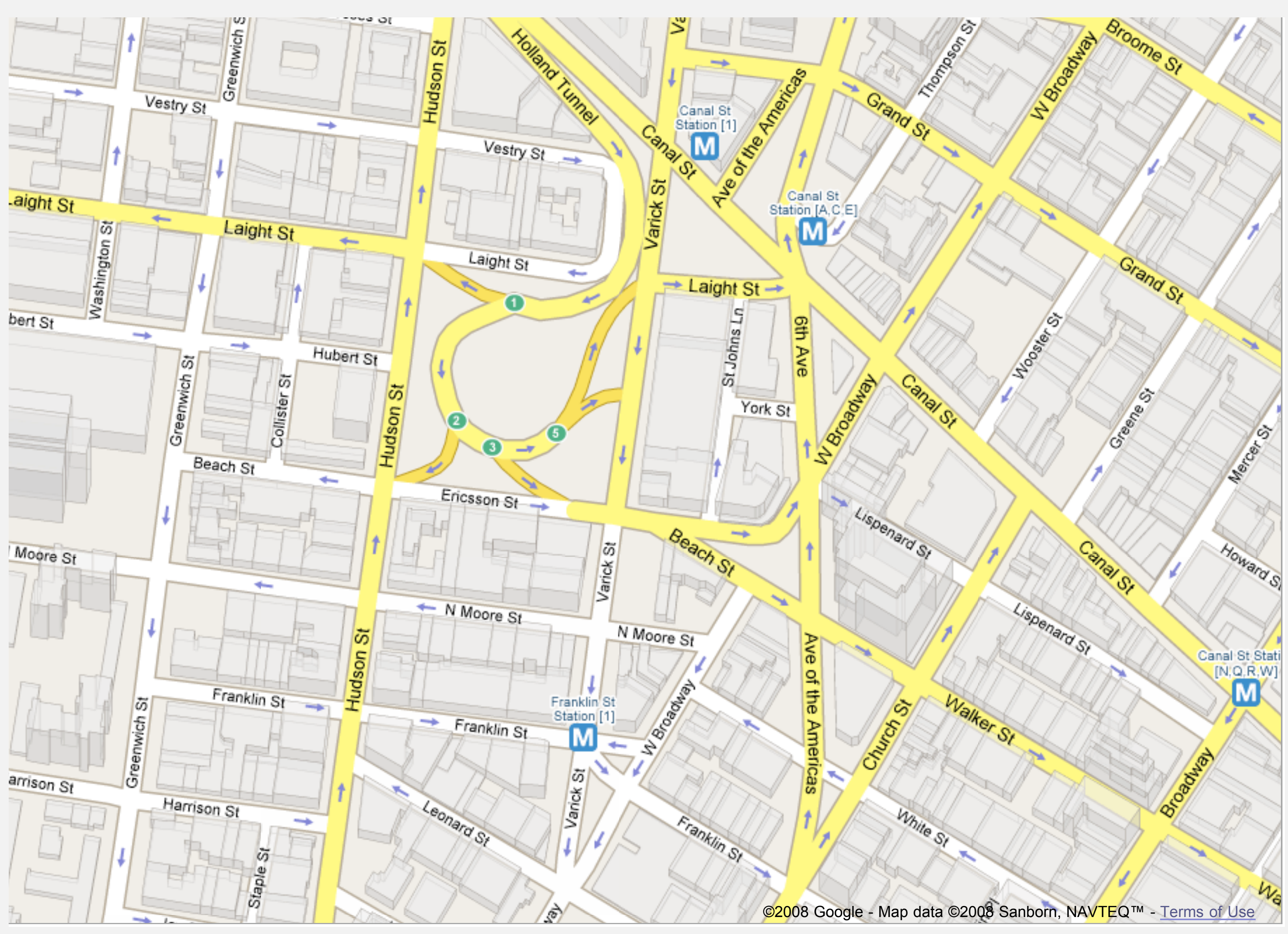
indegree = 2

directed path from 0 to 2

directed cycle

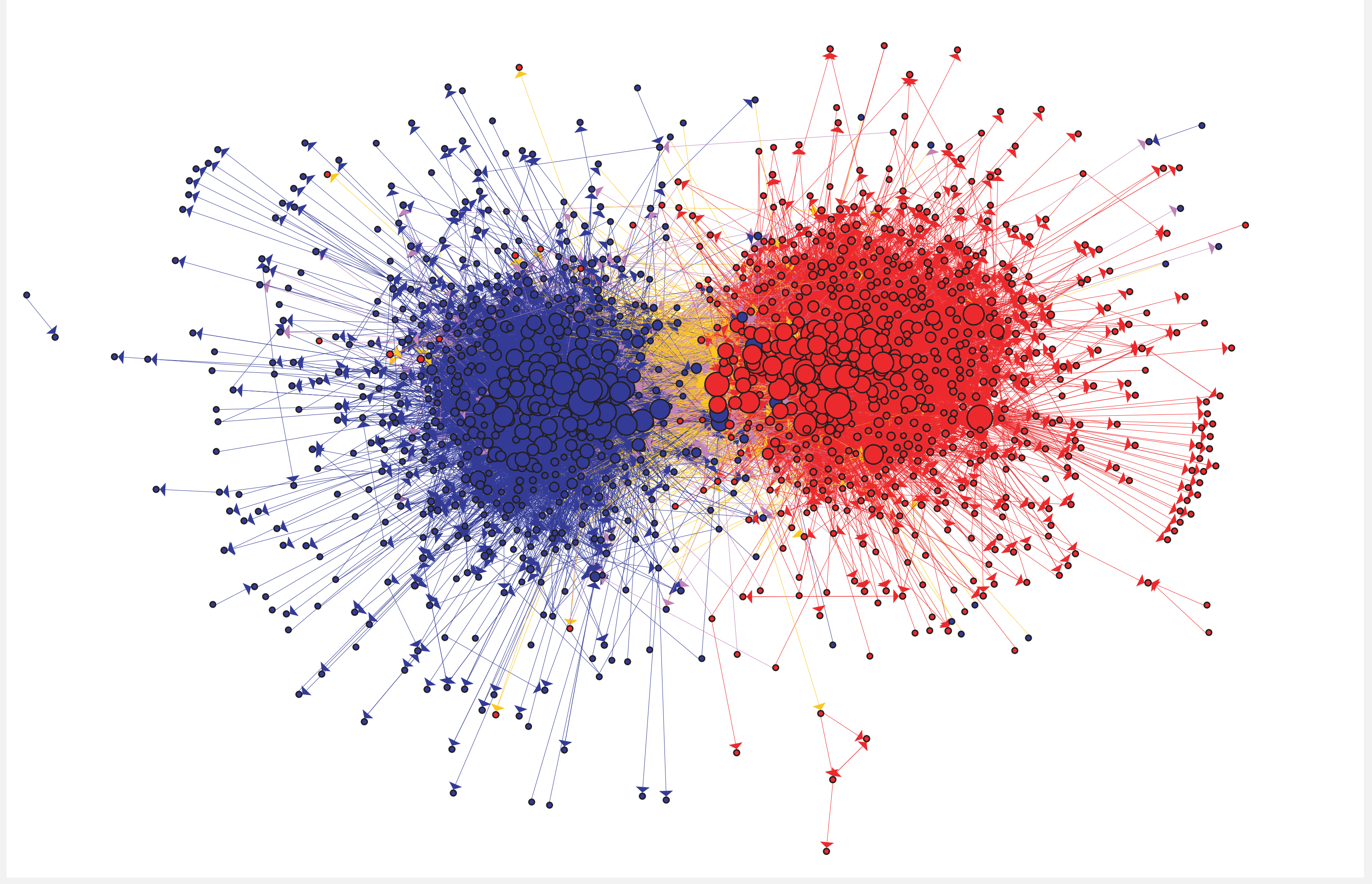
Road network

Vertex = intersection; edge = one-way street.



Political blogosphere graph

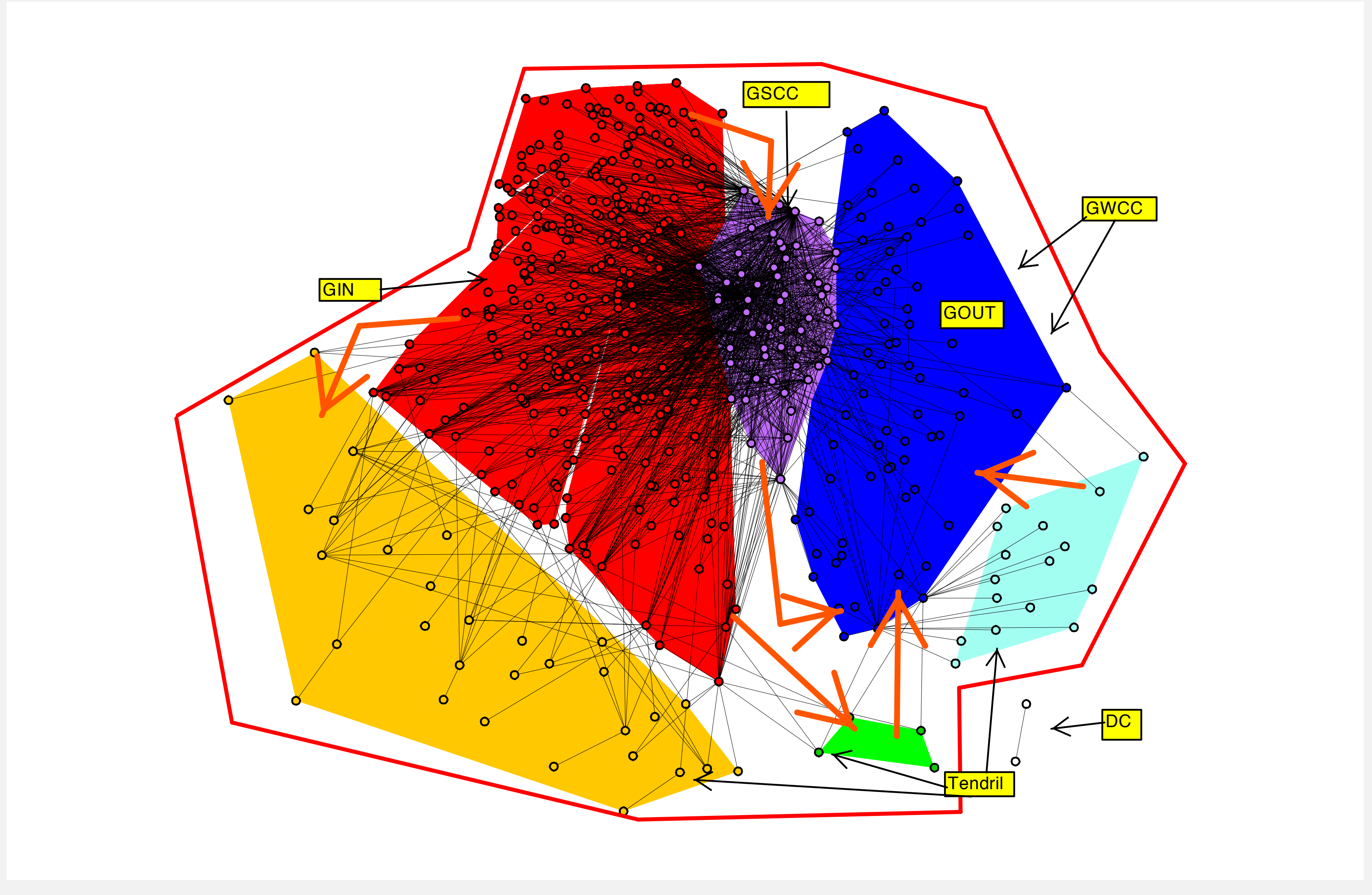
Vertex = political blog; edge = link.



The Political Blogosphere and the 2004 U.S. Election: Divided They Blog, Adamic and Glance, 2005

Overnight interbank loan graph

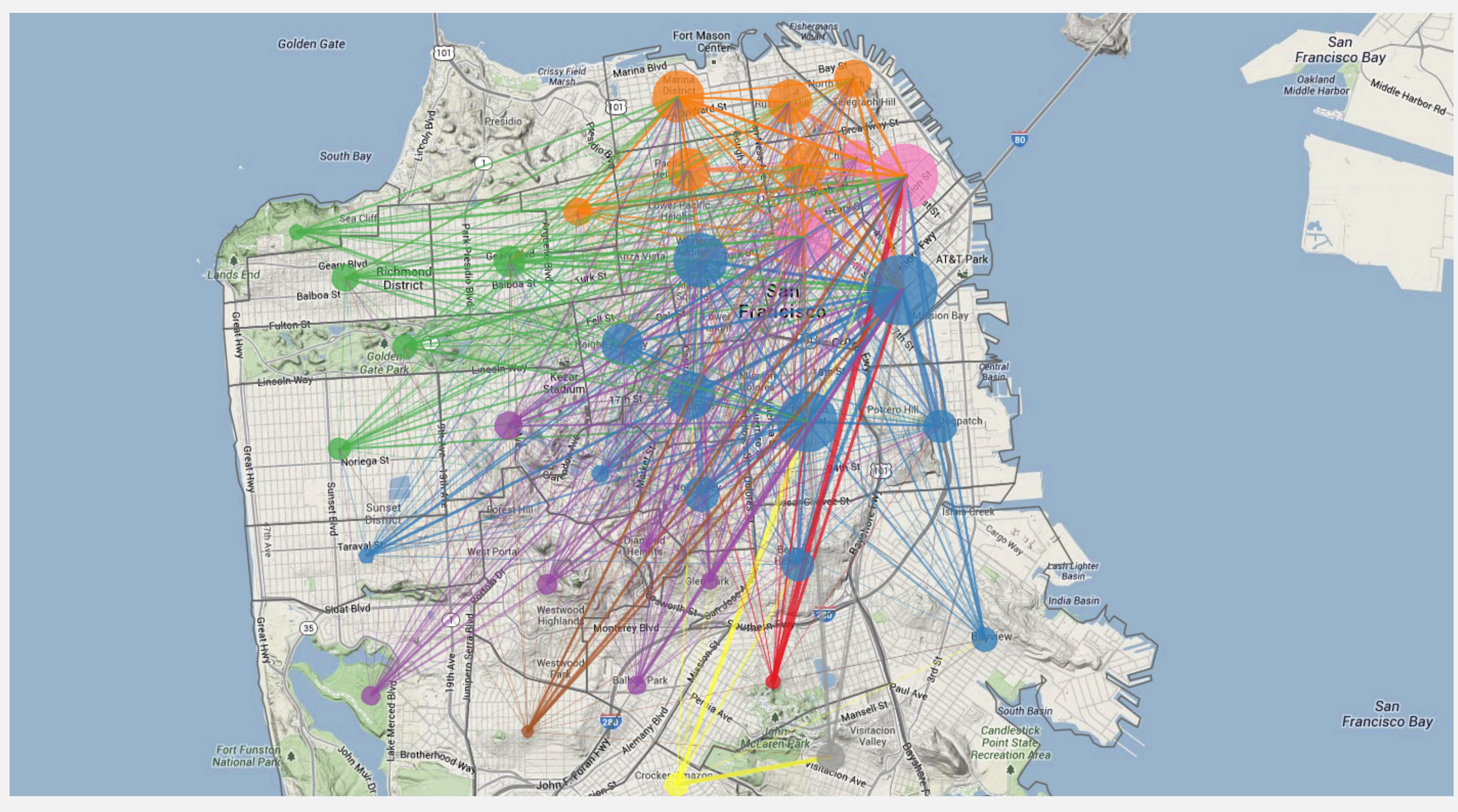
Vertex = bank; edge = overnight loan.



The Topology of the Federal Funds Market, Bech and Atalay, 2008

Uber taxi graph

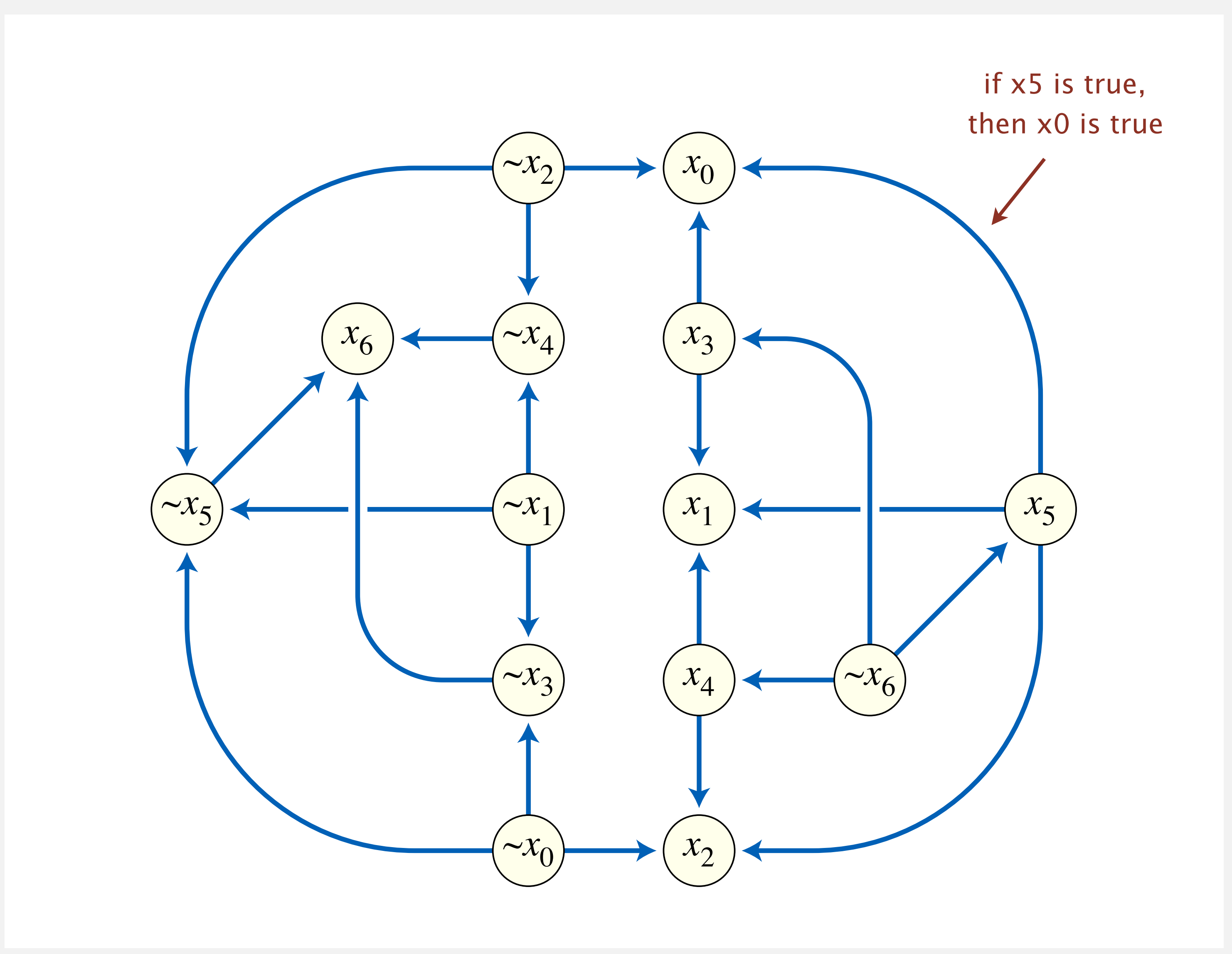
Vertex = taxi pickup; edge = taxi ride.



http://blog.uber.com/2012/01/09/uberdata-san-franciscomics/

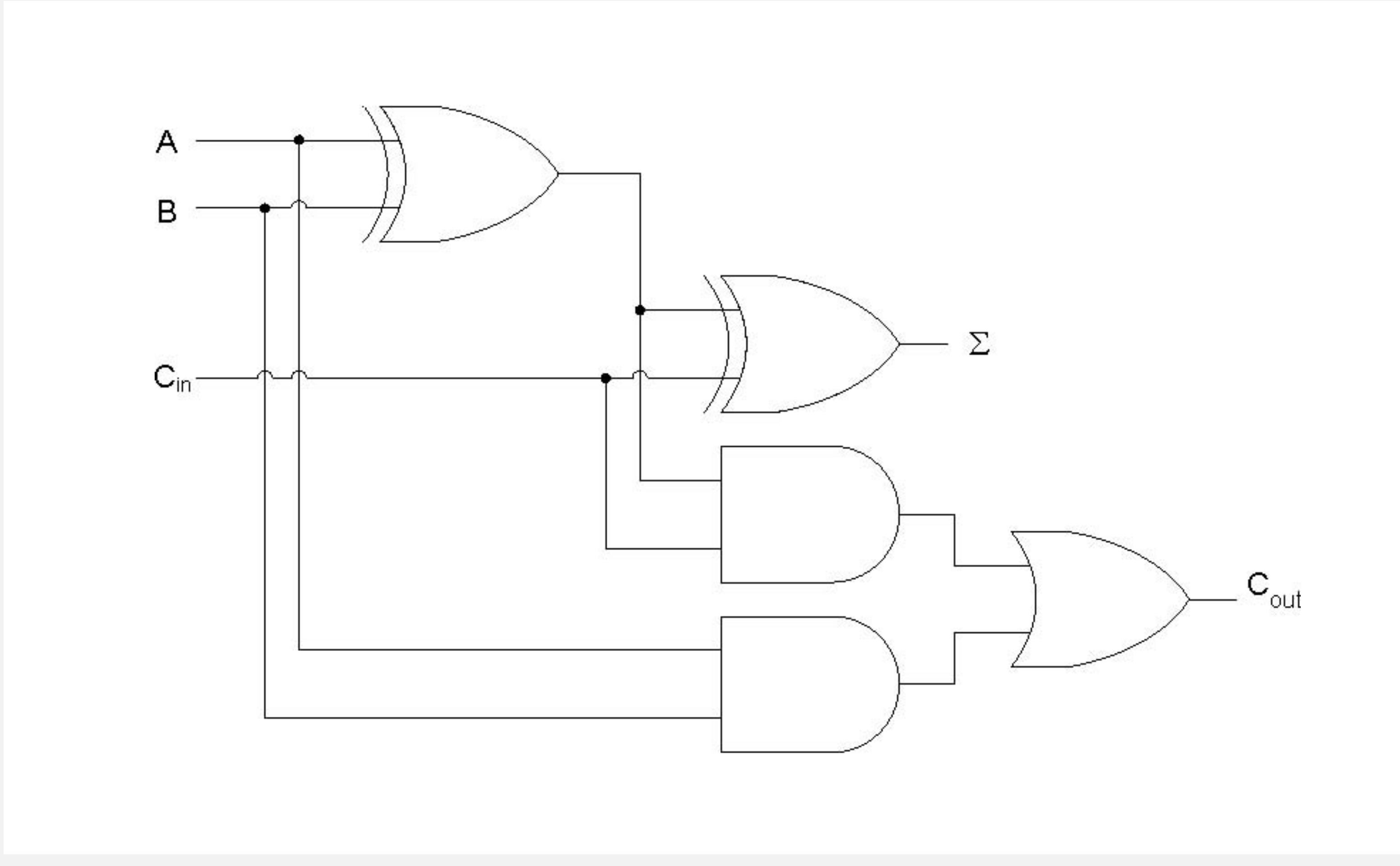
Implication graph

Vertex = variable; edge = logical implication.



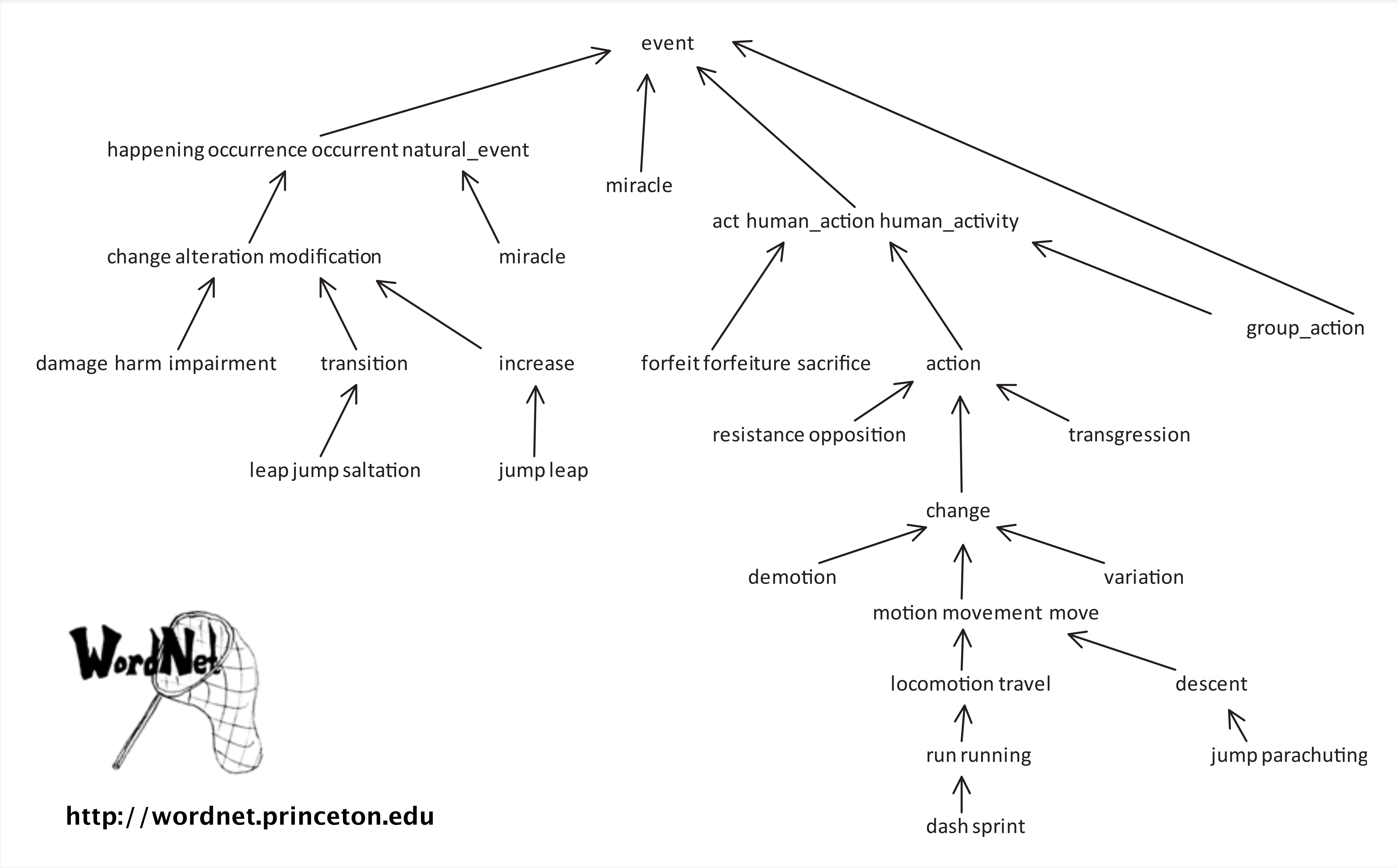
Combinational circuit

Vertex = logical gate; edge = wire.



WordNet graph

Vertex = synset; edge = hypernym relationship.



Digraph applications

digraph	vertex	directed edge
transportation	street intersection	one-way street
web	web page	hyperlink
food web	species	predator-prey relationship
WordNet	synset	hypernym
scheduling	task	precedence constraint
financial	bank	transaction
cell phone	person	placed call
infectious disease	person	infection
game	board position	legal move
citation	journal article	citation
object graph	object	pointer
inheritance hierarchy	class	inherits from
control flow	code block	jump

Some digraph problems

problem	description
s→t path	*Is there a path from s to t ?*
shortest s→t path	*What is the shortest path from s to t ?*
directed cycle	*Is there a directed cycle in the graph ?*
topological sort	*Can the digraph be drawn so that all edges point upwards?*
strong connectivity	*Is there a directed path between all pairs of vertices ?*
transitive closure	*For which vertices v and w is there a directed path from v to w ?*
PageRank	*What is the importance of a web page ?*

Robert Sedgewick | Kevin Wayne

http://algs4.cs.princeton.edu

4.2 Directed Graphs

- introduction
- **digraph API**
- digraph search
- topological sort
- strong components

Digraph API

Almost identical to `Graph` API.

public class Digraph		
	`Digraph(int V)`	*create an empty digraph with V vertices*
	`Digraph(In in)`	*create a digraph from input stream*
`void`	`addEdge(int v, int w)`	*add a directed edge v→w*
`Iterable<Integer>`	`adj(int v)`	*vertices pointing from v*
`int`	`V()`	*number of vertices*
`int`	`E()`	*number of edges*
`Digraph`	`reverse()`	*reverse of this digraph*
`String`	`toString()`	*string representation*

Digraph API

Edge list

tinyDG.txt

V → 13

E → 22 — عدد الـ edge الي بقرادهم . + باتجاه واحد

```
13
22
 4  2
 2  3
 3  2
 6  0
 0  1
 2  0
11 12
12  9
 9 10
 9 11
 7  9
10 12
11  4
 4  3
 3  5
 6  8
 8  6
 ⋮
```

run time : O(V+E)

بس اخلص اعد عدد اللاينات الي اعطتني اياها يساوي عدد الخطوط الي حطيتها، واعد عدد الـ vortise هل يساوي الي رسمتها. علشان نشوف صحيحة اولا .

```
% java Digraph tinyDG.txt
0->5
0->1
2->0
2->3
3->5
3->2
4->3
4->2
5->4
⋮
11->4
11->12
12->9
```

طريقة ثانية

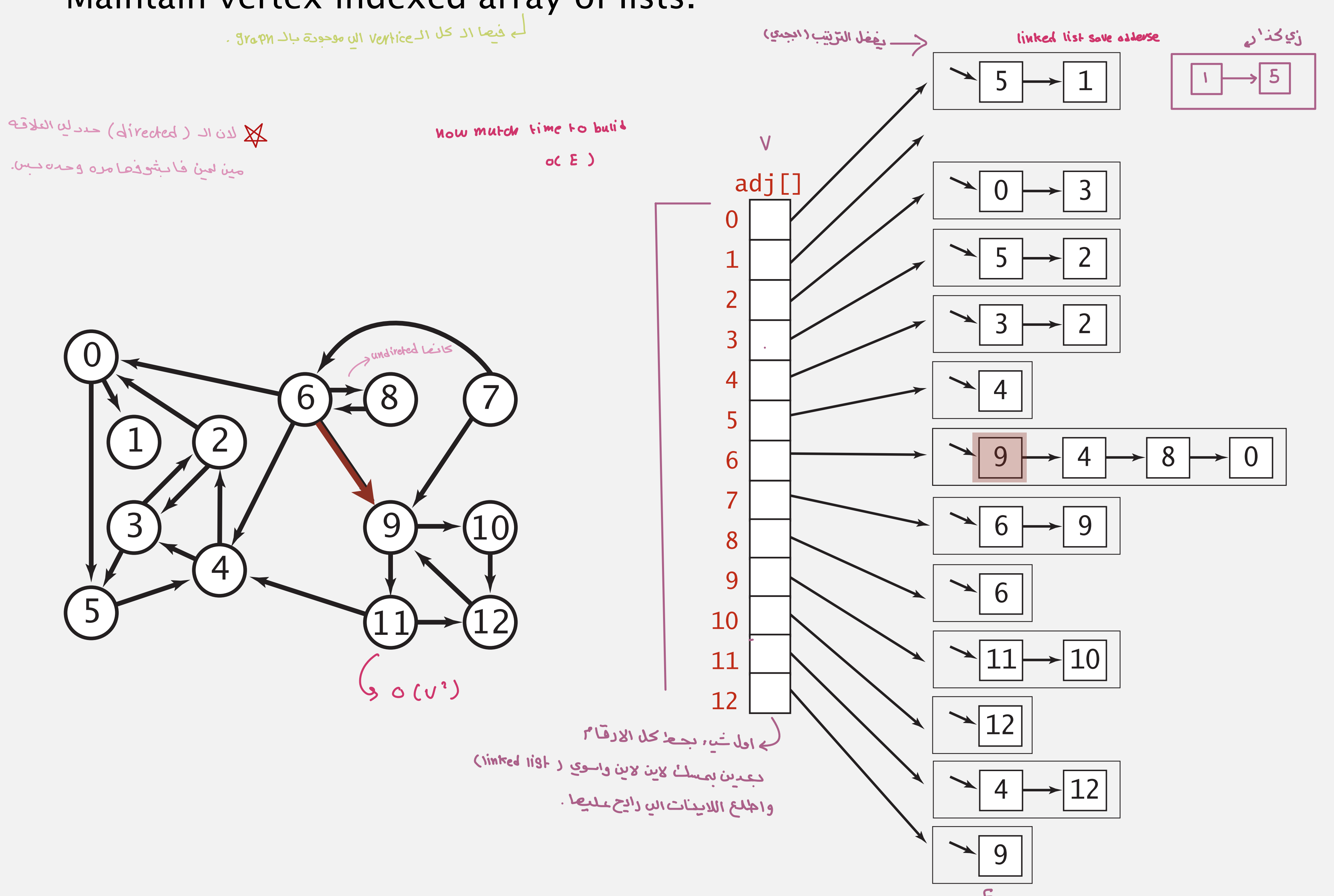
```
In in = new In(args[0]);
Digraph G = new Digraph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(v))
      StdOut.println(v + "->" + w);
```

read digraph from input stream

print out each edge (once)

Digraph representation: adjacency lists

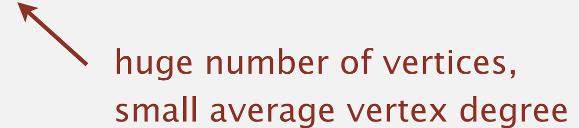
Maintain vertex-indexed array of lists.



Digraph representations

In practice. Use adjacency-lists representation.

- Algorithms based on iterating over vertices pointing from v.
- Real-world digraphs tend to be sparse.



representation	space	insert edge from v to w	edge from v to w?	iterate over vertices pointing from v?
list of edges	E	1	E	E
adjacency matrix	V^2	$1^{\dagger}$	1	V
adjacency lists	$E + V$	1	$outdegree(v)$	$outdegree(v)$

† disallows parallel edges

Adjacency-lists graph representation (review): Java implementation

```
public class Graph
{
   private final int V;
   private final Bag<Integer>[] adj;

   public Graph(int V)
   {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);
      adj[w].add(v);
   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists

create empty graph with V vertices

add edge v–w

iterator for vertices adjacent to v

* undirected Graph

* directed اذا بنسوي بس one

Adjacency-lists digraph representation: Java implementation

```
public class Digraph
{
   private final int V;
   private final Bag<Integer>[] adj;

   public Digraph(int V)
   {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);

   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists

create empty digraph with V vertices

add edge v→w

iterator for vertices pointing from v

only once

diercted Graph

Algorithms

Robert Sedgewick | Kevin Wayne

http://algs4.cs.princeton.edu

4.2 Directed Graphs

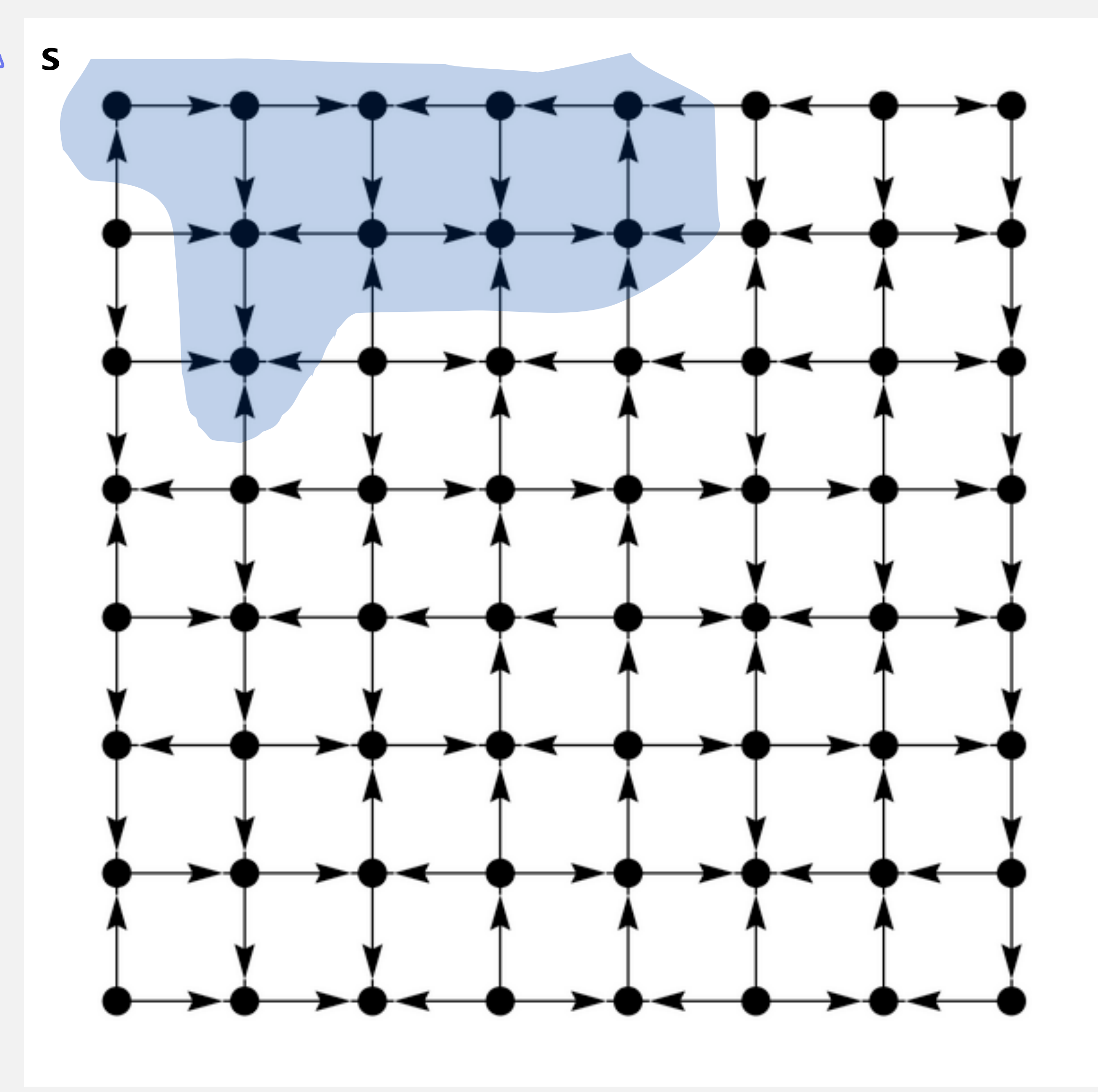
- introduction
- digraph API
- digraph search
- topological sort
- strong components

DFS

Reachability

Problem. Find all vertices reachable from s along a directed path.

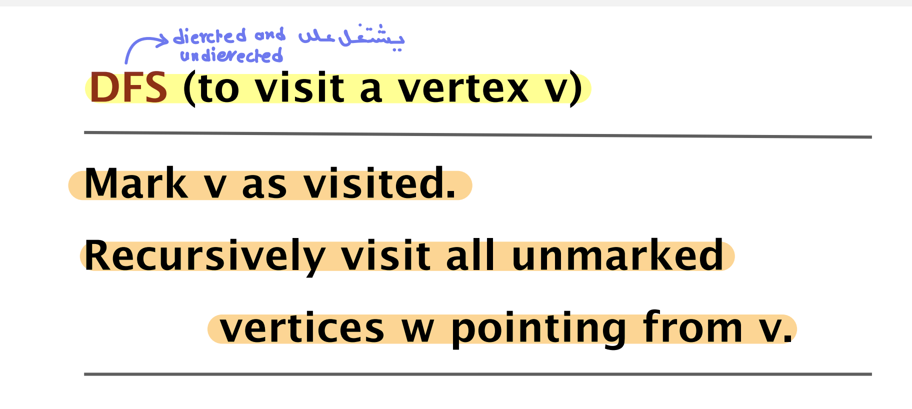
ننتبه للاتجاهات



Depth-first search in digraphs

Same method as for undirected graphs.

- Every undirected graph is a digraph (with edges in both directions).
- DFS is a digraph algorithm.



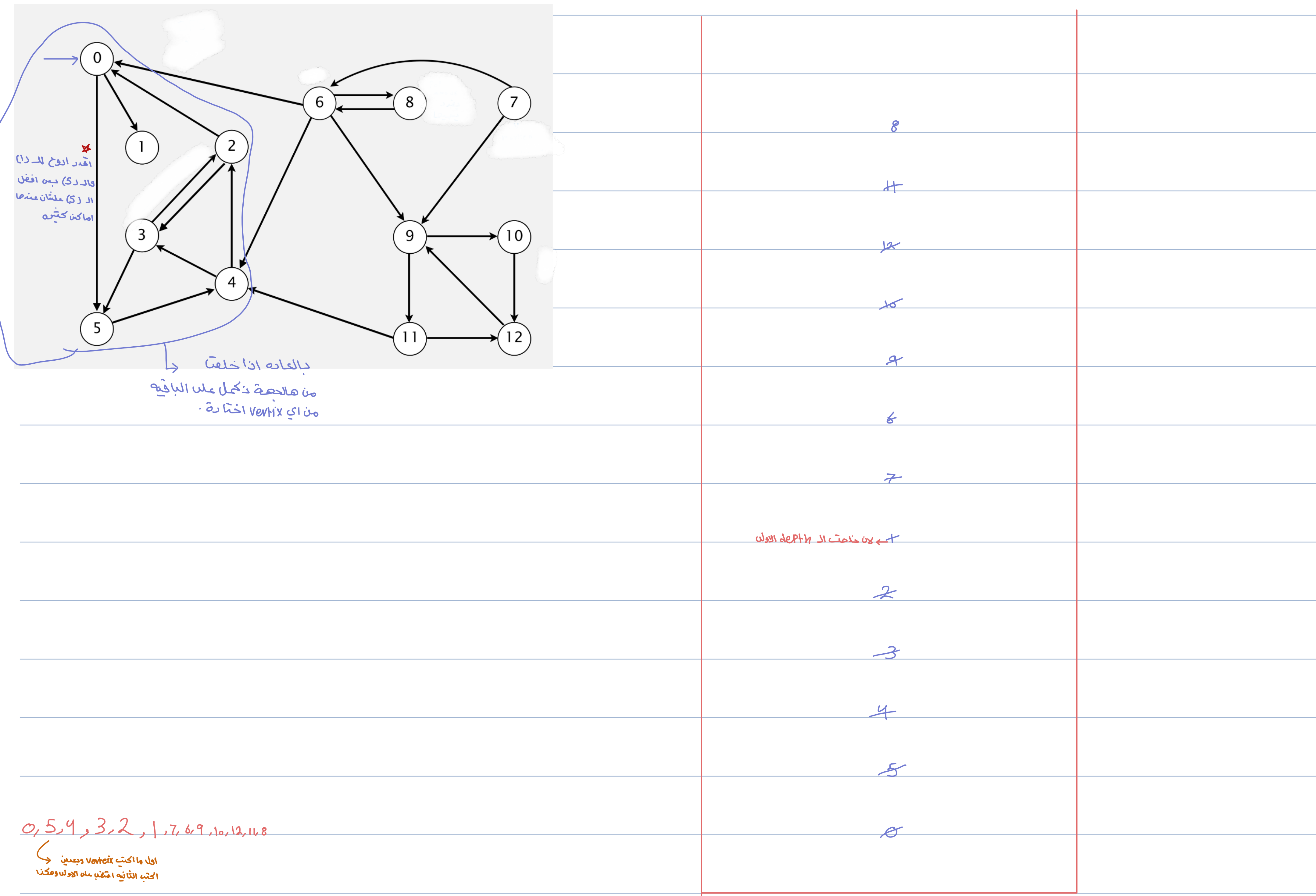
Depth-first search demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices pointing from v.

a directed graph

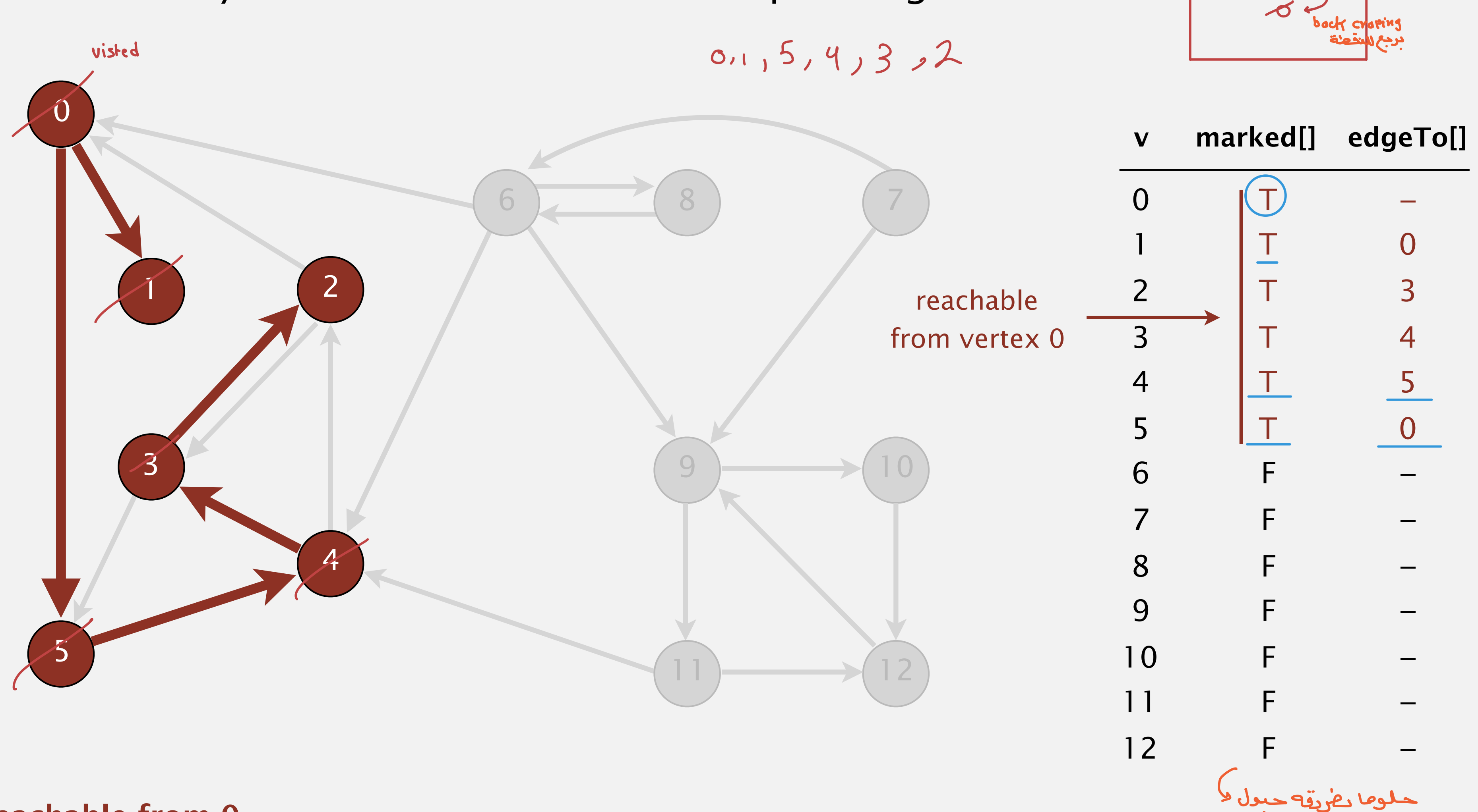
4→2
2→3
3→2
6→0
0→1
2→0
11→12
12→9
9→10
9→11
8→9
10→12
11→4
4→3
3→5
6→8
8→6
5→4
0→5
6→4
6→9
7→6


0
1
2
3
4
5
6
7
8
9
10
11
12
اقدر اروح للـ (1) والـ (5) بس افضل الـ (5) علشان عندها اماكن كثيره
بالعاده اذا خلصت من هالجهة نكمل على الباقيه من اي vertix اختاره .
8
11
12
10
9
6
7
1 ← لان خلصت الـ depth الاول
2
3
4
5
0
0, 5, 4, 3, 2, 1, 7, 6, 9, 10, 12, 11, 8
اول ما اكتب vorteix وبعدين الحتب الثانيه اشطب على الاول وهكذا

Depth-first search demo → stack نستخدم ال

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices pointing from v.



v	marked[]	edgeTo[]
0	T	–
1	T	0
2	T	3
3	T	4
4	T	5
5	T	0
6	F	–
7	F	–
8	F	–
9	F	–
10	F	–
11	F	–
12	F	–

reachable from 0

Depth-first search (in undirected graphs)

Recall code for undirected graphs.

اتجاه الطريق ممكن
استخدم اللاين رايح او اللاين راجع
باتجاهين .

```
public class DepthFirstSearch
{
   private boolean[] marked;

   public DepthFirstSearch(Graph G, int s)
   {
      marked = new boolean[G.V()];
      dfs(G, s);
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w]) dfs(G, w);
   }

   public boolean visited(int v)
   {  return marked[v];  }
}
```

- `private boolean[] marked;` — true if connected to s
- constructor — constructor marks vertices connected to s
- `dfs` — recursive DFS does the work
- `visited` — client can ask whether any vertex is connected to s

Depth-first search (in directed graphs)

Code for directed graphs identical to undirected one.

[substitute `Digraph` for `Graph`]

```
public class DirectedDFS
{
   private boolean[] marked;

   public DirectedDFS(Digraph G, int s)
   {
      marked = new boolean[G.V()];
      dfs(G, s);
   }

   private void dfs(Digraph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w]) dfs(G, w);
   }

   public boolean visited(int v)
   {  return marked[v];  }
}
```

true if path from s

constructor marks vertices reachable from s

recursive DFS does the work

client can ask whether any vertex is reachable from s

Reachability application: program control-flow analysis

Every program is a digraph.

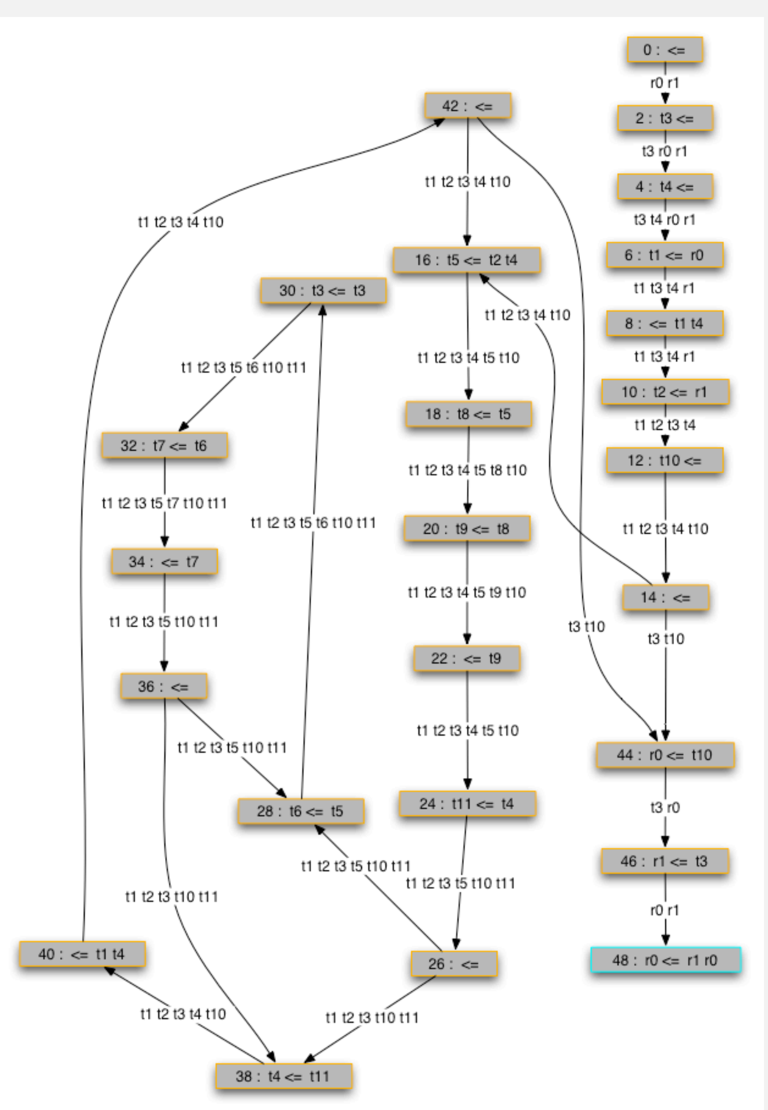
- Vertex = basic block of instructions (straight-line program).
- Edge = jump.

Dead-code elimination.

Find (and remove) unreachable code.

Infinite-loop detection.

Determine whether exit is unreachable.



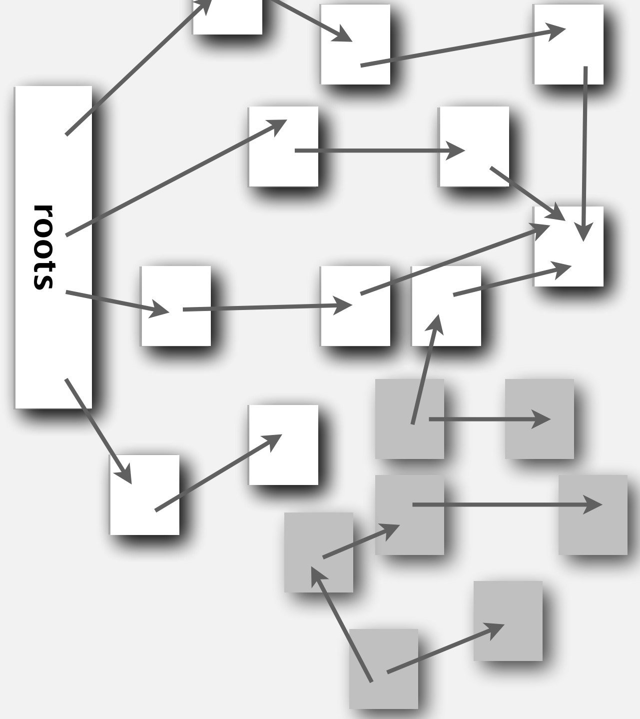
Reachability application: mark-sweep garbage collector

Every data structure is a digraph.

- Vertex = object.
- Edge = reference.

Roots. Objects known to be directly accessible by program (e.g., stack).

Reachable objects. Objects indirectly accessible by program (starting at a root and following a chain of pointers).

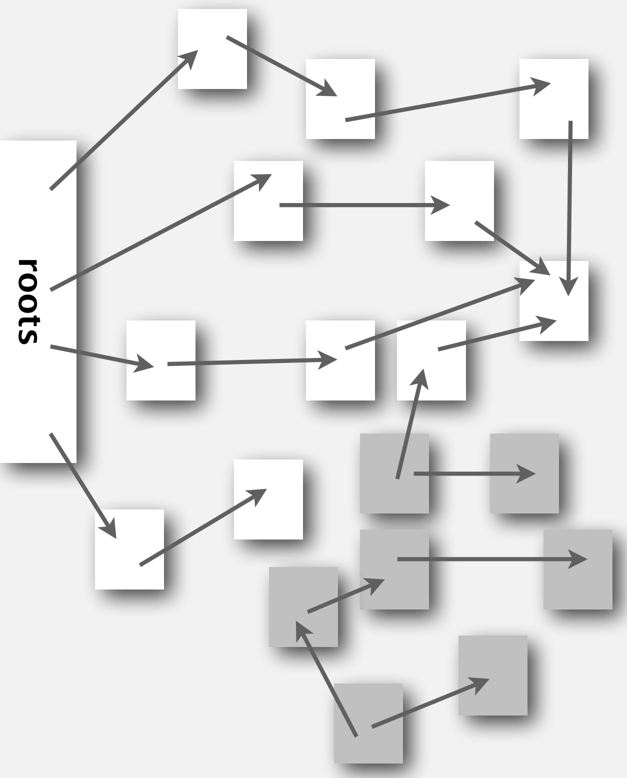


Reachability application: mark-sweep garbage collector

Mark-sweep algorithm. [McCarthy, 1960]

- Mark: mark all reachable objects.
- Sweep: if object is unmarked, it is garbage (so add to free list).

Memory cost. Uses 1 extra mark bit per object (plus DFS stack).



Depth-first search in digraphs summary

DFS enables direct solution of simple digraph problems.

- Reachability.
- Path finding.
- Topological sort.
- Directed cycle detection.

Basis for solving difficult digraph problems.

- 2-satisfiability.
- Directed Euler path.
- Strongly-connected components.

SIAM J. Comput.
Vol. 1, No. 2, June 1972

DEPTH-FIRST SEARCH AND LINEAR GRAPH ALGORITHMS*

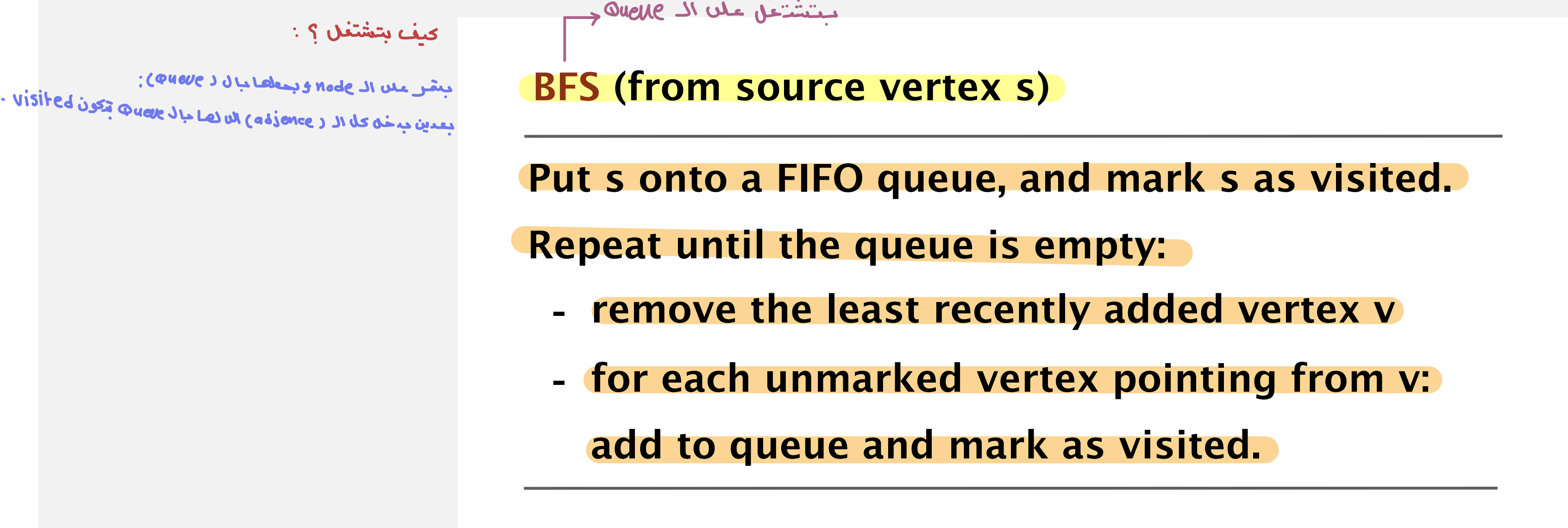
ROBERT TARJAN†

Abstract. The value of depth-first search or "backtracking" as a technique for solving problems is illustrated by two examples. An improved version of an algorithm for finding the strongly connected components of a directed graph and an algorithm for finding the biconnected components of an undirect graph are presented. The space and time requirements of both algorithms are bounded by $k_1V + k_2E + k_3$ for some constants k_1, k_2, and k_3, where V is the number of vertices and E is the number of edges of the graph being examined.

Breadth-first search in digraphs

Same method as for undirected graphs.

- Every undirected graph is a digraph (with edges in both directions).
- BFS is a digraph algorithm.



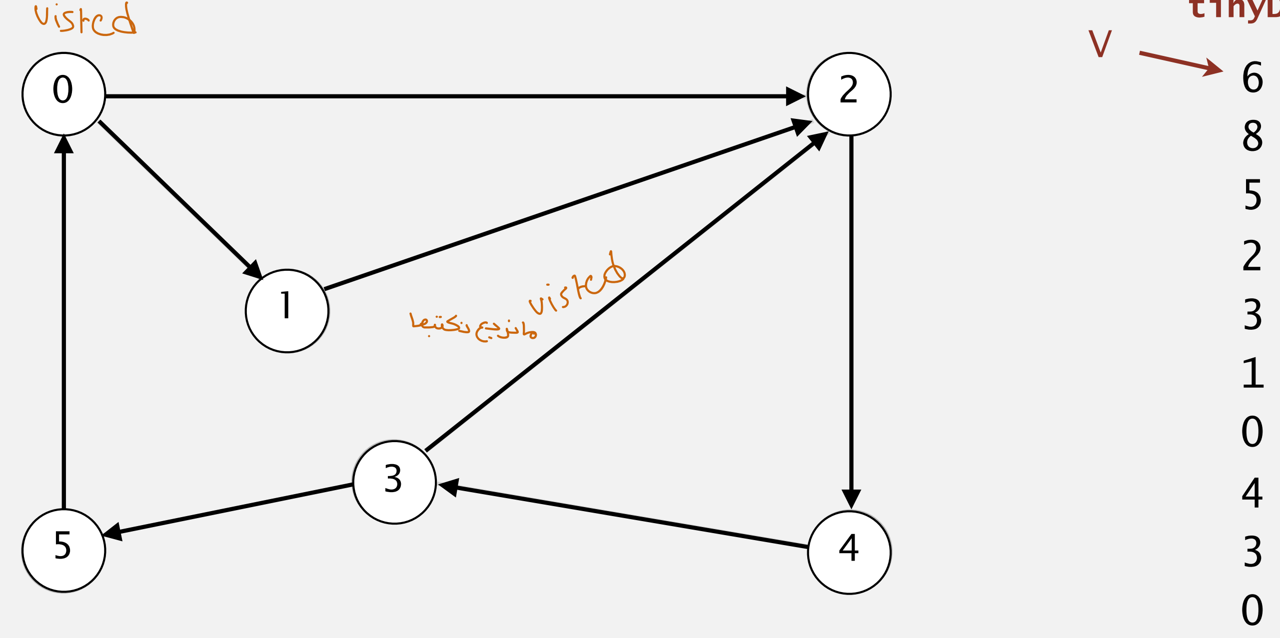
Proposition. BFS computes shortest paths (fewest number of edges) from s to all other vertices in a digraph in time proportional to $E + V$.

Directed breadth-first search demo

bfS + use Queue
والاسم بحدد الخيارات.

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices pointing from v and mark them.

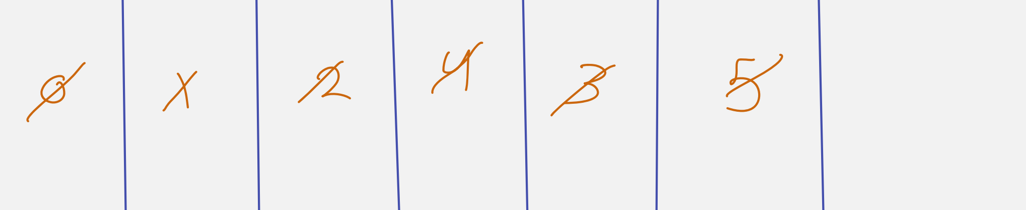


tinyDG2.txt

V → 6
E → 8

```
5 0
2 4
3 2
1 2
0 1
4 3
3 5
0 2
```

0, 1, 2, 4, 3, 5

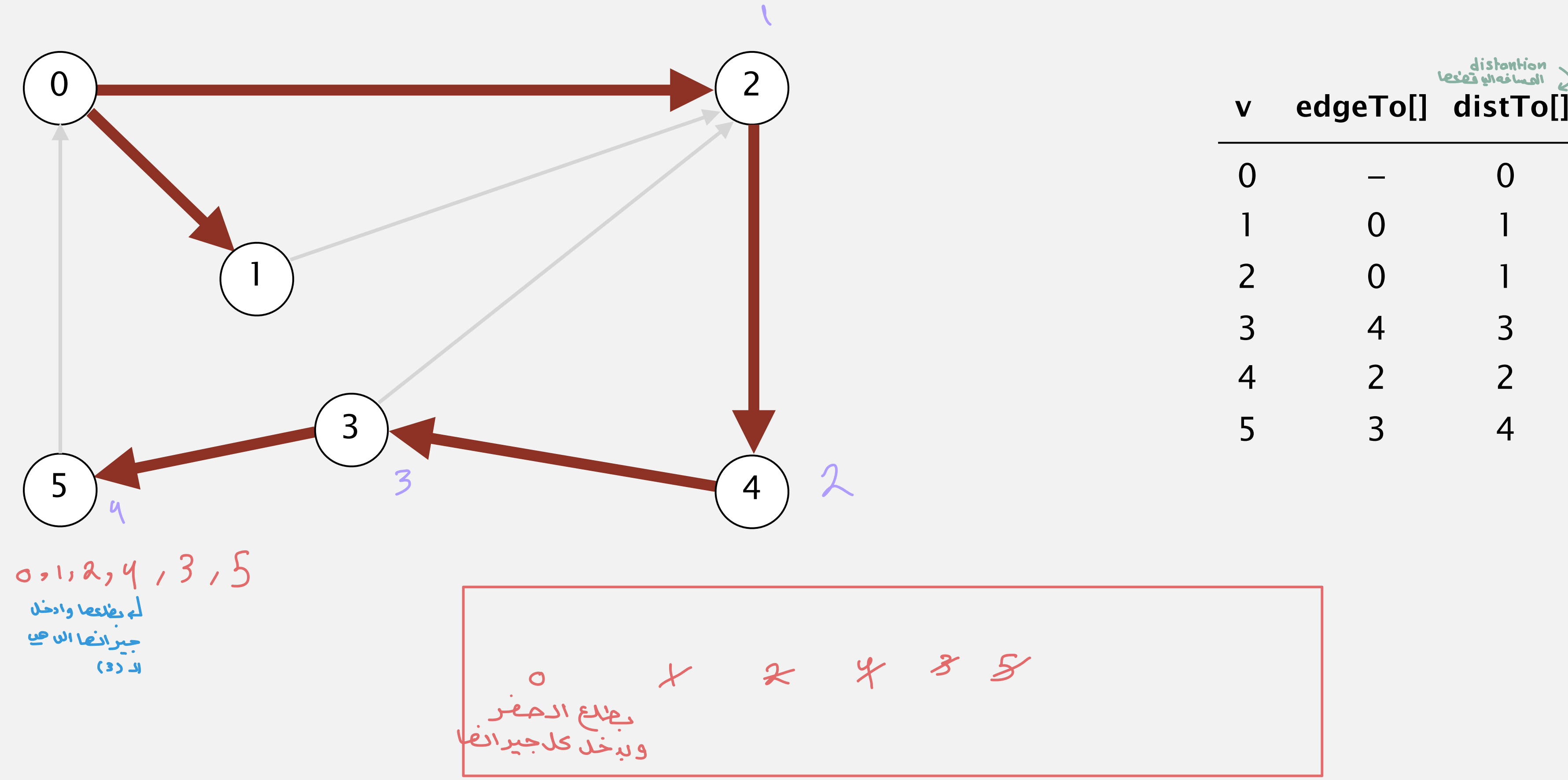


graph G

Directed breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices pointing from v and mark them.



v	edgeTo[]	distTo[]
0	–	0
1	0	1
2	0	1
3	4	3
4	2	2
5	3	4

done

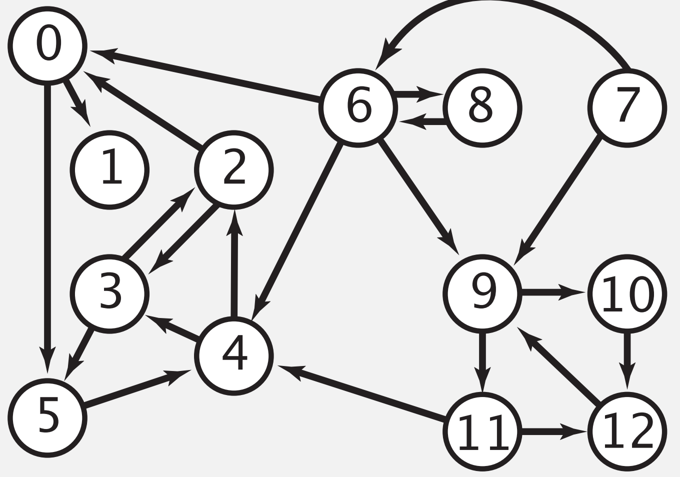
Multiple-source shortest paths

Multiple-source shortest paths. Given a digraph and a set of source vertices, find shortest path from any vertex in the set to each other vertex.

مرات يعطيني النقطه اللي ابدا
منها اكثر من وحدة.

Ex. $S = \{ 1, 7, 10 \}$.

- Shortest path to 4 is $7 \to 6 \to 4$.
- Shortest path to 5 is $7 \to 6 \to 0 \to 5$. 3
- Shortest path to 12 is $10 \to 12$.
- ...



Q. How to implement multi-source shortest paths algorithm?

A. Use BFS, but initialize by enqueuing all source vertices.

Q4. Show the result of Depth First Search following directed weighted graph. Consider the highest weighted edge for choosing the r

← بيختار اعلى ثقل او وزن

← وزنه

← لان مالها مؤشرات بنكتبها

ماحدود نبدا من الصفر.

0,2,6,4,5,1,7,3

3

7 ← لان عنده وزن اثقل

1 ← كذا خلصت اللفه كامله

5

4

6

2

0 ← ماجي اقول وين خيارات الصفر

Q5. Show the result of Breadth First Search following directed weighted graph. Consider the highest weighted edge for choosing the next vertex.

directed weighted : تخيف لها وزن ثقل
هنا يحدد اروح وين يعني الثقل الاصغر
او الاكبر. (معيار)

موب علشان مافيه شيء ياشر عليها
لان لازم تمر على كل ال vertex.

0, 2, 1, 4, 6, 5, 7, 3

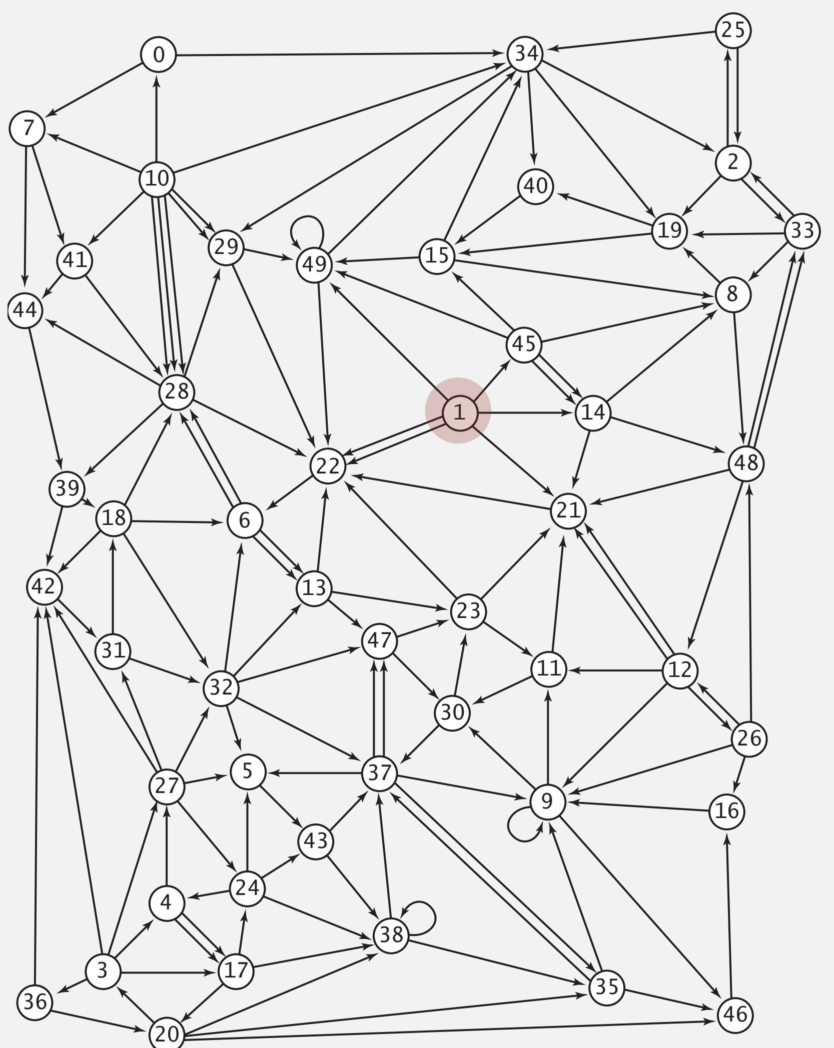
0	2	1	4	6	5	7	3	

Breadth-first search in digraphs application: web crawler

Goal. Crawl web, starting from some root web page, say www.psu.edu.sa

Solution. [BFS with implicit digraph]

- Choose root web page as source s.
- Maintain a Queue of websites to explore.
- Maintain a SET of discovered websites.
- Dequeue the next website and enqueue websites to which it links (provided you haven't done so before).



Q. Why not use DFS?

Bare-bones web crawler: Java implementation

```
Queue<String> queue = new Queue<String>();
SET<String> marked = new SET<String>();

String root = "http://www.princeton.edu";
queue.enqueue(root);
marked.add(root);

while (!queue.isEmpty())
{
   String v = queue.dequeue();
   StdOut.println(v);
   In in = new In(v);
   String input = in.readAll();

   String regexp = "http://(\\w+\\.)+(\\w+)";
   Pattern pattern = Pattern.compile(regexp);
   Matcher matcher = pattern.matcher(input);
   while (matcher.find())
   {
      String w = matcher.group();
      if (!marked.contains(w))
      {
         marked.add(w);
         queue.enqueue(w);
      }
   }
}
```

queue of websites to crawl

set of marked websites

start crawling from root website

read in raw html from next website in queue

use regular expression to find all URLs in website of form `http://xxx.yyy.zzz` [crude pattern misses relative URLs]

if unmarked, mark it and put on the queue

Web crawler output

BFS crawl →queue

```
http://www.princeton.edu
http://www.w3.org
http://ogp.me
http://giving.princeton.edu
http://www.princetonartmuseum.org
http://www.goprincetontigers.com
http://library.princeton.edu
http://helpdesk.princeton.edu
http://tigernet.princeton.edu
http://alumni.princeton.edu
http://gradschool.princeton.edu
http://vimeo.com
http://princetonusg.com
http://artmuseum.princeton.edu
http://jobs.princeton.edu
http://odoc.princeton.edu
http://blogs.princeton.edu
http://www.facebook.com
http://twitter.com
http://www.youtube.com
http://deimos.apple.com
http://qeprize.org
http://en.wikipedia.org
...
```

DFS crawl →stack

```
http://www.princeton.edu
http://deimos.apple.com
http://www.youtube.com
http://www.google.com
http://news.google.com
http://csi.gstatic.com
http://googlenewsblog.blogspot.com
http://labs.google.com
http://groups.google.com
http://img1.blogblog.com
http://feeds.feedburner.com
http:/buttons.googlesyndication.com
http://fusion.google.com
http://insidesearch.blogspot.com
http://agoogleaday.com
http://static.googleusercontent.com
http://searchresearch1.blogspot.com
http://feedburner.google.com
http://www.dot.ca.gov
http://www.TahoeRoads.com
http://www.LakeTahoeTransit.com
http://www.laketahoe.com
http://ethel.tahoeguide.com
...
```

Algorithms

Robert Sedgewick | Kevin Wayne

http://algs4.cs.princeton.edu

4.2 Directed Graphs

- *introduction*
- *digraph API*
- *digraph search*
- *topological sort* → بيرتب على حسب الزمن
- *strong components*

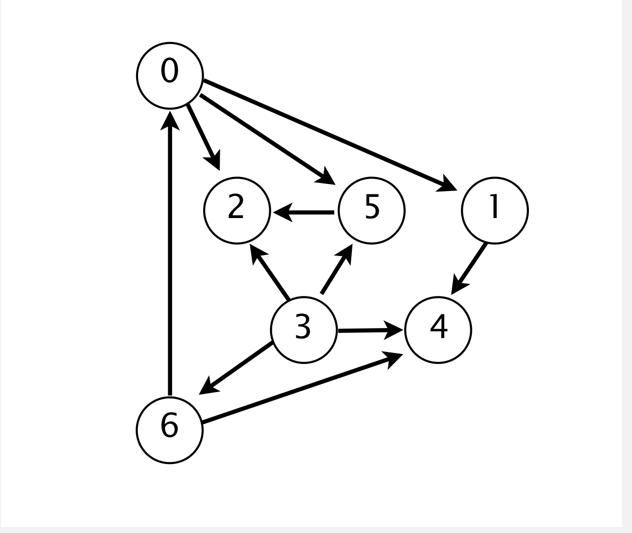
Precedence scheduling

Goal. Given a set of tasks to be completed with precedence constraints, in which order should we schedule the tasks?

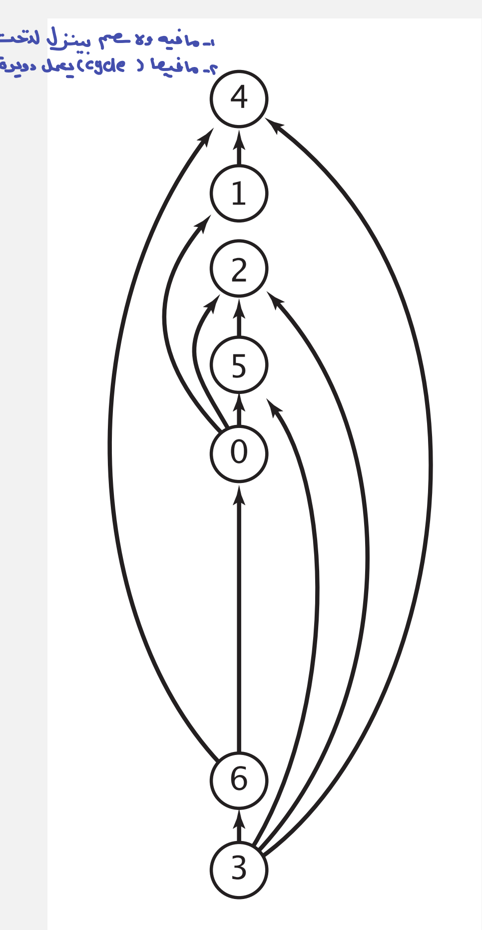
Digraph model. vertex = task; edge = precedence constraint.

0. Algorithms
1. Complexity Theory
2. Artificial Intelligence
3. Intro to CS
4. Cryptography
5. Scientific Computing
6. Advanced Programming

tasks



precedence constraint graph



feasible schedule

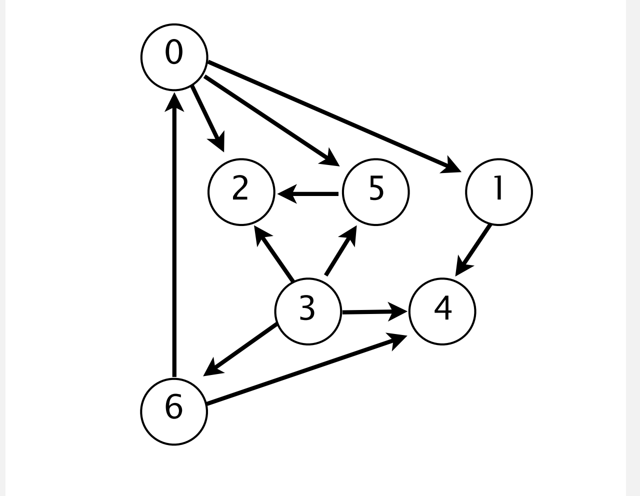
Topological sort

DAG. Directed acyclic graph.

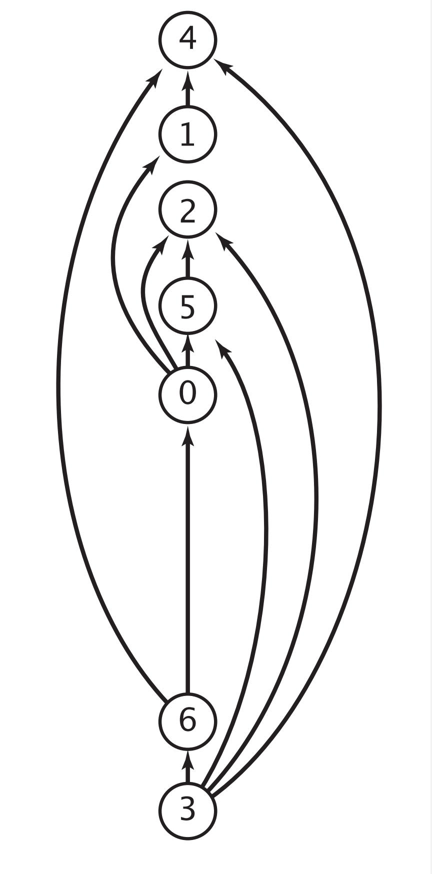
Topological sort. Redraw DAG so all edges point upwards.

0→5	0→2
0→1	3→6
3→5	3→4
5→2	6→4
6→0	3→2
1→4	

directed edges



DAG

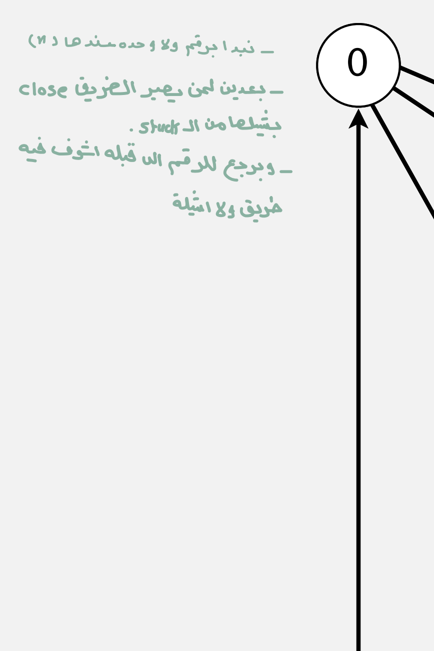


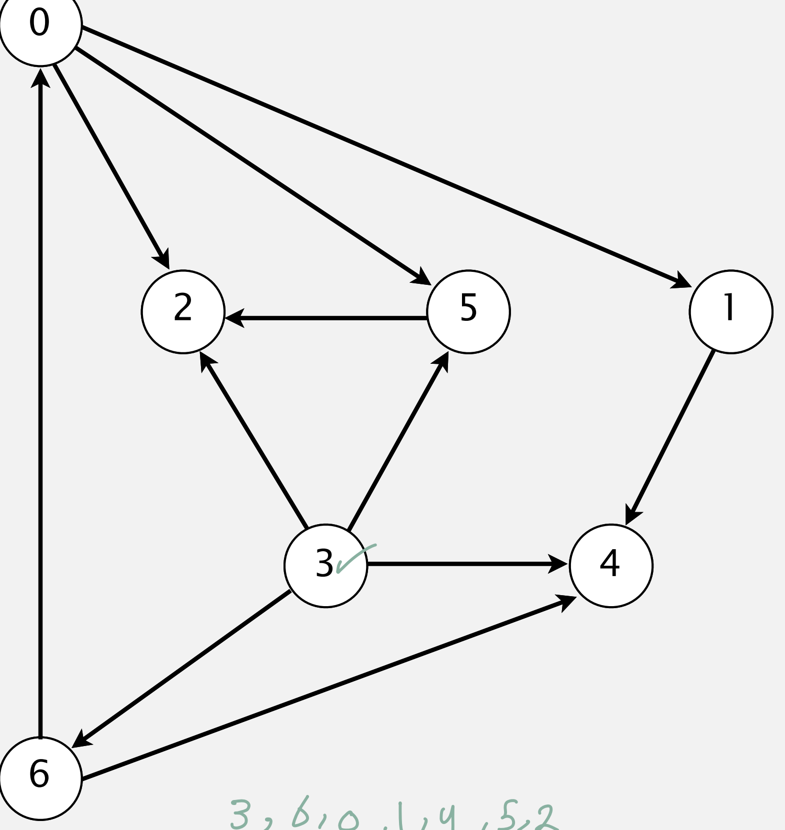
topological order

Solution. DFS. What else?

Topological sort demo

- Run depth-first search.
- Return vertices in reverse postorder.





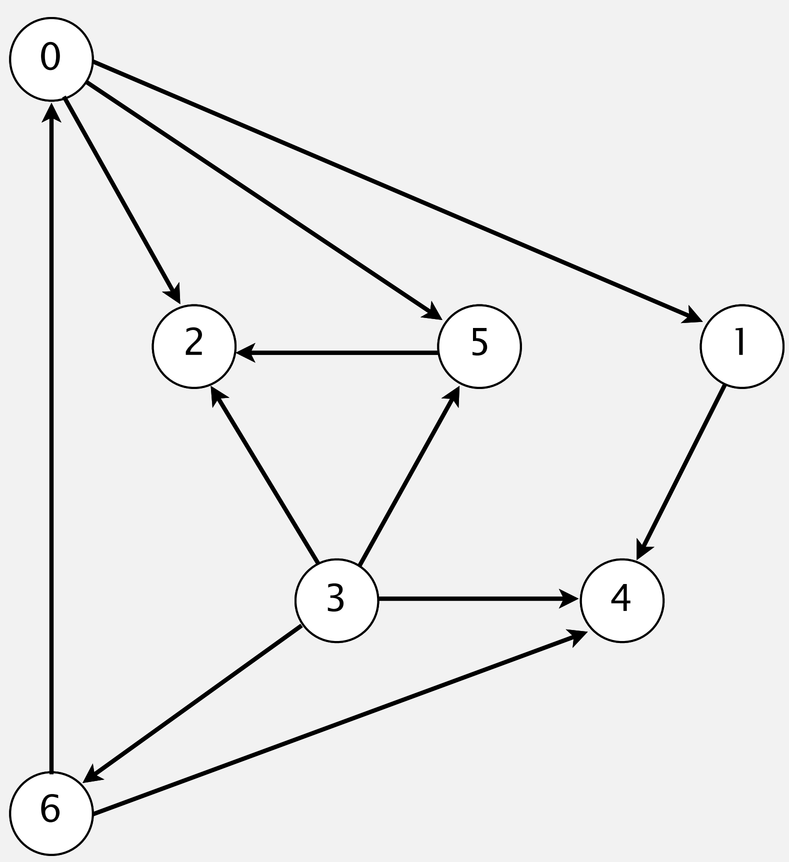
a directed acyclic graph

tinyDAG7.txt

```
7
11
 0  5
 0  2
 0  1
 3  6
 3  5
 3  4
 5  2
 6  4
 6  0
 3  2
```

Topological sort demo

- Run depth-first search.
- Return vertices in reverse postorder.



postorder

4 1 2 5 0 6 3

topological order

3 6 0 5 2 1 4

done

Depth-first search order

```
public class DepthFirstOrder
{
   private boolean[] marked;
   private Stack<Integer> reversePostorder;

   public DepthFirstOrder(Digraph G)
   {
      reversePostorder = new Stack<Integer>();
      marked = new boolean[G.V()];
      for (int v = 0; v < G.V(); v++)
         if (!marked[v]) dfs(G, v);
   }

   private void dfs(Digraph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w]) dfs(G, w);
      reversePostorder.push(v);
   }

   public Iterable<Integer> reversePostorder()
   {  return reversePostorder;   }
}
```

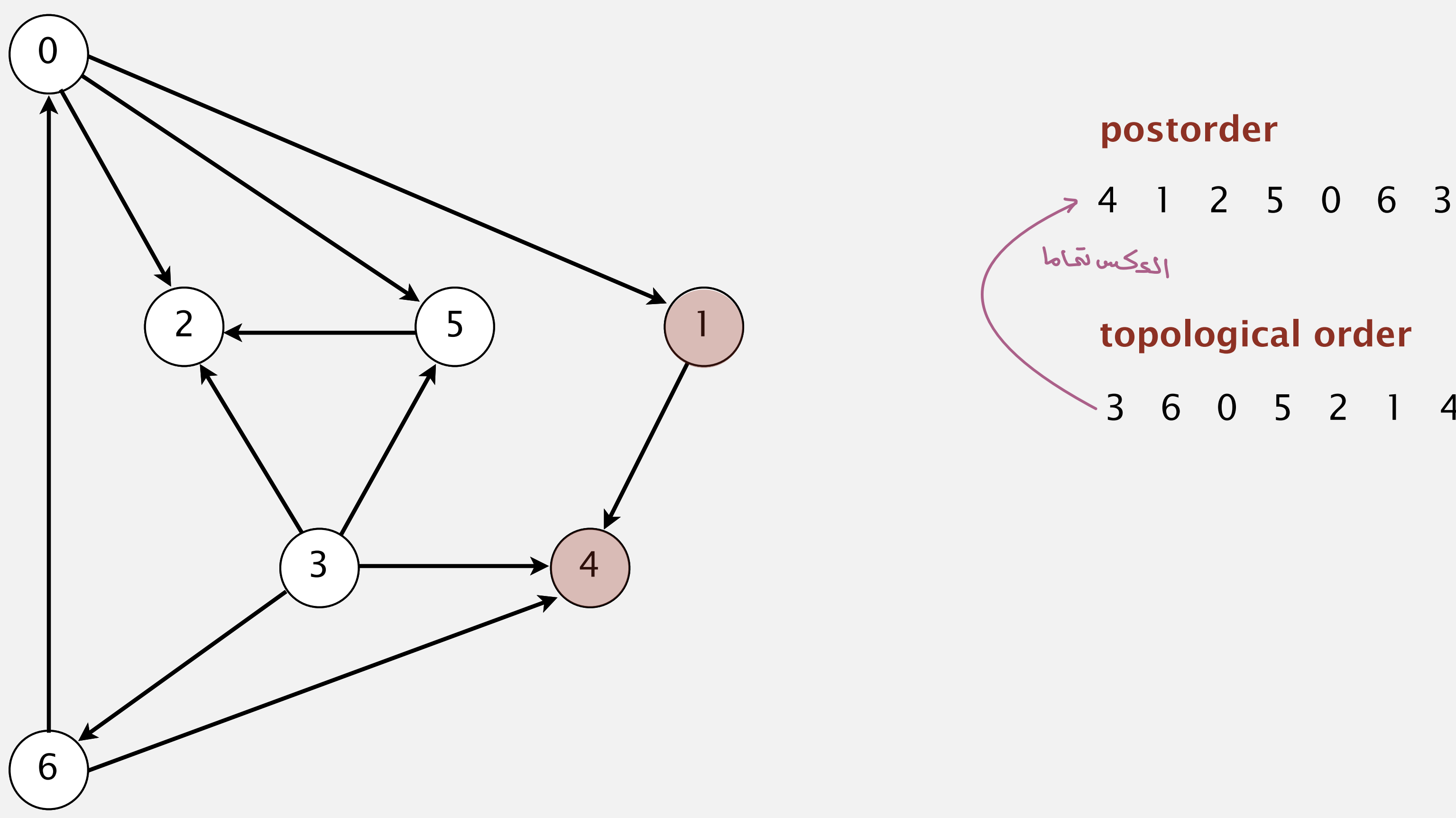
بيكونوا معكوسين بال stack
لازم اقلبهم علشان يطلعون مضبوطين

returns all vertices in "reverse DFS postorder"

Topological sort in a DAG: intuition

Why does topological sort algorithm work?

- First vertex in postorder has outdegree 0.
- Second-to-last vertex in postorder can only point to last vertex.
- ...



Topological sort in a DAG: correctness proof

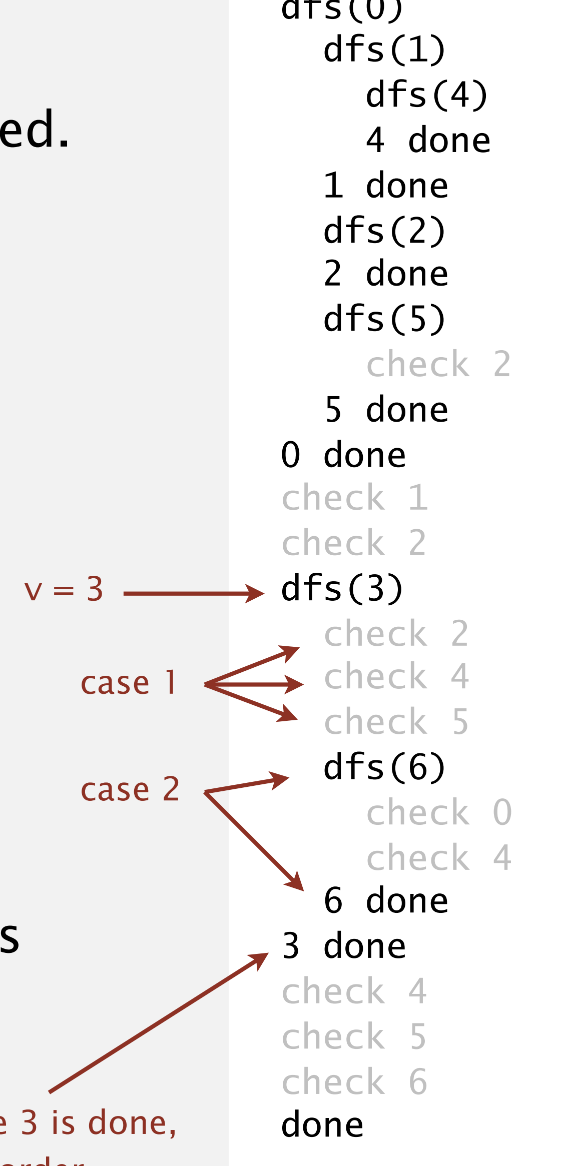
Proposition. Reverse DFS postorder of a DAG is a topological order.

Pf. Consider any edge v→w. When `dfs(v)` is called:

- Case 1: `dfs(w)` has already been called and returned.
  Thus, w was done before v.

- Case 2: `dfs(w)` has not yet been called.
  `dfs(w)` will get called directly or indirectly
  by `dfs(v)` and will finish before `dfs(v)`.
  Thus, w will be done before v.

- Case 3: `dfs(w)` has already been called,
  but has not yet returned.
  Can't happen in a DAG: function call stack contains
  path from w to v, so v→w would complete a cycle.



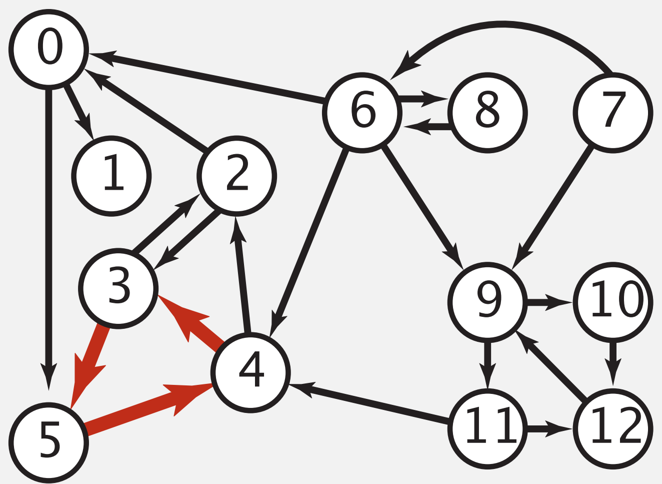
Directed cycle detection

Proposition. A digraph has a topological order iff no directed cycle.

Pf.

- If directed cycle, topological order impossible.
- If no directed cycle, DFS-based algorithm finds a topological order.

≠ same vertex



a digraph with a directed cycle

Goal. Given a digraph, find a directed cycle.

Solution. DFS. What else? See textbook.

Directed cycle detection application: precedence scheduling

Scheduling. Given a set of tasks to be completed with precedence constraints, in what order should we schedule the tasks?

PAGE 3

DEPARTMENT	COURSE	DESCRIPTION	PREREQS
COMPUTER SCIENCE	CPSC 432	INTERMEDIATE COMPILER DESIGN, WITH A FOCUS ON DEPENDENCY RESOLUTION.	CPSC 432

http://xkcd.com/754

Remark. A directed cycle implies scheduling problem is infeasible.

Directed cycle detection application: cyclic inheritance

The Java compiler does cycle detection.

```
public class A extends B
{
   ...
}
```

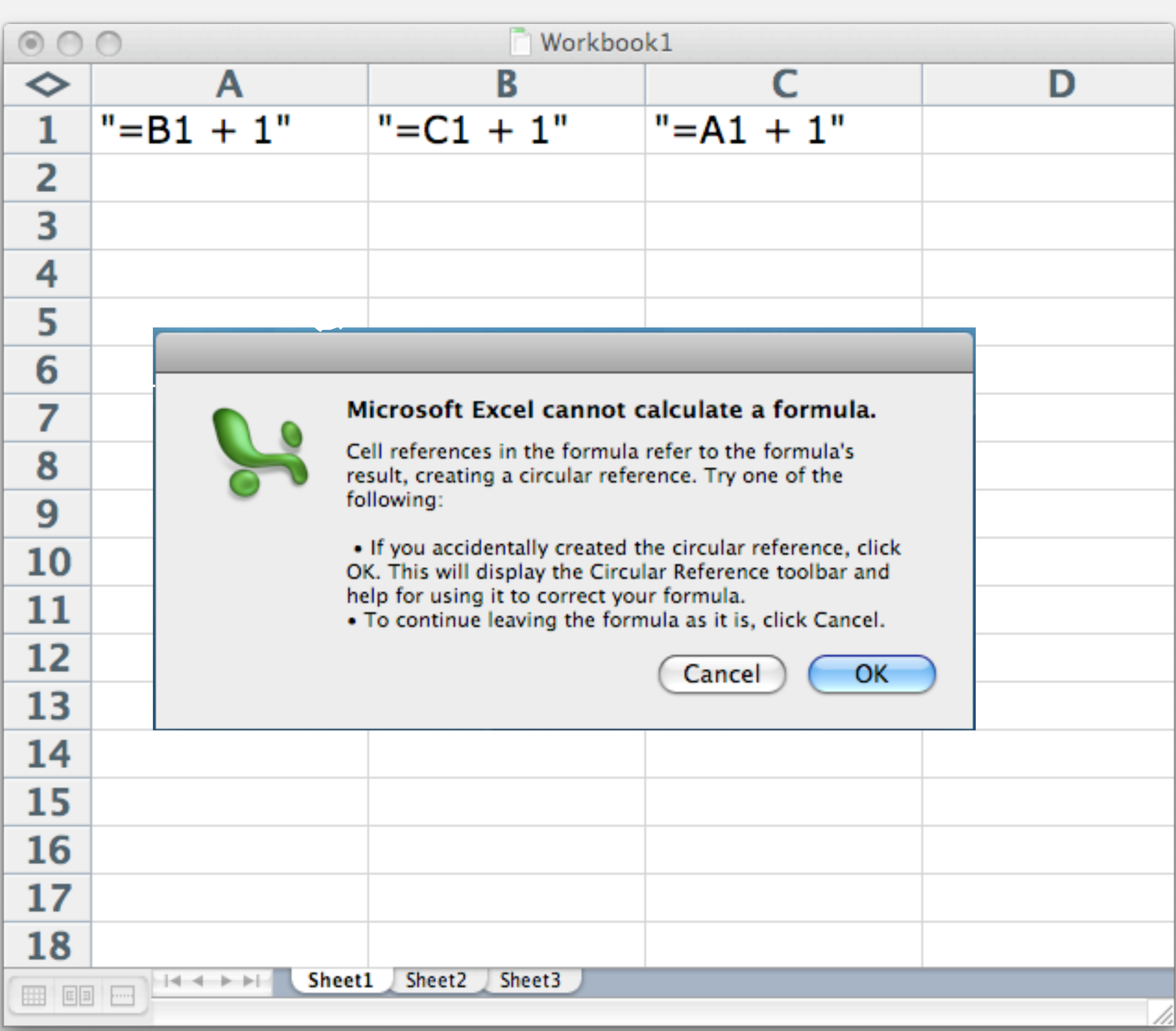
```
public class B extends C
{
   ...
}
```

```
public class C extends A
{
   ...
}
```

```
% javac A.java
A.java:1: cyclic inheritance
involving A
public class A extends B { }
       ^
1 error
```

Directed cycle detection application: spreadsheet recalculation

Microsoft Excel does cycle detection (and has a circular reference toolbar!)



Depth-first search orders

Observation. DFS visits each vertex exactly once. The order in which it does so can be important.

Orderings.

- Preorder: order in which `dfs()` is called.
- Postorder: order in which `dfs()` returns.
- Reverse postorder: reverse order in which `dfs()` returns.

```
private void dfs(Graph G, int v)
{
   marked[v] = true;
   preorder.enqueue(v);
   for (int w : G.adj(v))
      if (!marked[w]) dfs(G, w);
   postorder.enqueue(v);
   reversePostorder.push(v);
}
```

Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

4.2 DIRECTED GRAPHS

- *introduction*
- *digraph API*
- *digraph search*
- *topological sort*
- ***strong components***

Strongly-connected components → كلهم مربوطين ببعض

Def. Vertices v and w are **strongly connected** if there is both a directed path from v to w **and** a directed path from w to v.

Key property. Strong connectivity is an **equivalence relation**:

- v is strongly connected to v.
- If v is strongly connected to w, then w is strongly connected to v.
- If v is strongly connected to w and w to x, then v is strongly connected to x.

Def. A **strong component** is a maximal subset of strongly-connected vertices.

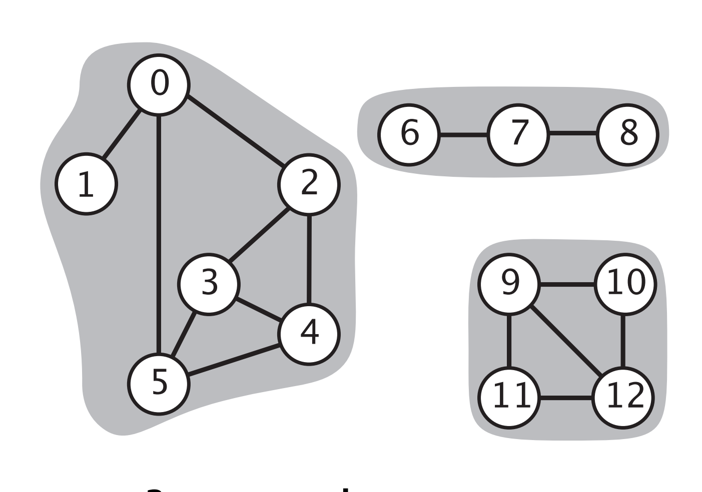
runtime O(V+E)

5 strongly-connected components

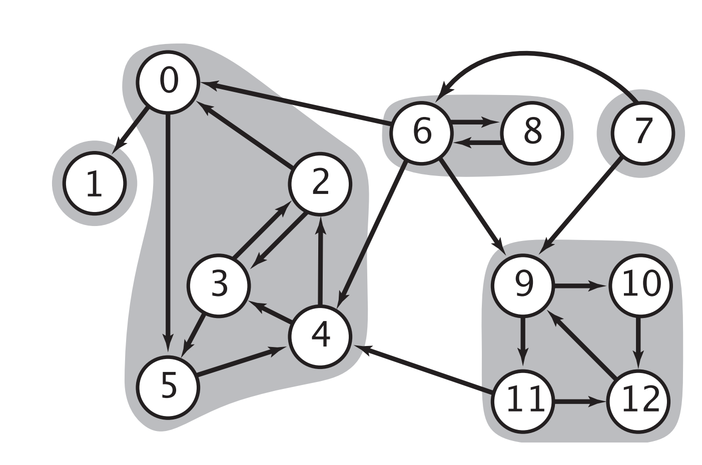
Connected components vs. strongly-connected components

v and w are **connected** if there is a path between v and w

v and w are **strongly connected** if there is both a directed path from v to w and a directed path from w to v



3 connected components



5 strongly-connected components

connected component id (easy to compute with DFS)

	0	1	2	3	4	5	6	7	8	9	10	11	12
id[]	0	0	0	0	0	0	1	1	1	2	2	2	2

```
public boolean connected(int v, int w)
{  return id[v] == id[w];  }
```

constant-time client connectivity query

strongly-connected component id (how to compute?)

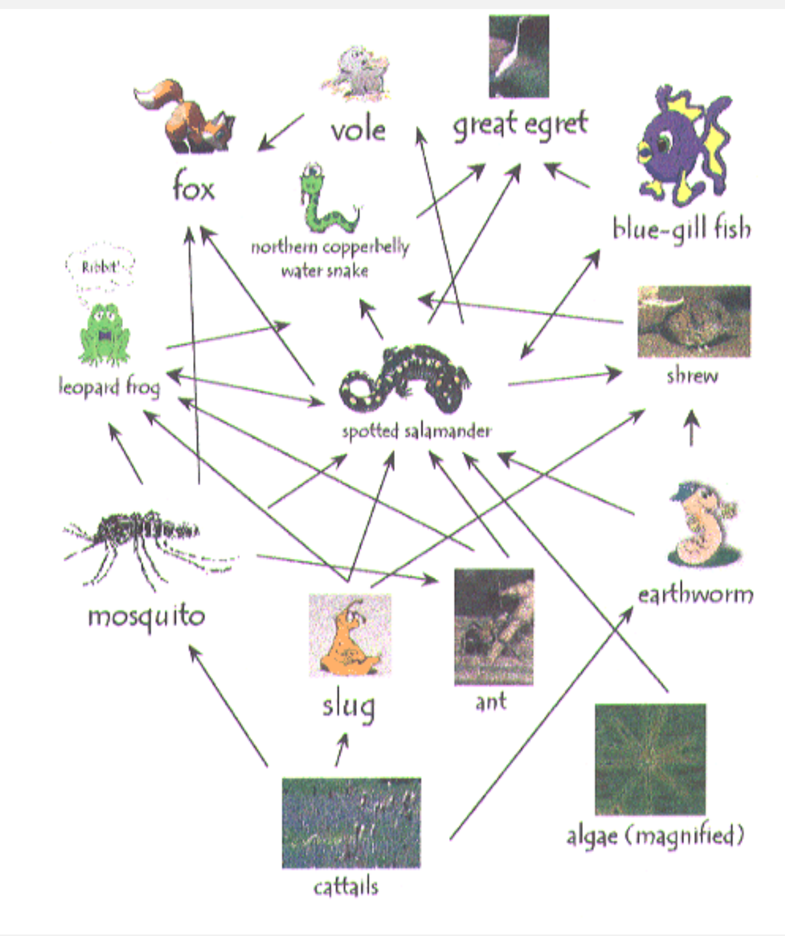
	0	1	2	3	4	5	6	7	8	9	10	11	12
id[]	1	0	1	1	1	1	3	4	3	2	2	2	2

```
public boolean stronglyConnected(int v, int w)
{  return id[v] == id[w];  }
```

constant-time client strong-connectivity query

Strong component application: ecological food webs

Food web graph. Vertex = species; edge = from producer to consumer.



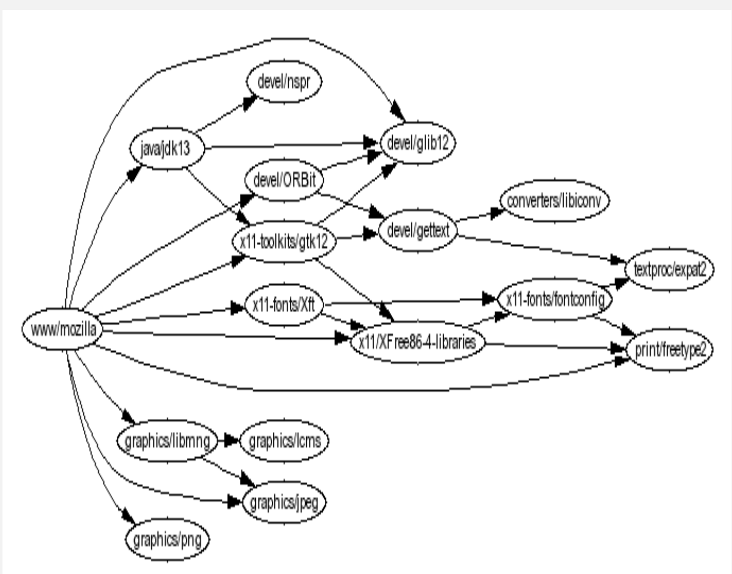
http://www.twingroves.district96.k12.il.us/Wetlands/Salamander/SalGraphics/salfoodweb.gif

Strong component. Subset of species with common energy flow.

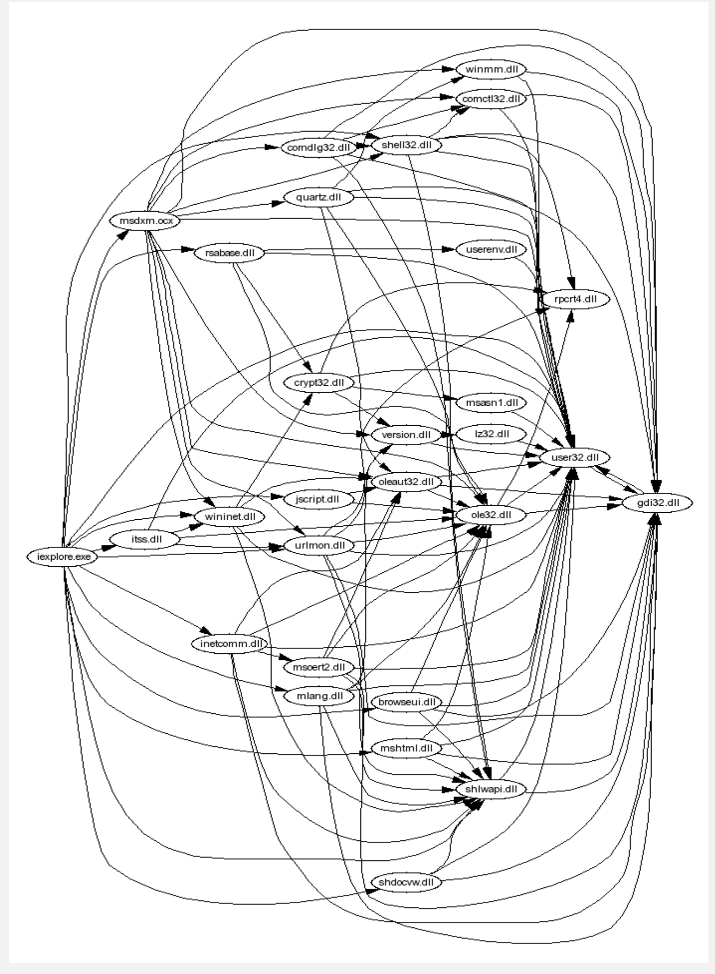
Strong component application: software modules

Software module dependency graph.

- Vertex = software module.
- Edge: from module to dependency.



Firefox



Internet Explorer

Strong component. Subset of mutually interacting modules.

Approach 1. Package strong components together.

Approach 2. Use to improve design!

Strong components algorithms: brief history

1960s: Core OR problem.

- Widely studied; some practical algorithms.
- Complexity not understood.

1972: linear-time DFS algorithm (Tarjan).

- Classic algorithm.
- Level of difficulty: Algs4++.
- Demonstrated broad applicability and importance of DFS.

1980s: easy two-pass linear-time algorithm (Kosaraju-Sharir).

- Forgot notes for lecture; developed algorithm in order to teach it!
- Later found in Russian scientific literature (1972).

1990s: more easy linear-time algorithms.

- Gabow: fixed old OR algorithm.
- Cheriyan-Mehlhorn: needed one-pass algorithm for LEDA.

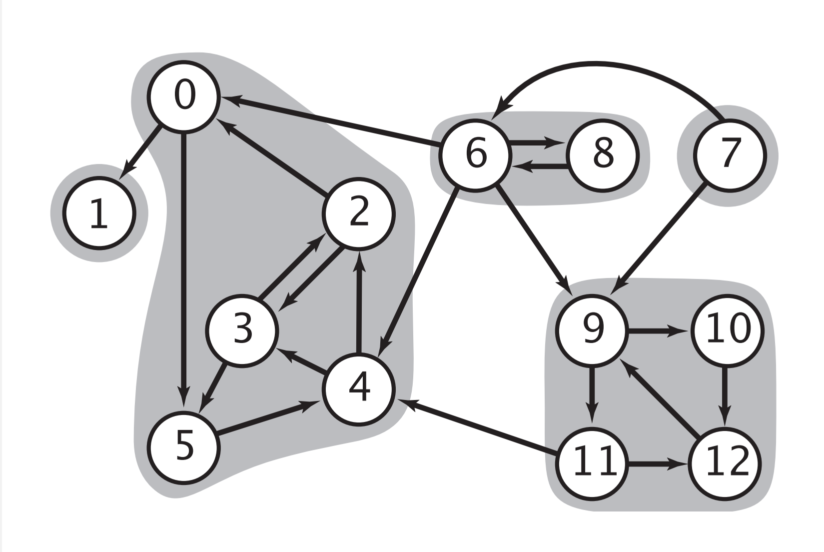
Kosaraju-Sharir algorithm: intuition

Reverse graph. Strong components in G are same as in G^R.

Kernel DAG. Contract each strong component into a single vertex.

Idea.

- Compute topological order (reverse postorder) in kernel DAG.
- Run DFS, considering vertices in reverse topological order.



digraph G and its strong components

first vertex is a sink
(has no edges pointing from it)

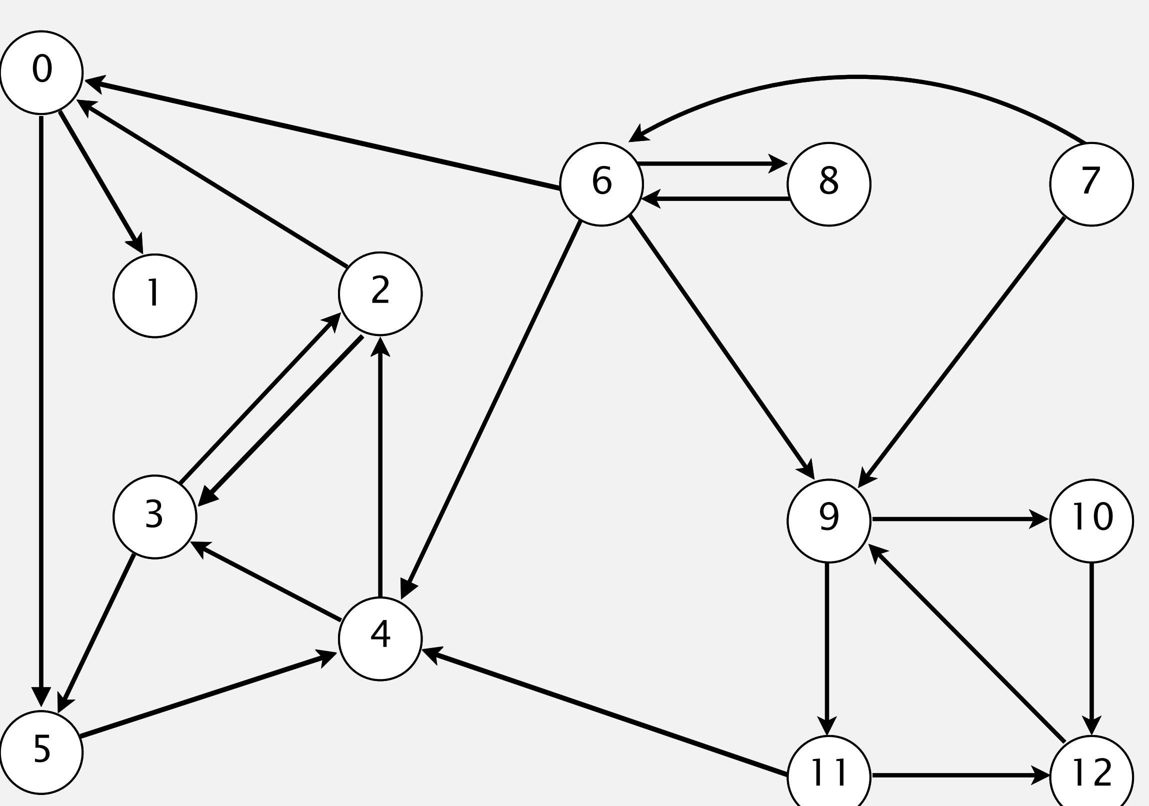
1 E
0 2 3 5 4 D
9 10 11 12 C
6 8 B
7 A

kernel DAG of G (topological order: A B C D E)

Kosaraju-Sharir algorithm demo

Phase 1. Compute reverse postorder in G^R.

Phase 2. Run DFS in G, visiting unmarked vertices in reverse postorder of G^R.

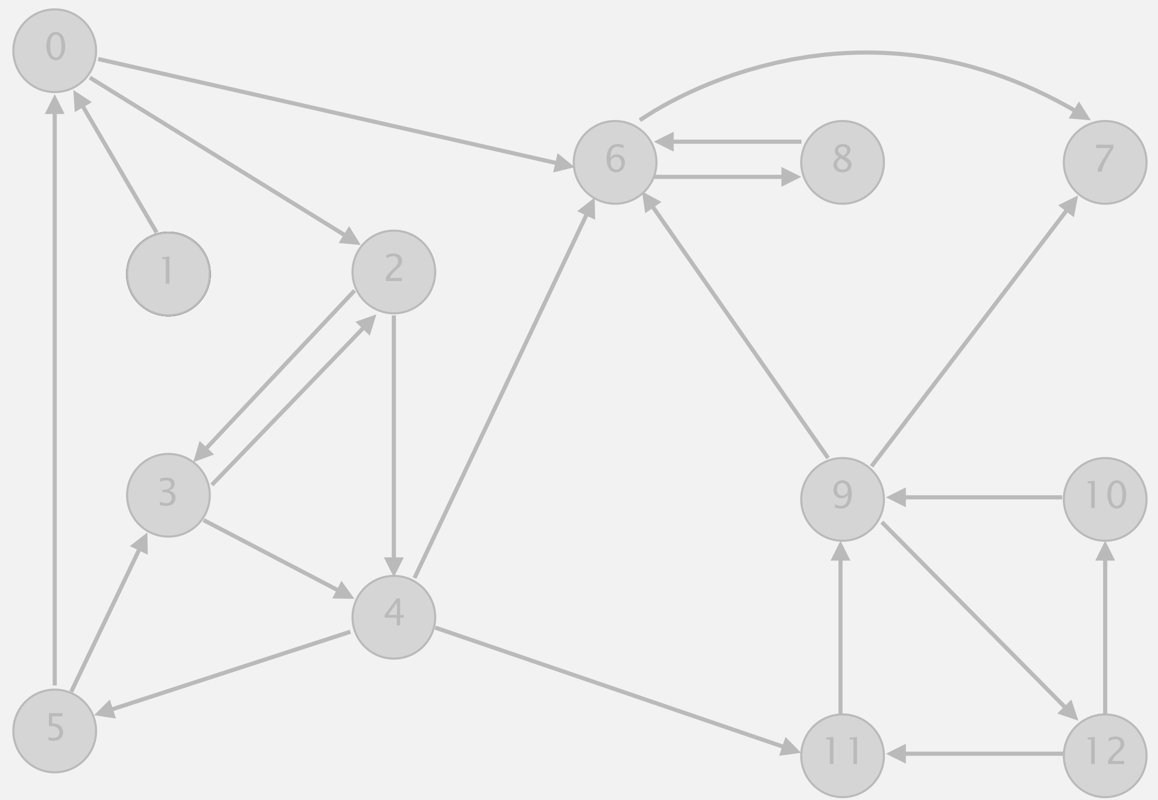


digraph G

Kosaraju-Sharir algorithm demo

Phase 1. Compute reverse postorder in G^R.

1 0 2 4 5 3 11 9 12 10 6 7 8

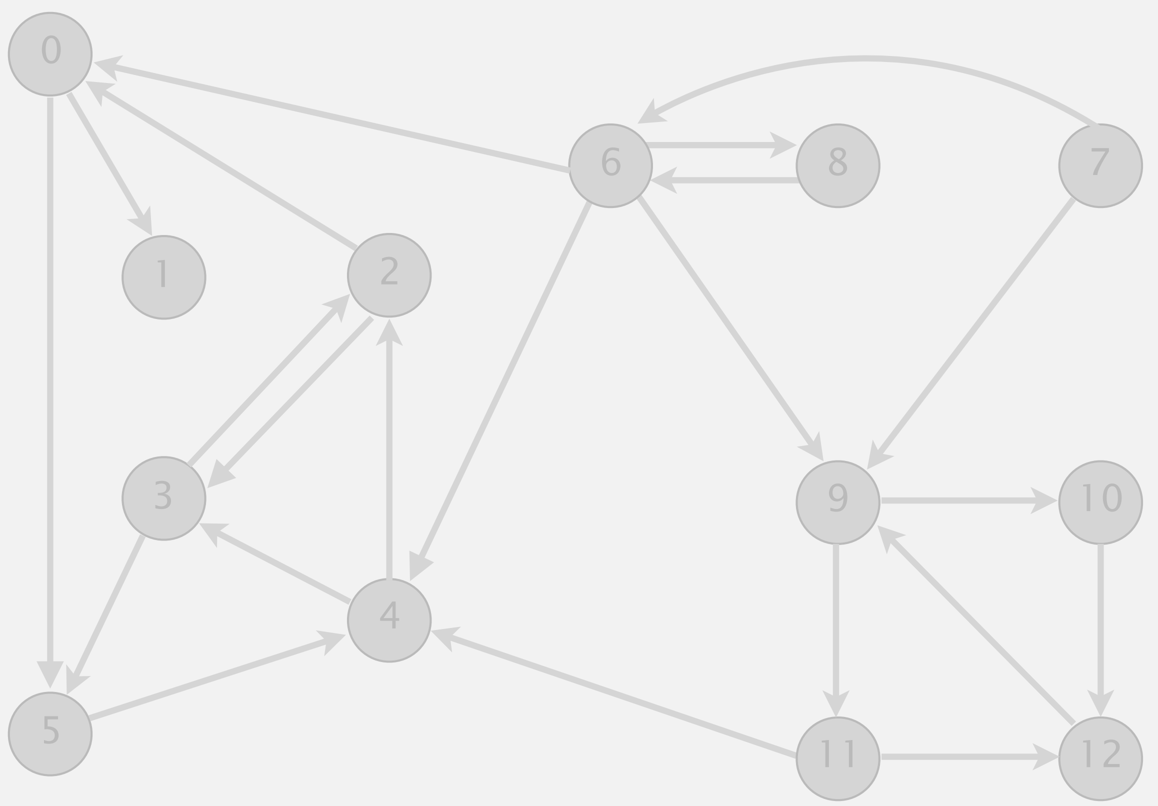


reverse digraph G^R

Kosaraju-Sharir algorithm demo

Phase 2. Run DFS in G, visiting unmarked vertices in reverse postorder of G^R.

1 0 2 4 5 3 11 9 12 10 6 7 8



v	id[]
0	1
1	0
2	1
3	1
4	1
5	1
6	3
7	4
8	3
9	2
10	2
11	2
12	2

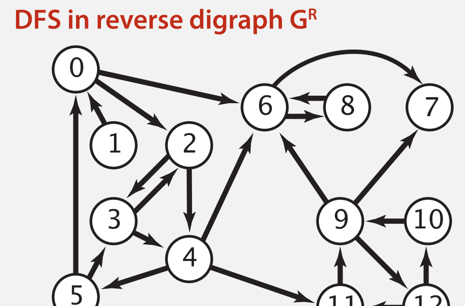
done

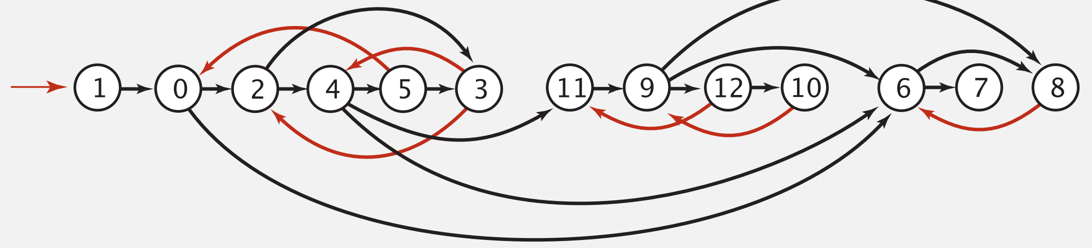
Kosaraju-Sharir algorithm

Simple (but mysterious) algorithm for computing strong components.

- Phase 1: run DFS on G^R to compute reverse postorder.
- Phase 2: run DFS on G, considering vertices in order given by first DFS.

DFS in reverse digraph G^R





check unmarked vertices in the order
0 1 2 3 4 5 6 7 8 9 10 11 12

reverse postorder for use in second `dfs()`
1 0 2 4 5 3 11 9 12 10 6 7 8

```
dfs(0)
| dfs(6)
| | dfs(8)
| | | check 6
| | 8 done
| | dfs(7)
| | 7 done
| 6 done
| dfs(2)
| | dfs(4)
| | | dfs(11)
| | | | dfs(9)
| | | | | dfs(12)
| | | | | | check 11
| | | | | | dfs(10)
| | | | | | | check 9
| | | | | | 10 done
| | | | | 12 done
| | | | | check 7
| | | | | check 6
...
```

Kosaraju-Sharir algorithm

Simple (but mysterious) algorithm for computing strong components.

- Phase 1: run DFS on G^R to compute reverse postorder.
- Phase 2: run DFS on G, considering vertices in order given by first DFS.

DFS in original digraph G

check unmarked vertices in the order

1 0 2 4 5 3 11 9 12 10 6 7 8

```
dfs(1)
1 done
```

```
dfs(0)
| dfs(5)
| | dfs(4)
| | | dfs(3)
| | | | check 5
| | | | dfs(2)
| | | | | check 0
| | | | | check 3
| | | | 2 done
| | | 3 done
| | | check 2
| | 4 done
| 5 done
| check 1
0 done
check 2
check 4
check 5
check 3
```

```
dfs(11)
| check 4
| dfs(12)
| | dfs(9)
| | | check 11
| | | dfs(10)
| | | | check 12
| | | 10 done
| | 9 done
| 12 done
11 done
check 9
check 12
check 10
```

```
dfs(6)
| check 9
| check 4
| dfs(8)
| | check 6
| 8 done
| check 0
6 done
```

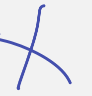
```
dfs(7)
| check 6
| check 9
7 done
check 8
```

Kosaraju-Sharir algorithm

Proposition. Kosaraju-Sharir algorithm computes the strong components of a digraph in time proportional to $E + V$.

Pf.

- Running time: bottleneck is running DFS twice (and computing G^R).
- Correctness: tricky, see textbook (2nd printing).
- Implementation: easy!



Connected components in an undirected graph (with DFS)

```
public class CC
{
   private boolean marked[];
   private int[] id;
   private int count;

   public CC(Graph G)
   {
      marked = new boolean[G.V()];
      id = new int[G.V()];

      for (int v = 0; v < G.V(); v++)
      {
         if (!marked[v])
         {
            dfs(G, v);
            count++;
         }
      }
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      id[v] = count;
      for (int w : G.adj(v))
         if (!marked[w])
            dfs(G, w);
   }

   public boolean connected(int v, int w)
   {  return id[v] == id[w];  }
}
```

Strong components in a digraph (with two DFSs)

```
public class KosarajuSharirSCC
{
   private boolean marked[];
   private int[] id;
   private int count;

   public KosarajuSharirSCC(Digraph G)
   {
      marked = new boolean[G.V()];
      id = new int[G.V()];
      DepthFirstOrder dfs = new DepthFirstOrder(G.reverse());
      for (int v : dfs.reversePostorder())
      {
         if (!marked[v])
         {
            dfs(G, v);
            count++;
         }
      }
   }

   private void dfs(Digraph G, int v)
   {
      marked[v] = true;
      id[v] = count;
      for (int w : G.adj(v))
         if (!marked[w])
            dfs(G, w);
   }

   public boolean stronglyConnected(int v, int w)
   {  return id[v] == id[w];  }
}
```

Digraph-processing summary: algorithms of the day